RAILWAY TRAFFIC VIBRATIONS: GENERATION AND PROPAGATION - THEORETICAL ASPECTS

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ABSTRACT

The interest in vibrations due to railway traffic is increasing in all developed countries, and it requires to develop both experimental and theoretical studies. In fact, innovative track can reduce the transmission of vibrations toward buildings and people, but also a better knowledge of the physical phenomenon can be useful to apply other methodologies, like a better control of contact surface characteristics.

Theoretical mechanical models, based on the analysis of dynamic interaction between wheel and rail, and between track and formation soil, are the key tool to understand the phenomenon and evaluate interventions.

In the paper, after a review of principles of vibration theory, two different calculation models are presented: the first one is a mathematical model for the analysis of dynamic loads caused by rail and wheel irregularities, the second one is useful to study the transmission of vibrations in the railway track and soil.

The models, which can be used in sequence, are valid for various applications, in particular concerning the analysis of the role of different system components (wheels, rail, track) and their importance in the generation and propagation problems of railway vibrations.

Keywords: railway, vibrations, modelling, contact surface characteristics

1. INTRODUCTION

The modern conception of the system of communication networks, in developed countries, gives a big relevance to railways. In many cases, railways are the most convenient, fast and safe transportation system, both for goods and passengers. In fact, the main advantages of the train are related to costs, attitude to massive transports, energy meter, environmental preservation and economic efficiency. These are current topics for transportation planning and infrastructural design, and for the scientific research too.

Since the railway traffic is greatly growing, it needs a special attention for looking at the new problems and requirements coming from this situation. In particular, the railway infrastructures are difficult to harmonize with the surroundings, because of their extension, physical characteristics, exercise conditions and operational problems. The rails create long and inaccessible barriers, and produce a separation between two parts of a territory, in which people have to do their normal activities. That is why people often dislike the construction of new railway infrastructure, and there are problems also in the normal exercise of existing railways.

Since some years ago, the main difficulties related to the agreements with population have descended from the occupancy of the areas, from the interferences with other networks, and especially from the noises and vibrations produced by the railway traffic. In regards to these two last themes, the studies performed by the researchers could produce some improvements, because the new technologies and a better knowledge of the problem are able to reduce the levels of noises and vibrations transmitted toward people and buildings in the environment, near the railways.

About vibrations, recently, the international literature especially shows two main approaches: there are many papers regarding experimental studies – in particular measures of accelerations and recorded data processing, under various test conditions, and implementing special materials or devices in the track – and other ones, consisting in development of the theoretical models or computational tools, useful for better understanding the problem of vibrations.

Only as examples, among the experimental studies there are various testing surveys regarding the accelerations transmitted by rails and track realized under new design criteria or with innovative technologies. In particular it seems that good results can be achieved by means of non-conventional tracks, floating-mass systems, or dissipative elements. The usual problem of this kind of researches is their high cost and the operational difficulties, deriving from the need of a control over the train motion and the site and conditions in which the tests are performed. Additionally, because of the intrinsic variance of these test data, it needs a lot of recordings to perform the due statistic treatments.

On the other side, the theoretical studies try to realize analytical and computational models, able to reproduce – according to the physical and mathematical laws – the dynamic behaviour of the main parts of the complex railway system: in particular it is important to realize the models of the wheels, the rail line, the track and the formation soil, and to calculate the effects of their dynamic interactions. The objectives of these processes should be the quantitative evaluation of the reductions of vibrations which it is possible to obtain by means of a better control of the contact surface characteristics or other special interventions, in the design or construction of the infrastructure.

The advantages of this theoretical approach, principally, are its accessibility also without wide economic resources, and the good level of approximation of the real phenomena, which now it is possible to reach, thanks to advanced softwares and computational tools. However, also for this studies, it would be important to have valid data coming from real tests, to calibrate and validate the computational models.

The authors of this paper have developed a study concerning the complex phenomena involved in the generation and propagation of railway vibrations, by means of computational models able to reproduce the dynamic interactions between the system components. Considering the different facets of the topic, and the need of a systematic exposition, it is important to preface the principles of the vibrations theory and the analytical models proposal for the investigation, according to the international scientific literature; so, in this paper these theoretical aspects are presented, before of the exposition of the fundamental structure and main characteristics of the computational models which have been developed in the described research. Instead, the improvements of the models and their various applications are shown in another paper ("*Railway traffic vibrations generation and propagation: use of computational models*"), presented in the same congress session.

