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## NUMERICAL ANALYSIS TO DEFINE THE OPTIMUM INCLINATION OF EMBANKMENT SLOPES

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### **ABSTRACT**

It is known that embankment must have suitable scarp slope to assure road solid stability and so infrastructure durability and the inclination established for scarps must be verified through geotechnics methods. These methods correlate inclination of scarps with mechanical characteristics of soil with which the embankment is made of.

In this memoir, the author means to report the results of a study, carried out through finite element method, to find out a correlation between scarp slope and ground inclination.

The analysis objective is to identify the optimum inclination of embankment scarps versus soil mechanical characteristics, ground inclination and embankment height, so that:

1. to guarantee road solid stability and so infrastructure durability;
2. to limit the volumes of filling material;
3. to limit the ground surface taken up by the infrastructure;
4. to limit construction and expropriation costs.

The study fits in main themes of environmental impact of road infrastructure: it is oriented to find some rules to protect the integrity of road work and to minimize environmental impact (in terms of land occupation and volumes of filling material) produced by road construction.

*Keywords: scarp slope, ground inclination, shearing stress, soil shear strength.*

## 1. INTRODUCTION

Hydrogeological accidents are natural phenomenon which belong to the Earth dynamical evolution. In front of such events the man's action can bring to two opposite consequences: it can bring to slowing down through reforestation actions and hydraulic works etc., or it can bring to an acceleration or even to trig the accident caused by vegetation coverage destruction, or caused by the built in of road infrastructures without considering hydrogeological, geotechnical and in general environmental conditions in which they must be put in.

When these accidents happen, the causes are mostly due to the interference between the road net and hydrographical system: under dimensioned bridges, embankments which constitute a earth dam to natural runoff, etc....

Besides, landslides are hastened by road construction because the infrastructure weight modifies mechanical equilibrium of the ground mass. Often, the cause of accident is the inadequate evaluation of embankment scarps slope related to soil mechanical characteristics, and ground inclination.



**Figure 1** Examples of road solid collapse

As a matter of fact, it is well known that embankment must have suitable scarps slope to assure road solid stability and so infrastructure durability and the inclination established for scarps must be verified through geotechnics methods. These methods correlate inclination of scarps with mechanical characteristics of soil with which the embankment is made of.

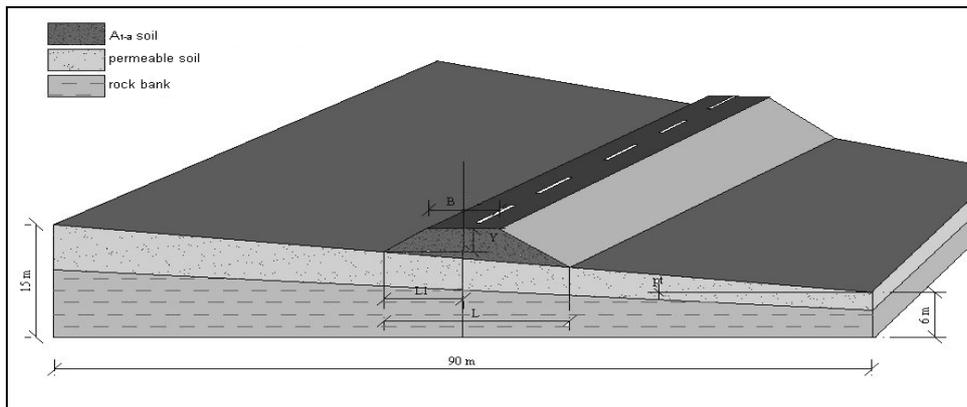
In this memoir, the author means to report the results of a study, carried out through finite element method (FEM), to find out a correlation between scarps slope and ground inclination.

The analysis objective is to identify the optimum inclination of embankment scarps slope versus soil mechanical characteristics, ground inclination and embankment height, so that:

1. to guarantee road solid stability and so infrastructure durability;
2. to limit the volumes of filling material;
3. to limit the ground surface taken up by the infrastructure;
4. to limit construction and expropriation costs.

## 2. ANALYSIS METHODS

Analysis was carried out by addressing a portion of terrain of differing mechanical characteristics, 90 m long with variable ground surface slope and variable depth of rocky strata. On this a highway embankment of varying height was constructed.



