EXPERIMENTAL CHARACTERIZATION OF ASPHALT FOR AN ELASTO-VISCO-PLASTIC CONSTITUTIVE MODEL

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ABSTRACT

This paper describes a range of uniaxial creep tests that have been undertaken for a proprietary polymer modified asphalt, the procedure used to determine the parameters required for an elasto-visco-plastic constitutive model.

Uniaxial compressive creep testing and creep recovery testing have been undertaken over a range of temperatures and stress conditions. Procedures used to determine the model parameters from the test data are detailed and parametric equations are developed to describe the model parameters as functions of the test conditions. Particular attention is given to the determination of the parameters related to visco-plastic flow and damage accumulation at high strain levels. The model has been implemented in to the CAPA-3D Finite Element (FE) program and preliminary verification has been undertaken.

Keywords: constitutive modelling, Asphalt concrete, Finite Element, Damage

1. INTRODUCTION

During the last few decades there has been an increasing trend in Europe towards the use of analytical methods of pavement design. The traditional empirical methods that they are replacing cannot respond rapidly to changes in traffic and relative material costs or innovative changes in methods for construction or materials. Analytical methods are more flexible and can respond to these changes more easily. The structural design of a pavement and the prediction of its long term performance are two complementary and closely linked tasks. Of course, there is a strong need to consider the behaviour of the pavement component materials and their deterioration mechanisms from the beginning of the design phase.

A particular sub-class of problems that is of interest to the pavement designer is related to situations where the loading is very slow moving or stationary (e.g. container loading, aircraft standing areas, etc). Under these circumstances, the long loading times can lead to high levels of permanent strain (deformation) in the asphalt and many of the standard predictive techniques are not appropriate. Since the rate of accumulation of permanent deformation is time dependent, the most suitable forms of constitutive model are based on viscoelasticity and/or viscoplasticity. The objective of the research described in this paper is to develop a constitutive model that can be used to predict permanent deformation under conditions where the loading is very slow-moving or stationary. The essential element in the prediction is a set of materials parameters that characterize the material and can be used in computer codes for predicting the magnitude of the rut depth. These parameters can be derived from appropriate laboratory tests carried out at specified loads and temperatures.

All these things considered, together with the continuing increase in computing power, it is now clear why more and more effort has recently been put into developing Finite Element (FE) codes for elasto-visco-plastic analysis of pavement materials, as in Scarpas et al. (Scarpas, 1977). In such codes, the behaviour of each element (into which the continuum under consideration has been subdivided) can be analyzed separately and the cumulative deformations of the element brought together to give a resultant deformation for the whole structure. An FE model can easily deal with non-linear material behaviour, time and temperature dependent materials under various loading conditions, and virtually any geometric condition, including pavement discontinuities. On the basis of the nodal displacements, stresses and strains can be computed at any location of the structure, and, therefore, the desired versatility can be attained for studying a wide range of pavement geometries and damage modes. In order to achieve this goal, a comprehensive constitutive model of asphalt mixtures is required, based on the development of constitutive equations that account for effects of viscoelasticity, viscoplasticity and growing damage in the mixture, as in Collop et al. (Collop, 1999).

The objective of this work is to combine the non-linear hereditary behaviour of bituminous mixtures as well as the effect of distributed damage into a single constitutive model for predicting stress-strain behaviour and growth of damage. The model has been implemented into the CAPA-3D FE program developed by Scarpas et al. (Scarpas, 1998) and Erkens et al. (Erkens, 2002).

2. MODEL DESCRIPTION

Even though an asphalt mixture is basically a multi-component material, the concept of a representative continuum has been widely accepted in describing its response to external loads and climatic effects. Furthermore, the measured mechanical characteristics of asphalt correspond to working conditions for which the material's response is very complex. As an approximation, the notion of deformable mechanics can be used and an asphalt mixture can be assumed to be visco-elastic, homogeneous and isotropic material, with plasticity. The mechanical properties of such a material can then be studied then separately, according to whether the deformations measured in creep/recovery tests are time dependent and recoverable or not, as in Lu et al. (Lu, 1997).The behaviour of an elasto-visco-plastic material closely resembles that of models built from discrete elastic and viscous elements, normally a number of spring and dashpot elements arranged in series and parallel as studied, for example, by Flugge (Flugge, 1967) and specified for asphalt mixtures by Santagata et al. (Santagata, 1996). The generalised Burger's model is adopted here, consistently with previous work within the framework of this research project. This model contains the Maxwell model and a number of Kelvin-Voigt models, each characterized by a time constant known as the relaxation time. In such a model the same stress is transmitted through each element, while the strains (and strain rates) are additive, giving:

$$\varepsilon(t) = \varepsilon_{EL}(t) + \varepsilon_{VE}(t) + \varepsilon_{VP}(t) \qquad \text{(Eq. 1)}$$