2. RAILWAY TRAFFIC VIBRATIONS

Vibration consist, essentially, in the effect produced by one or more oscillatory composed motions, which are generated and transmitted in physical bodies. The train-track-ground interaction phenomenon spreads over a broad frequency range: a few Hz for dynamic vehicle problems up to 10kHz for noise emission. So it is too hard to develop a single model to describe the interaction problem on the whole. This section discusses the main characteristics of vibration generated by trains.

2.1 Excitation Mechanisms

There are four main mechanisms for the generation of vibration from moving train. The first mechanism arises when successive axles of the train pass by a fixed observation point. The response of the observation point exhibits a peak when the simultaneous position of one of the wheels lies at the nearest track point to the observation point. It shows a trough when the nearest track point to the observation lies between two axles. The mechanism is known as the quasi-static effect and it is modelled by static forces moving along the track with the train speed. The effect is distinguishable close to the track and it significantly contributes to the low-frequency in the range 0-200 Hz [2,18].

The second is known as the parametric excitation mechanism and it is attributed to varying stiffness under moving wheels. The rail is discretely supported by sleepers at regular distance (typically 0.6m). A moving wheel over the rail experiences high stiffness at a sleeper and low stiffness between two sleepers. When the wheel moves with a constant speed over the rail, it applies a periodic force with time-periodicity equal to the sleeper spacing divided by the wheel speed. The force can be decomposed into Fourier series with principle frequency equal to the train speed divided by the sleeper spacing [22]. A measured acceleration spectra near railway track is presented by Auersh [3]. It shows that peaks appear at some distinctive frequencies such as the sleeper-pass frequency and also that the response can be large if the wheel-track resonance coincides with one of the sleeper-pass harmonics.

The third mechanism occurs due to dynamic loads caused by the vertical motion of the unsprung mass of the vehicle generated by rail and wheel unevenness or roughness. For an ideal case of smooth-wheels train, a mutual force occurs at the wheel-rail interface when a wheel moves over a continuous rail with a given wavelength-roughness. The mutual force has a frequency equal to the train speed divided by the wavelength and it is influenced by the inertial force of the train at the same frequency.

The fourth source of vibration is generated by high-speed train. In this mechanism very large amplitudes of vibration can be generated when train speeds approach or exceed either the speed of Rayleigh surface waves in the ground or the minimal phase speed of bending waves in the track.

2.1.1 Modes of propagation

Vibration propagates from the source to the receiver through motion of particles in the transmission path. It is very elaborate to model dynamic behaviour of ground due to passing trains for difficulty with ground parameter choice, which is a multi-phase granular continuum.

In many studies on train-induced ground vibration, ground is modelled as a layered isotropic elastic medium [15,19], in which two types of non-dispersive waves can freely propagate and these are called the body waves (Figure 1). The first is a dilatational wave, or a pressure wave (known as the P-wave). This is a longitudinal wave, where the wave-front and the medium particles move in the same direction. The second is the equivoluminal wave or the shear wave (known as the S-wave). This is a transverse wave, where the wave-front and the medium particles move perpendicular to each other.

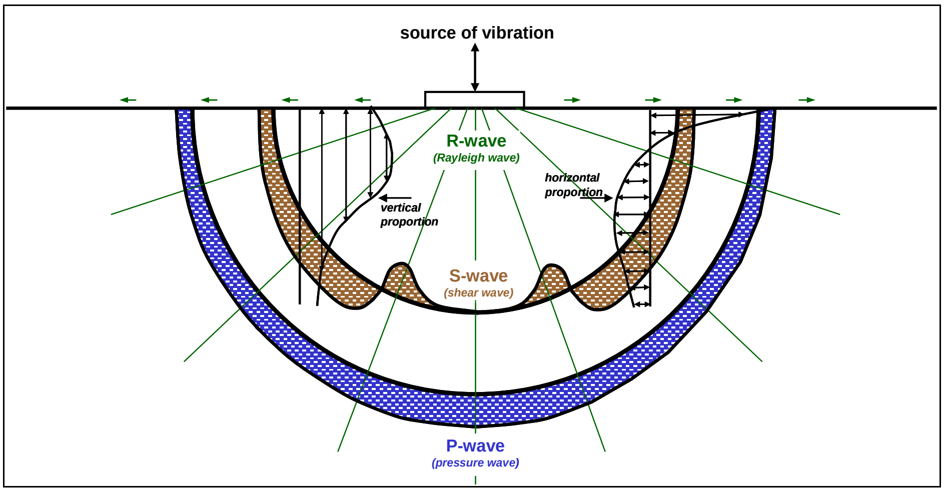

Figure 1 – Modes of propagation of ground vibration.