**Figure 2** The model in one of the cases considered

In the following table lists the magnitudes varied in order to make the model versatile and applicable to different case scenario.

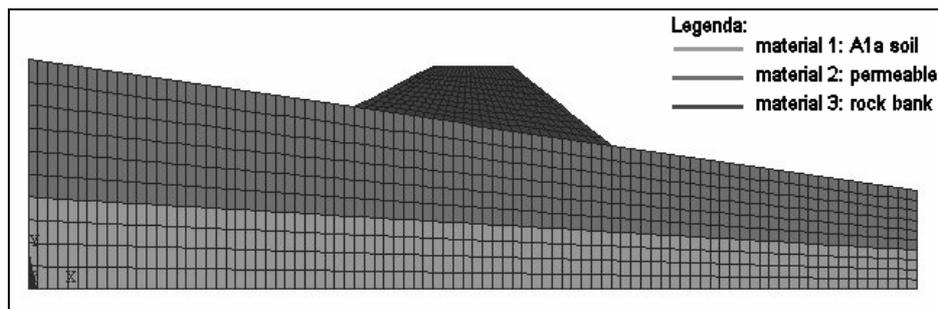
**Table 1: Values attributed to variable magnitudes**

pt = ground inclination	0%	15%	30%
ps = embankment scarps slope	3/2	2/3	1/2
y = embankment height [m]	3	6	9
qr = depth of rocky stratum measured downstream from the model [m]	-10	-6	-2
Dr = embankment soil density [kg/cm <sup>2</sup> ]	1800	1950	2200
Ds = embankment support terrain density [kg/cm <sup>2</sup> ]	1600	1800	2000
c = cohesion [Pa]	10000	20000	30000
φ = internal friction	20°	25°	30°

The assumption was made that the soil forming the embankment belonged to group A<sub>1</sub> with a Young's Modulus (E) of  $1 \times 10^8$  N/m<sup>2</sup>, a Poisson's ratio (ν) of 0.4 and finally variable density to simulate different compactations. The soil forming the embankment support terrain is characterized by Young's Modulus (E) of  $0.8 \times 10^8$  N/m<sup>2</sup>, a Poisson's ratio (ν) of 0.45 and finally density (ρ) variable. Furthermore it was assumed that the

stratum of permeable soil rested on a stratum of mica-schist of considerable rigidity ( $E = 79,3 \times 10^9 \text{ N/m}^2$ ). Its depth is one of the variables.

After model conceptualization, the domain was discretized into 900 elements which form the bed and 10 elements for L (overall embankment width: varying as a function of height), which make up the road embankment itself (see figure 3).



**Figure 3 Model discretization in one of the cases examined**

So that, 729 different configurations were analyzed varying values of geometric magnitudes and mechanical characteristics of soil.

To study the stress, displacement and strain status of the embankment and of the foundation soil in each different condition, Finite Element Method and ANSYS ® Software was utilized. Element Plane 42, used for modeling solid structures in two dimensions, was chosen.

In the finite-element method, a distributed physical system to be analysed is divided into a number (often large) of discrete elements.

Most or all of the model parameters have very direct relationships to the structure and material properties of the system.

A finite-element model generally has relatively few *free* parameters whose values need to be adjusted to fit the data. This assumes, of course, that the parameters are known *a priori* from other measurements.

In FEM analysis the behaviour of a particular type of element is analysed in terms of the loads and responses at discrete nodes. This analysis is often based on the Ritz-Rayleigh procedure which is based on the theorem of minimum potential energy in mechanics.

The result of the analysis of a typical element type is a small matrix relating a vector of nodal displacements to a vector of applied nodal forces.

The components of the matrix can be expressed as functions of the shape and properties of the element, and the values of the components for a particular element can then be obtained by substituting the appropriate shape and property parameter values into the formula.

Once the element matrices have been calculated, they are all combined together into one large matrix representing the whole complex system

In this case, constraints are represented by stopped displacement  $u_x$  on left and right boundaries of the model and by stopped displacement  $u_y$  on bottom boundary. Action are exclusively represented by ground mass weight.

### 3. RESULTS OBTAINED

The large elaborations have bring to these considerations.