where ε, ε_{EL}, ε_{VE} and ε_{VP} are the total, elastic, viscoelastic and viscoplastic strain components, respectively, and t is time. With the proposed approach, elastic, viscoelastic and viscoplastic strains can be evaluated individually, using the following equations:

$$\varepsilon_{EL}(t) = \frac{\sigma(t)}{E_0} \qquad \text{(Eq. 2)}$$

$$\varepsilon_{VE}(t) = J_{VE}(0) \cdot \sigma(t) + \int_0^t \frac{dJ_{VE}(t-t')}{d(t-t')} \cdot \sigma(t')dt' \qquad \text{(Eq. 3)}$$

$$\varepsilon_{VP}(t) = J_{VP}(0) \cdot \sigma(t) + \int_0^t \frac{dJ_{VP}(t-t')}{d(t-t')} \cdot \sigma(t')dt' \qquad \text{(Eq. 4)}$$

where E_0 is the modulus of elasticity and J_{VE} and J_{VP} are the viscoplastic and viscoelastic creep compliances, respectively, and t' is a dummy integration variable.

The strains due to elasticity are fully recoverable and time-independent. The strains due to viscoelasticity are time dependent: their magnitude depends on the load duration and on the rate of loading and unloading. These strains are fully recoverable. The strains due to viscoplasticity are permanent.

Integrating Equations 3 and 4 for $\sigma = \sigma_0$ (this is the case for a creep test), gives:

$$\varepsilon_{VP}(t) = \frac{\sigma_0}{\lambda_\infty} \cdot t \qquad \text{(Eq. 5)}$$

$$\varepsilon_{VE}(t) = \sigma_0 \cdot \sum_{i=1}^{n} \frac{1}{E_i} \cdot \left(1 - e^{\left(-\frac{t}{\tau}\right)} \right) \qquad \text{(Eq. 6)}$$

where

λ_∞ is the viscosity of the model, i is the i[th] Voigt element of the model, n is the total number of Voigt elements in the model, E_i is the modulus of elasticity of the i[th] Voigt element, $\tau_i = \lambda_1/E_1$ is the relaxation time of the i[th] Voigt element and λ_i is the viscosity of the i[th] Voigt element.

In terms of the total deformation, Equations 2, 5 and 6 can be combined to give:

$$\varepsilon(t) = \sigma(t) \cdot J(0) +' \int_0^t \sigma(t') \cdot \frac{dJ(t-t')}{d(t-t')} dt' \qquad \text{(Eq. 7)}$$

where

$$J(t) = \frac{1}{E_0} +' \frac{1}{\lambda_\infty} + \sum_{i=1}^{n} \frac{1}{E_i} \cdot \left(1 - e^{\left(-\frac{t}{\tau}\right)} \right) \qquad \text{(Eq. 8)}$$

J(t) is the total creep compliance of the model, here defined as the ratio between the measured strain and the applied stress without deterioration effects, when $\sigma(t) = \sigma_0$.

The viscoelastic and viscoplastic components were calculated using the Hereditary Integrals formulation. These formulas show how the viscoelastic and viscoplastic strain at any time depend on the entire stress history, with a fading memory.
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2.2 Viscoplastic stress dependence

Previous research by Collop et al. (Collop, 2001) has shown that at high stress levels, the steady-state (viscoplastic) strain-rate follows a power law relationship given by:

$$\frac{d\varepsilon}{dt} = K\sigma^n = K\sigma^{n-1}\sigma$$
$$= \frac{\sigma}{\lambda_\infty} \qquad \text{(Eq. 9)}$$

where K and n are material constants. Results from triaxial experiments presented by Collop *et al*. (Collop, 2001) have also shown that the steady-state strain rate not only depends on the overall stress level, but also on the test temperature and the degree of confinement. Consequently, based on these results, a model of the following form can be formulated for determining the equivalent viscosity of the viscoplastic element as a function of the stress conditions, temperature and degree of confinement:

$$\begin{aligned} \lambda_\infty &= \lambda_{uni}(T)\left(\frac{\sigma_e}{\sigma_0}\right)^{1-n} 10^{B(\eta+1/3)} \qquad && (\sigma_e > \sigma_0) \\ &= \lambda_{uni}(T)\, 10^{B(\eta+1/3)} && (\sigma_e \le \sigma_0) \end{aligned} \qquad \text{(Eq. 10)}$$

where σ_e is the Von Mises equivalent stress $\left(\sigma_e = \{3/2\, s_{ij} s_{ij}\}^{1/2}\right)$, s_{ij} is the deviatoric stress tensor $\left(s_{ij} = \sigma_{ij} - 1/3\,\delta_{ij}\sigma_{kk}\right)$, δ_{ij} is the Kronecker delta, p is the mean stress ($p=1/3\ \sigma_{kk}$), η is the stress ratio ($\eta=p/\sigma_e$) λ_{uni} is the uniaxial viscosity measured from a uniaxial compression test (where $\eta = -1/3$), T is temperature, and n, σ_0, B are material constants. Equation (10) is implemented numerically into CAPA-3D using the values of σ_e and η calculated at the beginning of the time step (i.e. at time t). The material parameters used by Collop *et al.* (Collop, 2001) are $\lambda_{uni}^{20C} = 60$ GPas, $\lambda_{uni}^{30C} = 5$ GPas, $\sigma_0 = 100$ kPa, $n=2$ and $B = -3.2$.