Both types of body waves decay at the surface and only Rayleigh wave propagates freely at the surface. The pressure wave is the fastest and arrives first, the shear wave comes next and finally Rayleigh wave arrives and it is associated with large amplitudes. Rayleigh wave (known as the R-wave or the surface wave) is non-dispersive and it is confined near the surface to a depth approximately equal to the wavelength. The particles move in elliptical shape in the same plane as the wave-front. The vertical component of particle motion is greater than the horizontal component; both components decay exponentially with depth. Away from the surface where the Rayleigh wave disappears, motion of particle is attributed to body waves.

The frequency range of interest is indicatively 20–200 Hz, since high frequency vibration, typical of noise emission, are drown out by track and soil [8,11].

2.2 Impact of Vibration

The vibration exposure level due to railway traffic cannot be represented in univocal way, because the different perception of the receivers, buildings and persons, so also rules and standards about the vibration effect on occupants of buildings and equipment are different [16].

There are many factors which influence human response to vibration, which include vibration magnitude, frequency, duration, vibration input position and other environmental influences. It is found that exposure to high frequency vibration with low amplitude for long periods of time affects concentration ability, while exposure to low frequency vibration with high amplitude for shorts period may cause muscular or internal organ injury.

In general, the effect of ground borne vibration is annoyance to people rather than damage to buildings [20]. So any actions to limit noise for person can ensure against structural damage to buildings, however it is important to prevent fatigue damage due to cyclical vibration in addition to worries for historical buildings.

2.3 Vibration Control System

There are many methods to decrease ground-born vibration, such as isolating the source, interrupting the vibration path or isolating the receiver *i.e.* the buildings.

The main advantages can be achieved by vibration reduction systems at the source which include low-stiffness train suspensions, continuous rail support, soft rail-pads and floating slab-track. Other vibration reduction systems such as rail grinding, rail welding and wheel truing also reduce vibration. However, they are generally considered as maintenance issues rather than vibration countermeasures.

Most of recent studies focus on dynamic isolation of track from support soil, by means of elastomeric materials like natural rubber. Floating-slab track is widely known as an effective measure of vibration isolation above all for underground tunnels.

In this type of railway track, sleepers are supported on a concrete slab (continuous cast-in-place slab or discontinuous pre-cast slabs), while rubber bearings or steel springs are accommodated between the slab end the trackbed. The key-point of these system is the value of the fundamental natural frequency of vehicle track combination, which is in general of the order of 10-20 Hz [20], in order to cut off vibration components with higher frequency. Example of floating-slab track is "*Milano Massivo*" system [1,9], consisting of 4 ton concrete slab resting on the base by means of four discrete elastomeric supports and lateral stoppers. The first natural frequency of the whole system is about 18 Hz, which is considered the best compromise between the smaller natural frequency, track functionality (damaged by too soft supports) and track feasibility (too heavy slabs).

For railways, use of ballast mat underneath the normal ballast is analogous to the use of floating-slab track in underground tunnels. Nelson [13] discusses different methods of vibration isolation in U.S. transit system, among these methods a ballast mat consisting of inverted natural rubber placed on a concrete base or tamped soil beneath the ballast. The results show vibration reductions limited to the frequency range in excess of about 30 Hz with highest reductions of the order of 10 dB, and little vibration reduction at frequency below 25 Hz, though the concrete based ballast appears to be slightly more effective than the soil based mat.

The selection of vibration mitigation technology is dependent on a number of factors, not the least of which cost and practicality. For this reason improving vibration prediction accuracy is of great importance in developing vibration control provisions.

3. MODELLING AND SIMULATION OF TRAIN–TRACK INTERACTION

Dynamic train-track interaction is either simulated in the frequency domain or in the time domain. Each approach offers both benefits and drawbacks when compared to the other. For example when interaction is solved in the frequency domain, the included model of train, track and wheel-rail rolling contact must be linear, however solution time are shorter compared to time domain methods. The latter, on the other, can account for non-linear rolling contact mechanics, state-dependent and randomly distributed properties. In the following, a survey of mathematical models for simulation of train track interaction is given.

3.1 Modelling of Components

This section discusses the literature of modelling of vehicle and track in reference to the different approach to the problem of train-track interaction and application fields.

3.1.1 Vehicle

Usually the vehicle is modelled as a multibody system where the wheelsets, bogie frames and vehicle body are considered as rigid bodies joined by means of linear suspension, spring-dashpot elements, which reproduce the dynamic and deformation behaviour. This kind of model are characterized by a lot of degrees of freedom (30 for simplest 3D model), and they are mainly used for dynamics and comfort vehicle survey. Diana et al. [8] present a model which considers the deformability of vehicle body and wheelset (151 degrees of freedom) to reproduce vehicle vibration up to 15 Hz.