About the horizontal stress ( $s_x$ ) it appears that:

1. the increase of the ground inclination produces a stress increasing at ground level localized above the embankment and in proximity to the base of toe of slope;
2. the variation of embankment scarps slope doesn't change horizontal stress distribution and values;
3. the increase of the embankment weight (or height) produces a stress increase under the embankment and in proximity to the material change surface;
4. the decrease of rock bank depth produces a decrease of maximum value of horizontal stress;
5. the increase of embankment soil density produces a stress increase in road solid;
6. likewise, the increase of ground soil density produces a stress increase in ground soil;

About the vertical stress ( $s_y$ ) it appears that:

1. the increase of the ground inclination produces a stress increase localized in rock bank bottom cause of the model configuration and the constraint localization;
2. the variation of embankment scarps slope produces a vertical stress increase;
3. obviously, the increase of the embankment weight (or height) produces a vertical stress increase;
4. the decrease of rock bank dept doesn't change vertical stress distribution and values;
5. the increase of embankment soil density doesn't produces any vertical stress variations;
6. the increase of ground soil density produces a stress increase in the same ground soil;

About the shearing stress ( $s_{xy}$ ) it appears that:

1. the increase of the ground inclination produces stress distribution variation which increases in proximity of down the slope toe (see figure below);
2. the decrease of embankment scarps slope produces a decrease of shearing stress value in proximity of down the slope toe (see figure below);
3. the increase of the embankment weight (or height) produces a shear stress value increase (see figure below);
4. the decrease of rock bank depth produces a reduction of shear stress values (see figure below);
5. the increase of embankment soil density produces shearing stress increase;
6. the increase of ground soil density doesn't produces any shearing stress increase;

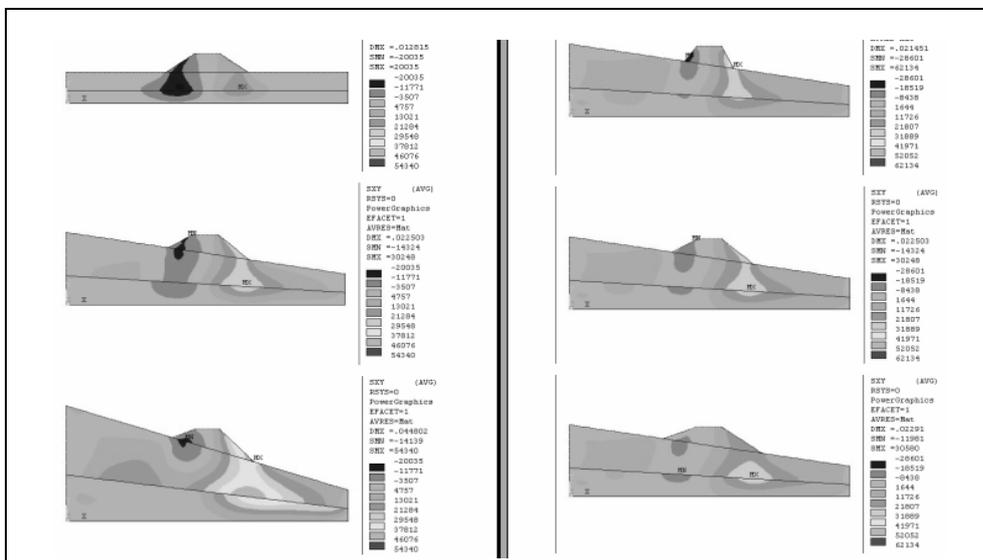


Figure 4 Shearing stress distribution versus ground inclination and embankment scarps slope.