It can be shown that Equation (10) models the behaviour reasonably well, although the temperature and stress ratio ranges are somewhat limited. It should also be noted that, although Equation (10) is in a form in which it can be applied to any stress state, the experimental data from which it was derived were only for negative stress ratios (i.e. compressive mean stress).

2.3 Viscoplastic damage

It can be seen from Equations (9) and (10) that a constant stress creep test will result in a constant value of effective viscosity and therefore a constant rate of permanent strain accumulation. However, it is well known that during the latter stages of a creep test (the tertiary creep phase), the strain rate increases dramatically, which implies that the viscosity decreases as the material becomes progressively damaged.

Various methods are in use to model "damage". A relatively simple approach is to use Continuum Damage Mechanics (CDM) (Murakami, 2000; Schapery, 2001). According to CDM, it is assumed that the damaged state at an instant t can be described by a scalar Damage Variable D(t) ($0 \le D(t) \le 1$), where the states D=0 and D=1 represent the undamaged and the completely damaged state, respectively. CDM also requires a damage evolution law to describe damage growth (Odqvist, 1974, Schapery, 1999). As in previous research by Collop *et al*.(Collop, 2003) or by Le May *et al*. (Le May, 1999), Rabotnov's theory (Rabotnov, 1969) has been used in this work giving:

$$\frac{d\varepsilon}{dt} = \frac{C_1 \sigma^n}{(1-D)^m} \qquad \text{(Eq. 11)}$$

$$\frac{dD}{dt} = \frac{C_2 \sigma^v}{(1-D)^\mu} \qquad \text{(Eq. 12)}$$

where C_1, C_2, n, m, v, μ are material constants that, in general, depend on temperature. This kind of system reveals a clearly defined phenomenological approach, based on the assumption that creep and deterioration do not proceed independently.

It can be seen that Equation 11 can be re-written in a similar form to Equation 9, giving:

$$\frac{d\varepsilon}{dt} = \frac{C_1 \sigma^{n-1} \sigma}{(1-D)^m} = \frac{1}{\lambda}\sigma \qquad \text{(Eq. 13)}$$

$$\lambda = \frac{\sigma^{1-n}(1-D)^m}{C_1} \qquad \text{(Eq. 14)}$$

In this way, the equivalent viscosity of the viscoplastic element can be determined as a function of stress conditions, temperature and degree of confinement. Taking into account the similitude between Equations 9 and 11, and also between Equations 10 and 14, Equation 10 can be modified, introducing an extra term accounting for cumulative damage giving:

$$\begin{aligned} \lambda_\infty &= \lambda_{uni}(T)\left(\frac{\sigma_e}{\sigma_0}\right)^{1-n} 10^{B(\eta+1/3)} (1-D)^m \qquad (\sigma_e > \sigma_0) \\ &= \lambda_{uni}(T)\, 10^{B(\eta+1/3)} (1-D)^m \qquad (\sigma_e \le \sigma_0) \end{aligned} \qquad \text{(Eq. 15)}$$

In order to be valid also for three-dimension problems, the evolution equation (Equation 12) can be rewritten in terms of Von Mises equivalent stress to give:

$$\frac{dD}{dt} = \left(\frac{\sigma_e}{\widetilde{C}}\right)^v \frac{1}{(1-D)^\mu} \qquad \text{(Eq. 16)}$$

where $\widetilde{C}$ is a material constant.

It has also been suggested that $v = (n+1)/2$ holds for a range of materials and temperatures in uniaxial creep further reducing the number of constants. Moreover, if it is assumed that *n*=*m*, the number of independent constants to be determined in Equation 16 decreases further. The previous assumptions lead to a damage law formulation that, after integrating and imposing the appropriate boundary condition, is given by:

$$D(t) = 1 - \left[1 - \left(\frac{\sigma_e}{\widetilde{C}}\right)^{\frac{n+1}{2}} \cdot (\mu+1) \cdot t\right]^{\frac{1}{\mu+1}} \qquad \text{(Eq. 17)}$$