In most application when only vertical train-track interaction is investigated a simple plane model of the bogie is sufficient [14], since vertical and transversal dynamics of the vehicle are independent, any actions in the vertical plane produce effects only in that plane.

For wheel-rail contact excitation above around 20 Hz, the dynamic behaviour of the vehicle body is decoupled from the bogie by the soft secondary suspension, and the same is normally true for the bogie components above the primary suspension. Consequently if the vertical wheel-rail contact force and track response are of primary interest, as in this study, a lumped parameter model of the unsprung mass of the wheelset is sufficient [21]. Obvious exceptions occur when vibration amplitudes in the wheel and rolling noise are to be calculated, in such cases a 3D model based on finite element method of the wheel is necessary.

3.1.2 Track

A simple and widely accepted model of track was first presented by Winkler, which consists of a single infinite beam supported on a foundation with constant stiffness. This model allows a relatively simple analytical solution. Besides is disregarded the contribution of single components of the track in addition to the real location of supports.

Most recently many authors have proposed different models which take into account inertial and elastic property of track components (rail-pads, sleepers and ballast), for which a beam rests on damped elastic layers. So a first hierarchy of track models is explained based on the numbers of layers in the model, Berardi [4] discusses the different response in frequency domain of different track typologies characterized by two or more layers. Two theories for the beam are used, Euler-Bernoulli and Timoshenko formulations. The former accounts only for the bending behaviour while the latter takes account also for shear deformations and rotary inertia of the beam. For a low frequency of excitation, where the propagating wave has a wavelength much greater than the beam-cross-sectional dimensions, both the formulations converge to the same solution. For a high frequency of excitation, Timoshenko formulation is necessary for an accurate solution [10].

Another important difference concerning if the model includes continuous or discrete supports [10]. The most striking difference between the two models regards system response to high frequency excitation, as can be seen by the trend of point receptance (inverse of dynamic stiffness) which is discussed afterwards.

Such models of track, on a ground modelled as a uniform stiffness or a rigid foundation, are usually used to investigate the effect of properties of the track, train speed, sleepers spacing on the wheel-rail interaction. To this objective, in the present study, two types of models are considered:

- *continuous* where rail-pad and ballast are represented by uniform damped elastic layers, while the sleepers are represented by a layer devoid of bending or shear stiffness and having uniform mass per unit length;
- *discontinuous* where the model is periodic and every discrete support consists of a spring-dashpot to account for rail-pad and ballast and a concentrated mass to account for a sleeper (Figure 2).

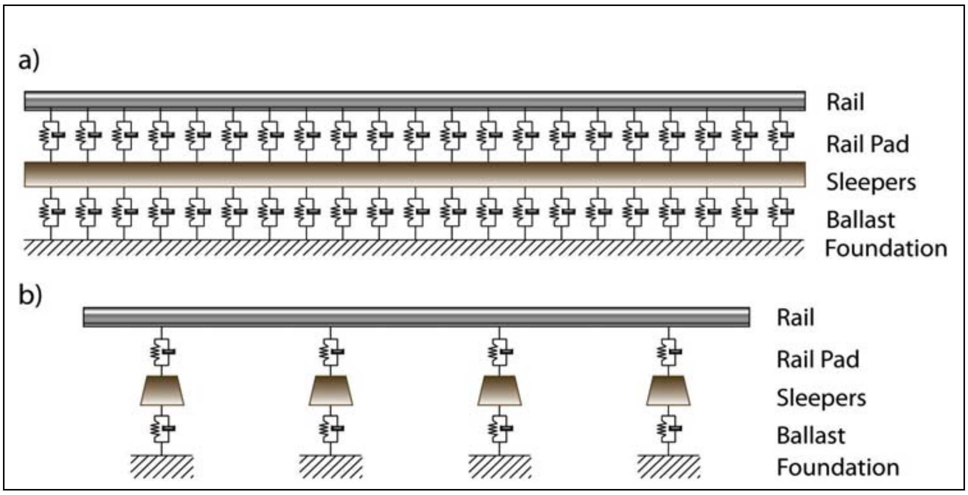

Figure 2 - Model of track: a) Continuous on a two damped elastic layers; b) Discontinuous on a discrete support

In the modelling of vibration from railways many researchers have developed analytical models using a beam resting on elastic half-space [11,12], where ground characteristics are represented by Young's modules, Poisson ratio, density, loss-factor and propagation speeds of P-S and Rayleigh waves. Sheng et al. [17] present a model of railway track modelled by a single rail beam of infinite length, while the ground is modelled as structure of three-dimensional viscoelastic layers overlying either a half-space or a rigid foundation. Coupling the differential equations of motion of the rail-ground system by taking into account the continuity of the displacements and equilibrium of stresses in the plane of contact, the effects on the layered ground of a constant or harmonic moving load are investigated.