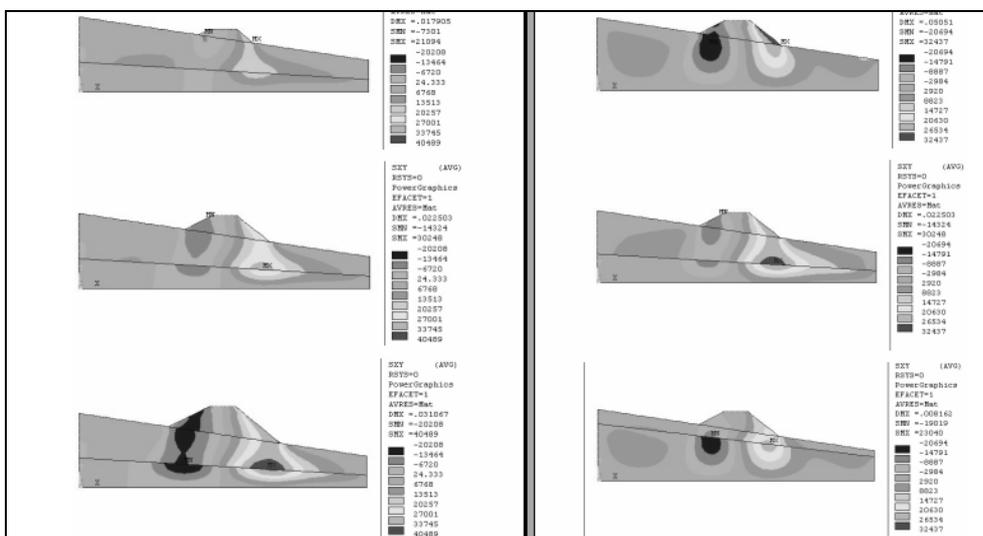


Figure 5 Shearing stress distribution versus embankment height and rock bank depth.

About the horizontal displacement ( $u_x$ ) it appears that:

1. the increase of the ground inclination produces a horizontal displacement values increase mostly in proximity to down the slope toe;

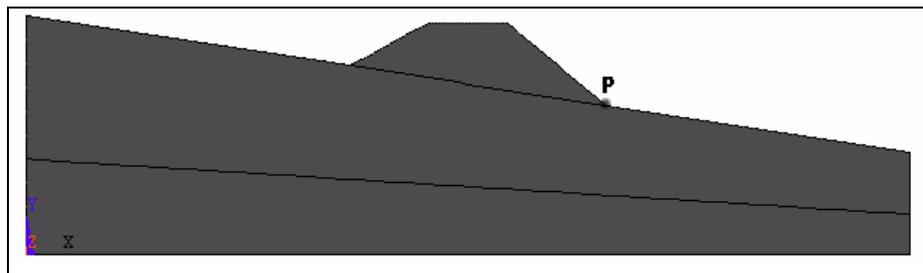
2. the variation of embankment scarps slopes doesn't change horizontal displacement distribution and values;
3. the increase of the embankment weight (or height) produces horizontal displacement values increase;
4. the decrease of rock bank depth produces a remarkable decrease of horizontal displacement which decreases near to zero when  $qr = -2m$ ;
5. the increase of embankment soil density produces a light horizontal displacement variations;
6. the increase of ground soil density produces a widening of the area in which horizontal displacement are maximum.

About the vertical displacement ( $u_y$ ) it appears that:

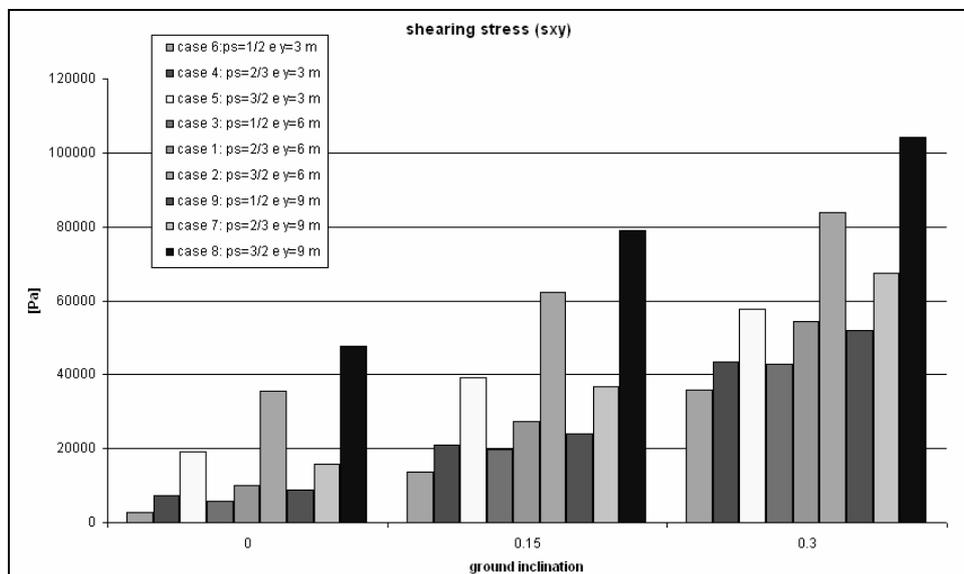
1. the increase of the ground inclination produces a vertical displacement values increase mostly in proximity to superior extremity of the model above the embankment;
2. the variation of embankment scarps slopes doesn't change vertical displacement distribution and values;
3. the increase of the embankment weight (or height) produces vertical displacement values increase;
4. the decrease of rock bank depth produces a remarkable decrease of vertical displacement which decreases near to zero when  $qr = -2m$ ;
5. the increase of embankment soil density produces a vertical displacement increase;
6. the increase of ground soil density produces a vertical displacement increase.