This closed-form solution can be used for fitting purposes, but it has to be used in an incremental way if introduced into an FE program. In incremental formulation, it can easily be shown that the damage increment is given by:

$$\Delta D = {}^{t+\Delta t}D - {}^{t}D \cong \left(\frac{\sigma_e + \Delta\sigma_e}{\widetilde{C}}\right)^{\frac{n+1}{2}} \cdot \frac{1}{(1-{}^{t}D)^{\mu}} \cdot \Delta t \qquad \text{(Eq. 18)}$$

By substituting Equation 7 into Equation 15, Equation 1 can be rewritten as in Equation 19, in a way that makes it possible to take into account the evolution of the damage:

$$\varepsilon(t) = \frac{\sigma_e}{E_0} + \frac{\sigma_e}{\lambda_{uni}(T)\left(\frac{\sigma_e}{\sigma_0}\right)^{1-n} 10^{B(\eta+1/3)}(1-D(t))^n} \cdot t + \sigma_e \cdot \sum_{i=1}^{n} \frac{1}{E_i} \cdot \left(1 - e^{\left(-\frac{t}{\tau}\right)}\right) \qquad \text{(Eq. 19)}$$

3. EXPERIMENTAL WORK

A test that is frequently used for evaluating time-dependent properties of asphalt mixtures is the uniaxial compression test, in which a specimen is subjected to loading/unloading cycles, with the applied stress kept constant over the loading period. Uniaxial creep compression tests were carried out on laboratory prepared specimens of dense AC 0/16 with a proprietary polymer modified bitumen. The samples were prepared with a diameter of 100 mm and a height of 120 mm using the gyratory compactor. The density of the specimens was kept as constant as possible, at a target value of 2,405 kg/m3 corresponding to 100% Marshall density. A total of 61 specimens were prepared and tested.

Creep (uniaxial compression) tests were performed at three temperatures (20, 40 and 60 °C) and various stress levels depending on the test temperature (0.5 to 6.0 MPa). Two types of test were performed: loading until failure (Figure 1) (assuming 10% axial strain as a failure criterion) and loading followed by unloading at some point during the steady-state phase where the deformation rate is approximately constant (Figure 2). Axial deformation was measured using load-line displacement and radial deformation was measured using three LVDT’s equally spaced around the circumference of the specimen at mid-height. A friction reduction system was used between the asphalt specimen and the test plates, which consisted of two thin plastic sheets separated with silicon grease.