Numerical models are the ultimate choice to fully investigate the problem of ground vibration. The importance of these models is that they account for specific details of the prototype, but this is at the expense of the computational time. Anyway the continued development in computer storage and speed facilitates attracts the rail industry to use numerical methods such as the FEM and the BEM, by means of commercial FE-programs, see [6,15] for example.

3.1.3 Wheel-Rail contact

Wheel and rail transmit on the each other interaction forces by the contact patch. To account for the elasticity in the wheel-rail contact, several authors [14, 21] adopt Hertzian models in the hypothesis of perfect elastic isotropic bodies, dimensions of contact patch small compared to principal radii of contact surfaces, forces normal to surfaces and small deformations compared to bodies dimensions. The force transmitted in the contact is given by the equation:

$$F_c = k_H \cdot \delta^{3/2} \qquad \text{(Eq. 1)}$$

where $\delta = y_w - y_r - r$ is the approach of the centres of the two elastic bodies in contact (r is the excitation input, due to the surfaces irregularities of wheel and rail). The (Eq. 1) is valid when δ is > 0; when δ is < 0, indeed, $F_c = 0$. k_H is a constant which depends on contact geometry and mechanical characteristics of the materials (see Figure 3).

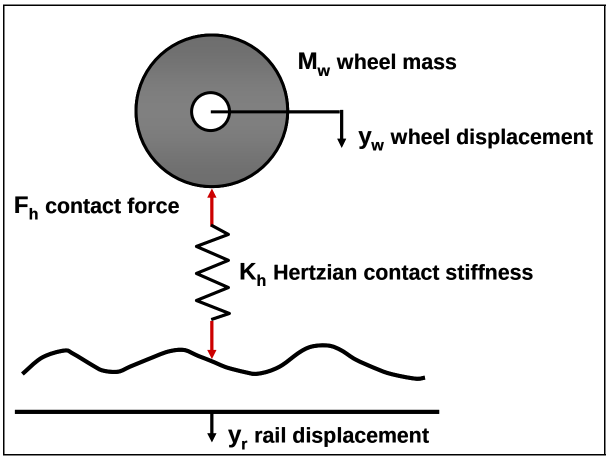

Figure 3 - wheel-rail contact model.

3.2 Track Dynamics

The dynamic behaviour of a system on the whole is represented by his modal parameters: eigenvalues and eigenvectors. The former parameter show system's natural frequency vibration while the latter delineate the shape of every modes of vibration. Anyway, in civil engineering application is usually enough to know resonant frequencies and filtering capacity of the system.

System response to a generic known excitation can resolved by Fourier algorithm, typical of control discipline with computation of harmonic transfer function [7]

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \qquad \text{(Eq. 2)}$$

where numerator and denominator of eq. 2 are, respectively, input and output signal Fourier transform, and ω is the angular frequency. This function in the supposition of linear behaviour of components depends only on points of measure.

In order to analyze track behaviour in the frequency domain, the first step is to determine "frequency response function" (FRF, receptance) of the structures when excited by a point load. In Figure 4 and Figure 5 are illustrated the vertical receptance both for continuous and discrete track models presented in section 3.1.2 calculated by means of the *Harmonic Analysis* with ANSYS code. For continuous model the FRF shows two regions of locally high receptance, once approximately 160 Hz and the other at about 900 Hz, separated by a region of low receptance around 400 Hz. This behaviour may be understood if we assume, for the moment, that the rail has zero bending stiffness and consider the motion of unit length of track, the schematization is comparable to a two degree-of-freedom system, so it is clear that two resonant frequency are to be expected [5,10]. At the lower of these, the rail and sleeper are vibrating in phase on the flexibility of the ballast, while at the higher they vibrate out of phase on the flexibility of the rail-pads. Between the two resonances, there is an antiresonance where the rail would be stationary while the sleepers act as a vibration absorbers.