Afterwards, shearing stresses, calculated in a referred point P on different configurations, have been compared. The results are shown in the figure 7.

P represents the model point in which shearing stress assumes highest values on different configurations (see figure 6).



**Figure 6 The model with the referred point P**



**Figure 7 Shearing stresses versus ground inclination on different configurations.**

The histogram shows that high scarps slope produces the highest increase of shearing stress. Then, soil shear strength, calculated in a referred point P on different configurations, have been compared. The soil shear strength, as known, is expressed by the following equation:

$$\tau = c + \sigma \tan g\varphi \quad (\text{Eq. 1})$$

where:

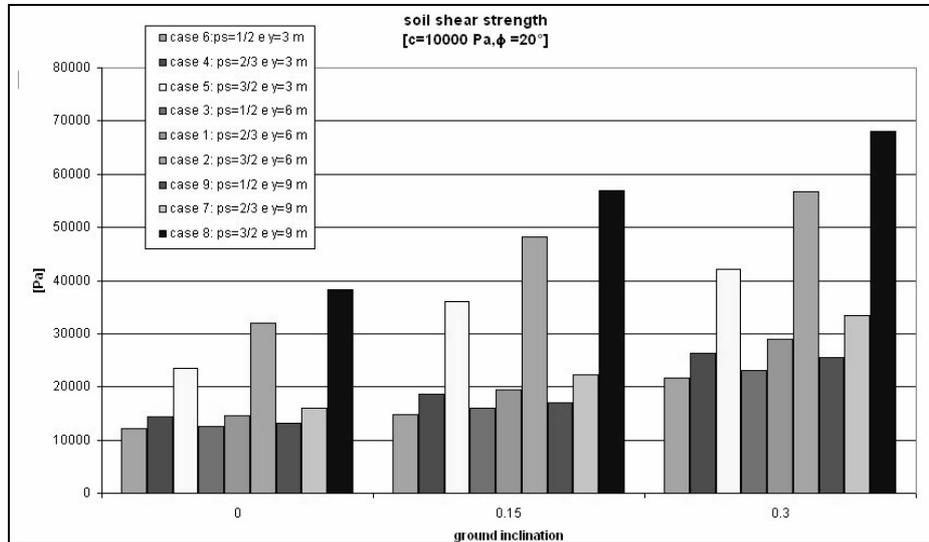
$\tau$  = soil shear strength;

$c$  = cohesion;

$\varphi$  = internal friction.

To determine soil shear strength three different type of soil have been considered, each of them characterized by cohesion and internal friction values which are listed in table 1

High scarps slope produces the highest increase of soil shear strength, too, as the histogram on figure 8 shows.



**Figure 8 Soil shear strength versus ground inclination on different configurations.**

So that, it isn't always true that having slight scarps slope is the best solution: it is necessary to consider all the different magnitudes to define the optimum inclination of embankment slopes.

So, shearing stresses have been compared with soil shear strength. Actually, crisis situation happen when shearing stresses are larger than soil shear strength. .

These analysis have led to following results.

When **pt=0%** road solid stability is assured independently from embankment scarps slope and from soil type, but only for  $y \leq 9$  m. In the case of  $y \geq 9$  m it is necessary that embankment scarps slope (ps) is less or equal to 2/3 to assure stability for all soil type.