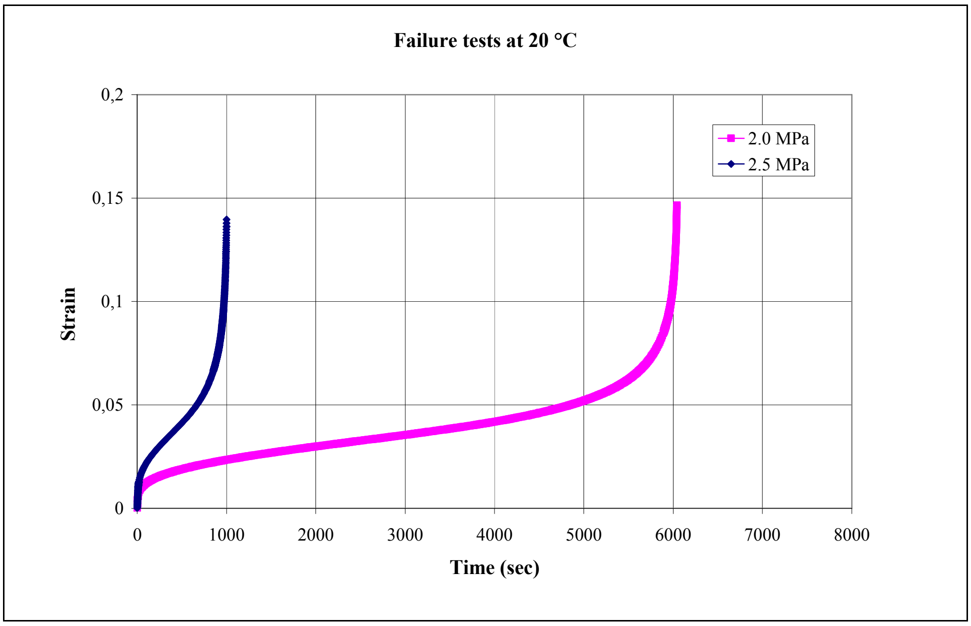

Figure 1 Creep tests till failure, for different stress levels

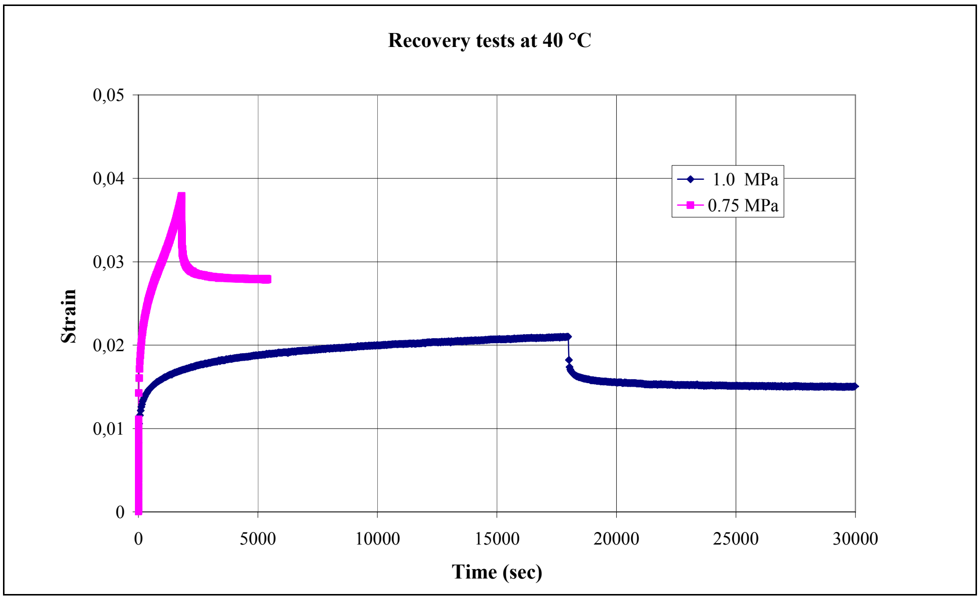

Figure 2 Creep tests with recovery, for different stress levels.

For each combination of load and temperature, recovery test were preceded by failure test carried out in analogous conditions, in order to be able to identify the range of the steady state, so that it is possible to avoid the risk of bringing on any damage into the material when performing recovery tests.

3.1 Model fitting procedure

With one Voigt element in the model, 7 parameters need to be determined. Four of them are related to the classical Burger's model (E_0, λ_∞, E_1 and λ_1), another two are necessary in order to calibrate the damage growth ($\widetilde{C}$ and μ) , while the last one, n, takes into account the stress-based nonlinearity introduced into the viscoplastic component of the model. Unloading tests were used to determine the classical Burger's model parameters, while failure tests were used to determine additional damage parameters. Both of them, failure and recovery test, were used to determine the last parameter, related to the non-linear stress dependent behaviour of the asphalt mixture.

Substituting Equations 2, 5 and 6, Equation 1 can be rewritten as:

$$\varepsilon(t) = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{\lambda_\infty} \cdot t + \sigma_0 \cdot \sum_{i=1}^{n} \frac{1}{E_i} \cdot \left(1 - e^{\left(-\frac{t}{\tau} \right)} \right) \qquad \text{(Eq. 20)}$$

Using a non-linear least squares fitting method, it is possible to determine the above parameters, from the region where the load is held constant.

The instantaneous elastic modulus of the model E_0, was directly calculated as the slope of the stress-strain curve during the time when the load is applied at a constant rate until the target stress level is reached, as shown in Figure 3.

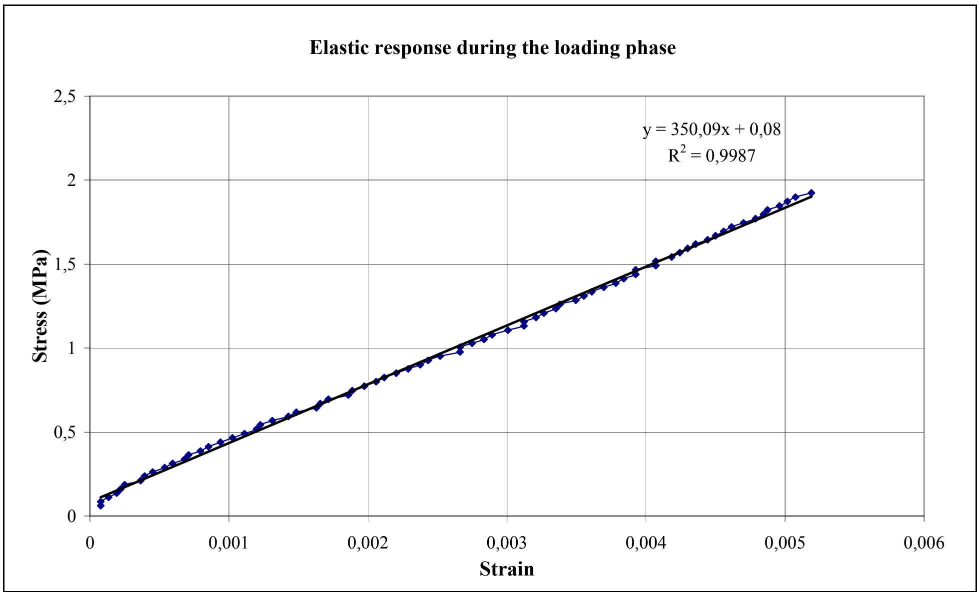

Figure 3 Stress vs. strain, during the application of the load.