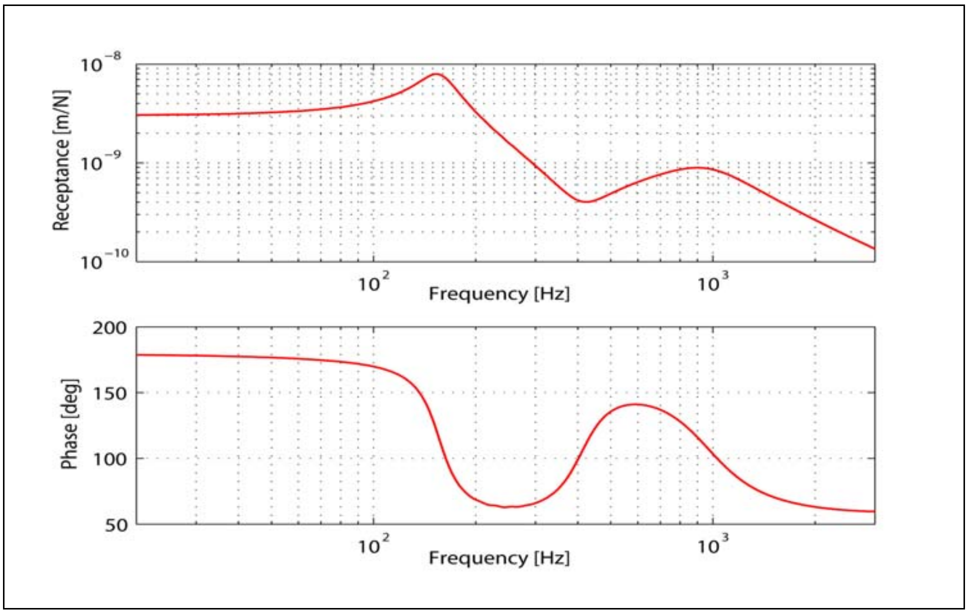

Figure 4 - Vertical direct receptance of track on a continuous two-layers support

For discrete support model the point receptance is calculated at different positions of applied load and displacement response within a sleeper bay. The trend is, in general, the same for the three chosen positions and is also much the same as the receptance calculated for continuous model. The most important differences is in the region of strong resonant activity at a frequency of about 1000 Hz, the so called "pinned-pinned" resonance frequency, at which the rail vibrates with a wavelength equal twice the sleepers spacing with nodes above the sleepers.

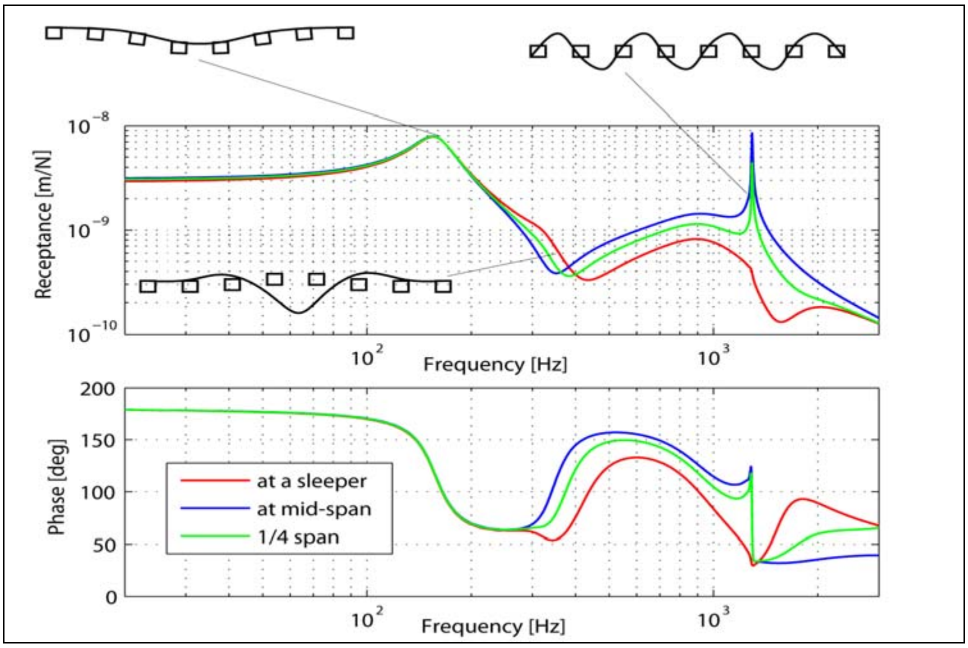

Figure 5 - Vertical receptance of a discretely supported track at different positions

3.3 Train-Track interaction in the Time Domain

Two main types of model are generally used to study wheel-rail interactions (Figure 6), the most realistic and accurate accounts for the wheel is moving along the rail containing wheel/rail irregularities.