When **pt=15%**, embankment stability is assured independently from embankment scarps slope and from the height, when the soil has a high cohesion ( $c= 30000$  [Pa]). If the cohesion is lower ( $c= 20000$  [Pa]), and when  $y=3$  m, soil shear strength is always larger than shearing stress independently from embankment scarps slope. For  $c = 10000$  [Pa], stability is still assured when  $\phi = 30^\circ$  and  $ps = 2/3$ , or when  $\phi = 25^\circ$  and  $ps = 3/2$ , or  $\phi = 20^\circ$  and  $ps = 1/2$ .

But, when  $y = 6$  m and  $c = 10000$  Pa stability is assured only when  $\phi = 30^\circ$  and  $ps = 3/2$ . If cohesion rises to 20000 Pa stability is assured when  $\phi = 20^\circ$  and  $ps = 2/3$ ; while if  $\phi = 20^\circ$  and it is necessary to have  $ps = 3/2$  it is needed an higher cohesion to assure stability.

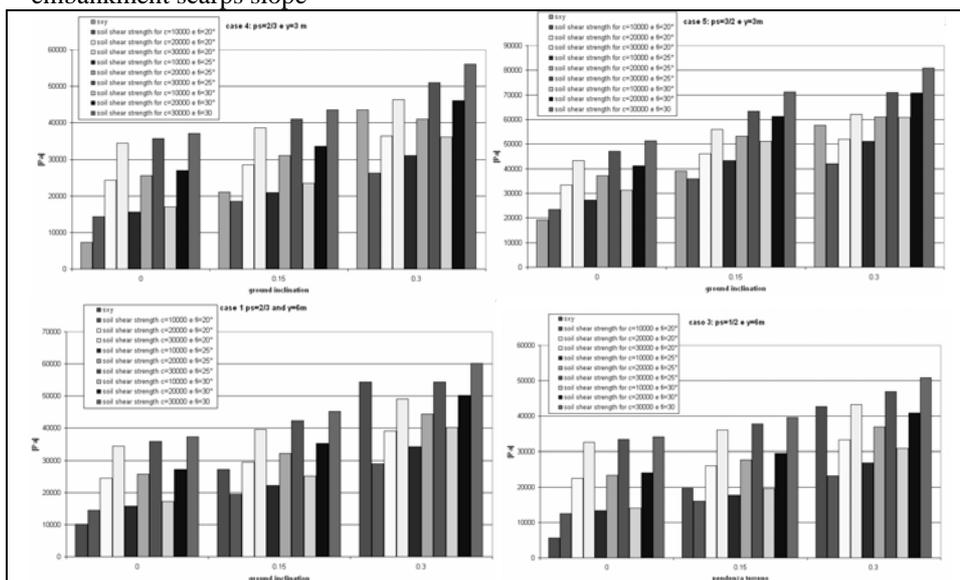
Even when  $y = 9$  m and  $c = 10000$  Pa, stability is assured only when  $\phi = 30^\circ$  and  $ps = 3/2$ . If cohesion rises to 20000 Pa stability is assured when  $\phi = 30^\circ$  and  $ps = 2/3$  or it could be  $= 3/2$ ; while if  $ps = 1/2$  it is sufficient  $\phi = 20^\circ$ .

At last, when **pt=30%**, if  $y = 3$  m and  $c = 30000$  Pa embankment stability is assured independently from embankment scarps slope and from internal friction. If  $c = 20000$  Pa stability is assured when  $\phi = 30^\circ$  and  $ps = 2/3$  or  $3/2$  or  $1/2$ . if  $c = 10000$  Pa stability

is not assured in any case when  $ps = 2/3$  while it is assured only when  $\varphi = 30^\circ$  and  $ps = 3/2$ .

If  $y=6$  m to assure the mass stability embankment scarps slope must be decided in function of soil characteristics: if  $c=30000$  Pa and  $\varphi=30^\circ$   $ps$  can be even  $3/2$ , while if  $c=30000$  Pa and  $\varphi=20^\circ$  or  $25^\circ$  it is necessary to have  $ps = 1/2$ .