During this phase, Imposing a constant stress rate C and evaluating the Hereditary Integral in Equation (7) for $\sigma(t)= C\cdot t$ gives:

$$\varepsilon(t)=\frac{C\cdot t}{E_0}+\frac{C\cdot t^2}{2\lambda_\infty}+\frac{C\cdot\tau}{\lambda_1}\left\{t\cdot e^{\frac{t}{\tau}}+\tau\cdot\left(1-e^{\frac{t}{\tau}}\right)\right\} \qquad \text{(Eq. 21)}$$

It was found that, as expected, the elastic modulus E_0 does not depend on the stress level but only on temperature (see Figure 4):

$$E_0=\frac{a}{T} \quad (22) \qquad \text{(Eq. 22)}$$

where a = 4964.

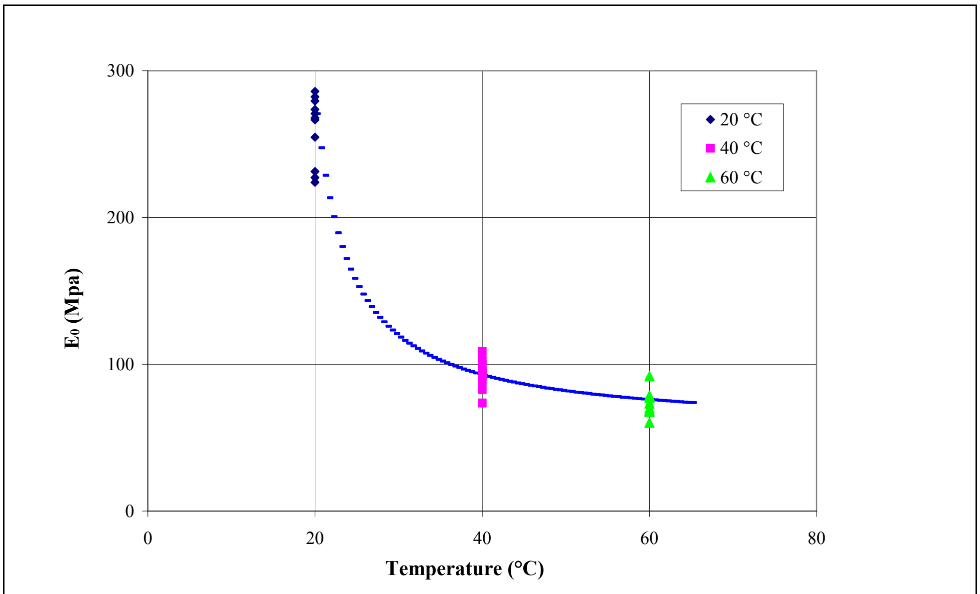

Figure 4 Elastic modulus as a function of temperature from creep testing.

The other parameters of the model not related to damage growth (i.e. λ_∞, E_1 and λ_1) have been determined based on the nonlinear least square fitting method, fitting to the phase where the load is held constant in the recovery tests. The variation of the viscoelastic parameter E_1 with stress and temperature (see Figure 5) was found to be given by:

$$E_1(T,\sigma)=106.89\cdot\sigma_e+2.025\cdot T-100.22 \qquad \text{(Eq. 23)}$$

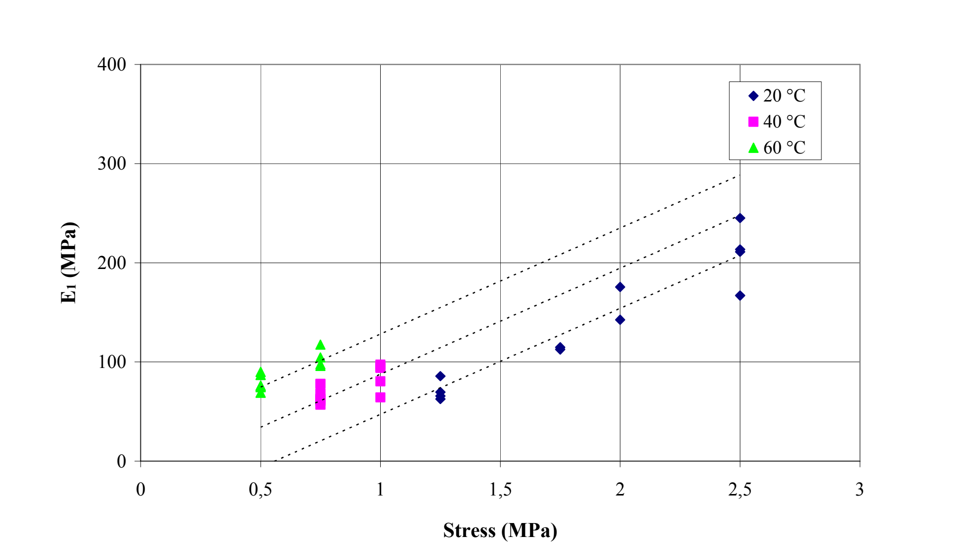

Figure 5 E_1 as a function of stress from creep testing.