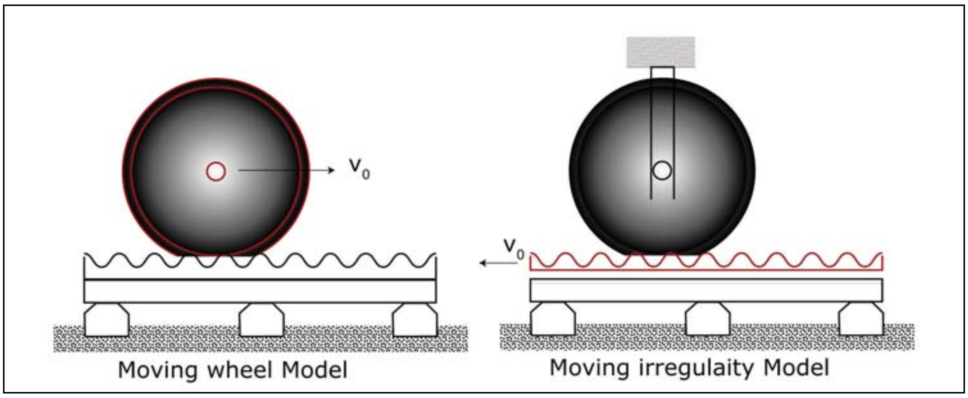

Figure 6 - Wheel-Rail interaction model

This kind of model, often referred to as "moving wheel model", is normally used in the time domain. The other is a "moving irregularities model", for which the vehicle remains in a fixed position on the rail, and a imaginary strip containing the irregularities is pulled at a steady speed between wheel and rail. The latter model is the common choice when interaction is solved in the frequency domain.

Here both model are used to investigate track response to wheel-rail irregularities according to continuous and discrete track models. In the interaction system (Figure 7) the wheel and track coupling are considered to be linear; the wheel is regarded as a mass Mw including the unsprung mass of the bogie, as the natural frequency of the vehicle suspension system is much lower than that of the wheel/track vibration, the vehicle is simplified as a static load acting on the wheel. The equation of motion for the wheel is

$$M_W \cdot \ddot{x}_W = (W - f) \qquad \text{(Eq. 3)}$$

where f is the contact force, which is assumed to follow the Hertz law

$$f = \begin{cases} C_H (x_W - x_r - r)^{3/2} & x_W - x_r - r > 0 \\ 0 & x_W - x_r - r < 0 \end{cases} \qquad \text{(Eq. 4)}$$

where r represents the displacement excitation due to roughness, wheel flat or other irregularities.

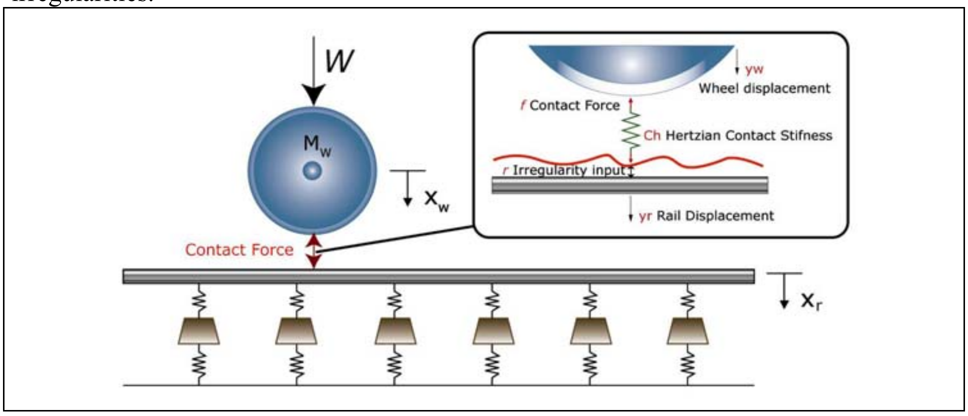

Figure 7 - Wheel-Track interaction model

For the impact problem due to the wheel and rail discontinuities, a time domain model is needed because in such case loss of contact often occurs. In order to allow for non linear analysis of the wheel/rail interaction, an equivalent multiple degree of freedom (MDOF) model for track dynamics is developed. The frequency response functions (rail receptance) presented in section 3.2, are approximated using a transfer function in the form of a ratio of two polynomials in the Laplace variable s, by means of the "*invfreqs*" function in the Signal Processing Toolbox of MATLAB®, which return the real numerator and denominator coefficient vectors b and a of the transfer function, who complex frequency response approximates the required response.