If  $y = 9$  m stability is assured only when  $c=30000$  Pa and  $\varphi=30^\circ$  independently from embankment scarps slope



**Figure 9 Histograms which represent shearing stress and soil shearing strength [case 4:  $ps=2/3$  and  $y=3$  m; case 5:  $ps=3/2$  and  $y=3$  m;] [case 1:  $ps=2/3$  and  $y = 6$  m; case 3:  $ps=1/2$  and  $y = 6$  m]**

#### 4. CONCLUSION

Hydrogeological accidents are natural phenomenon which belong to the Earth dynamical evolution. When these accidents happen, the causes are mostly due to the interference between the road net and hydrographical system: under dimensioned bridges, embankments which constitute a earth dam to natural runoff, etc....

Besides, landslides are hastened by road construction because the infrastructure weight modifies mechanical equilibrium of the ground mass. Often, the cause of accident is the inadequate evaluation of embankment scarps slope related to soil mechanical characteristics and ground inclination.

These considerations suggested the study which is reported in this paper. Actually, the analysis objective was to identify the optimum inclination of embankment scarps slope versus soil mechanical characteristics, ground inclination and embankment height.

The analysis was carried out through finite element method to determine shearing stress and soil shear strength, in particular. Shearing stresses and soil shear strengths,

calculated in a referred point P on different configurations, have been individually compared. This comparison showed that high scarps slope produces the highest increase of shearing stress and it produces the highest increase of soil shear strength, too.

So that, it isn't always true that having slight scarps slope is the best solution: it is necessary to consider all the different magnitudes to define the optimum inclination of embankment slopes.

So, shearing stresses, calculated in a referred point P on different configurations, have been compared with soil shear strength. Actually, crisis situation happen when shearing stresses are larger than soil shear strength.

These analysis have led to following results.

1. When **pt=0%** road solid stability is assured independently from embankment scarps slope and from soil type, but only for  $y \leq 9$  m. In the case of  $y \geq 9$  m it is necessary that embankment scarps slope ( $p_s$ ) is less or equal to  $2/3$  to assure stability for all soil type.
2. When **pt=15%**, embankment stability is assured independently from embankment scarps slope and from the height, when the soil has a high cohesion ( $c= 30000$  [Pa]). If the cohesion is lower ( $c= 20000$  [Pa]), and when  $y=3$  m, soil shear strength is always larger than shearing stress independently from embankment scarps slope. For  $c = 10000$  [Pa], stability is still assured when  $\varphi = 30^\circ$  and  $p_s = 2/3$ , or when  $\varphi = 25^\circ$  and  $p_s = 3/2$ , or  $\varphi = 20^\circ$  and  $p_s = 1/2$ .

But, when  $y = 6$  m and  $c = 10000$  Pa stability is assured only when  $\varphi = 30^\circ$  and  $p_s = 3/2$ . If cohesion rises to  $20000$  Pa stability is assured when  $\varphi = 20^\circ$  and  $p_s = 2/3$ ; while if  $\varphi = 20^\circ$  and it is necessary to have  $p_s = 3/2$  it is needed an higher cohesion to assure stability.

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3. At last, when **pt=30%**, if  $y = 3$  m and  $c = 30000$  Pa embankment stability is assured independently from embankment scarps slope and from internal friction. If  $c = 20000$  Pa stability is assured when  $\varphi = 30^\circ$  and  $p_s = 2/3$  or  $3/2$  or  $1/2$ . if  $c = 10000$  Pa stability is not assured in any case when  $p_s = 2/3$  while it is assured only when  $\varphi = 30^\circ$  and  $p_s = 3/2$ .

If  $y=6$  m to assure the mass stability embankment scarps slope must be decided in function of soil characteristics: if  $c=30000$  Pa and  $\varphi=30^\circ$   $p_s$  can be even  $3/2$ , while if  $c=30000$  Pa and  $\varphi=20^\circ$  or  $25^\circ$  it is necessary to have  $p_s=1/2$ .

If  $y = 9$  m stability is assured only when  $c=30000$  Pa and  $\varphi=30^\circ$  independently from embankment scarps slope.

The results of this theoretical study can be considered only a first step to define the optimum inclination of embankment slopes: the analysis should be widened to a larger record of cases and since the theoretical studies are limited, it would be necessary to verify the results veracity through an adequate experimental analysis.

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