The variation in the other viscoelastic parameter λ_1 with stress and temperature (see Figure 6) was found to be given by:

$$Log\lambda_1(T,\sigma) = (-0.0588 \cdot T + 0.4671) \cdot \sigma_e + 5.6957 \qquad \text{(Eq. 24)}$$

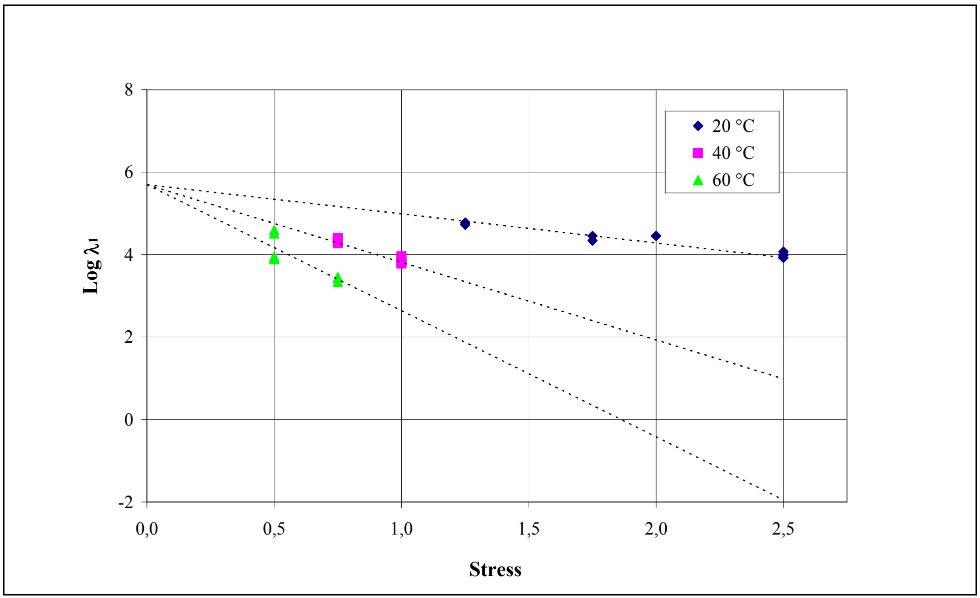

Figure 6 Parameter λ_1 as a function of stress from creep testing.

Combining the two of them together, it is possible to predict the dependence of the relaxation time $\tau_1 = \lambda_1/E_1$ on stress and temperature, as shown in Figure 7.

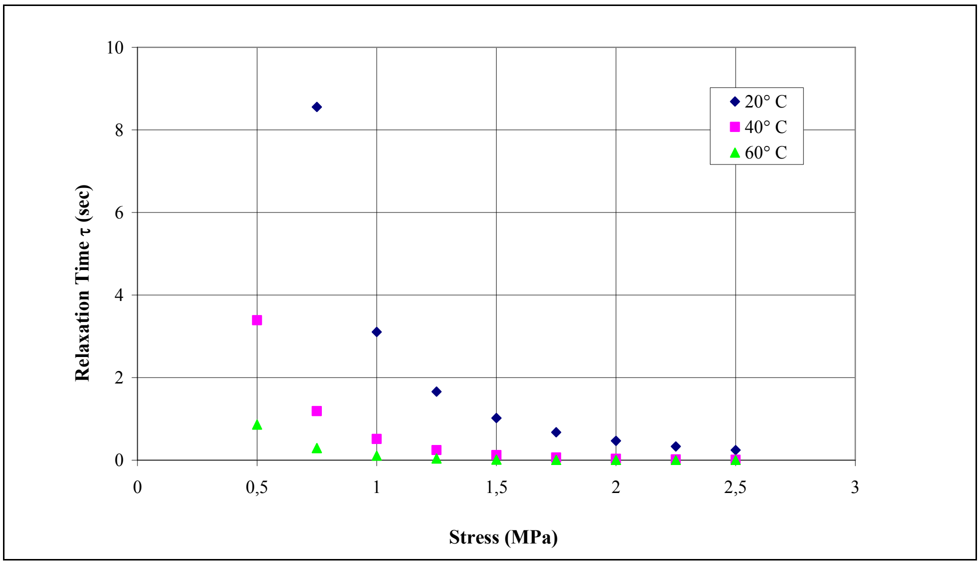

Figure 7 Relaxation time τ of the Voigt element as a function of stress.