$$H(s) = \frac{B(s)}{A(s)} = \frac{b_1 s^m + b_2 s^{m-1} + \ldots + b_m s + b_{m+1}}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \qquad \text{(Eq. 5)}$$

Because the track model is assumed to be linear the conventional system theory can be applied , transforming from the frequency domain to time domain the transfer function *H(s)*, to which corresponds an equivalent differential equation

$$(D^n + a_1 D^{n-1} + \ldots + a_{n-1} D + a_n) y(t) = (b_1 D^m + b_2 D^{m-1} + \ldots + b_m D + b_{m+1}) f(t) \qquad \text{(Eq. 6)}$$

coupling this equation with mass wheel trough contact stiffness, the mathematical model of interaction is set up.

For continuous track model the point receptance is calculated only in one point, so it corresponds to the moving irregularity model on the basis that structural wave speed in the rail is much higher than the train speed in the frequency range of interest [21]. The moving wheel model , instead, is set up according to the discrete track model, which has different receptance within a sleeper bay. Fixing the observation point on the moving wheel, the wheel can be considering to interact with a periodically time-varying system. The mathematical model can be set up conveying the space-varying dynamic stiffness (point receptance at different position) into time-varying parameters (a_i and b_i vectors coefficients of differential equation) depending on train speed. With this kind of model it can be simulated the periodically excitation at sleeper passing frequency (parametric excitation) when a wheel rolls over a track.

These results can be used as input data for the study of the complex phenomena that start after the generation of vibrations. In particular, the loads can be applied to a FE model (see Figure 8), to analyze the propagation under and around the track.

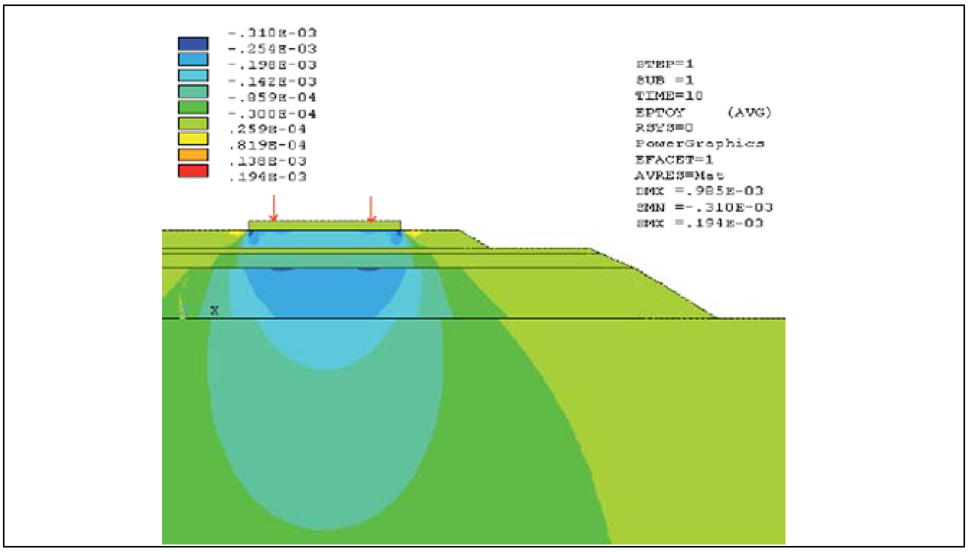

Figure 8 – example of FE model for the study of propagation phenomena.

These applications are reported in another paper (*“Railway traffic vibrations: generation and propagation - use of computational models”*), presented in the same congress session.

CONCLUSIONS

A simplified train-track interaction model has been developed to investigate the dynamic load do to the irregularities of wheel-rail interface.

At first, some recent theoretical and experimental studies were presented, focalizing the main possible applications in the reductions of vibrations.

Then, computational models able to reproduce the phenomena involved in the generation and propagation of railway vibration were presented. Literature studies regarding modelling of vehicle and track, in reference to different approaches to train-track interaction problem and their possible applications, have been discussed. In particular for the track, two model were considered: a continuous model (on a two damped elastic layers) and a discontinuous one (on a discrete support). For both models some results are presented and discussed.

Regarding the applications of these theoretical aspects, in another paper (“*Railway traffic vibrations: generation and propagation - use of computational models*”), presented in the same congress session, are shown various applications for different type of discontinuities, as short pitch corrugation or wheel-flat, and are also presented the differences between moving wheel and irregularity model.

In the same paper is then presented a FEM model of track-ground system, to which are applied the dynamic loads calculated with the interaction model presented here, to analyze the propagation modes of track vibration.

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