Together with E_1 and λ_1, the same fitting routine was applied to the constant load phase of the recovery tests in order to determine $\lambda\infty$ as a function of stress level and temperature (see Figure 8):

$$Log\lambda_{\infty} = -8.1395 \cdot Log\sigma_e - 0.1021T + 9.771 \qquad \text{(Eq. 25)}$$

The parameter n was determined from the viscoplastic stress dependence of the material. It can be seen from Equation 10 that if the equivalent viscosity $\lambda\infty$ is plotted against stress level on double logarithmic scales (see Figure 8) the gradient is given by 1 - n. This resulted in a value of n = 9.139.

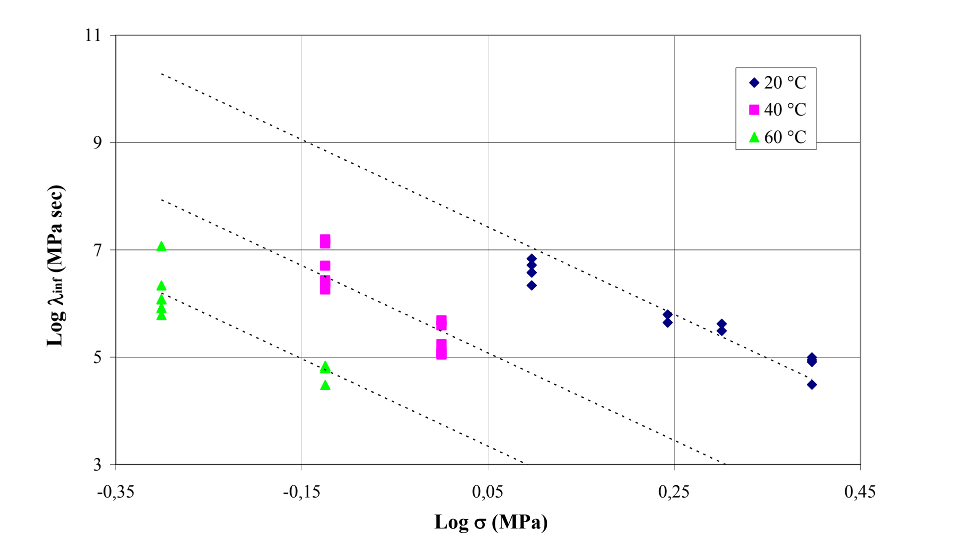

Figure 8 Parameter λ∞ as a function of stress from creep testing.

Failure tests were used to determine additional damage parameters. As far as $\tilde{C}$ is concerned, it was found that it depended only on temperature (see Figure 9):

$$\tilde{C} = -0.3784 \cdot T + 30.429 \quad \text{(Eq. 26)}$$

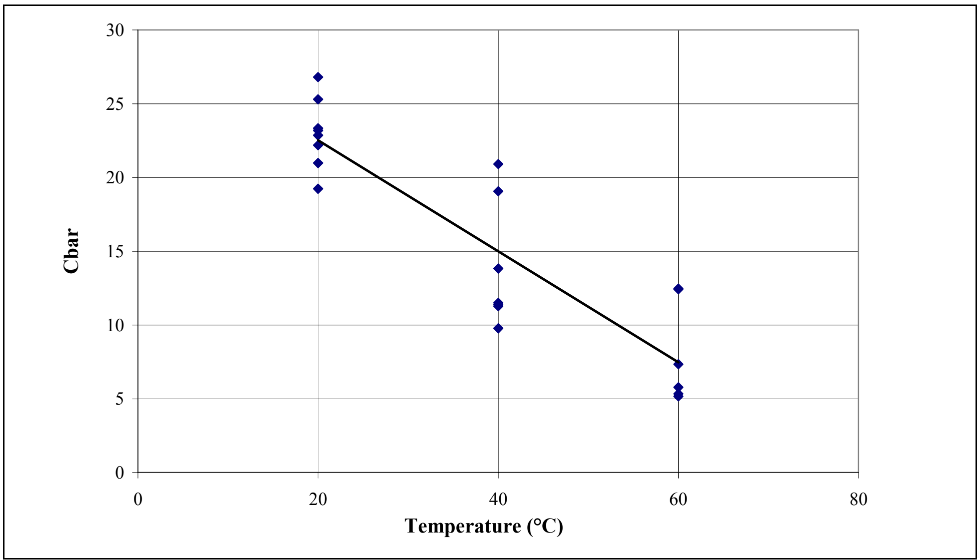

Figure 9 Parameter $\tilde{C}$ as a function of temperature.

The damage parameter μ was found to depend on both temperature and $\widetilde{C}$ (see Figure 10):

$$\mu = (0.078*T+0.3814)\cdot \widetilde{C} \qquad \text{(Eq. 27)}$$

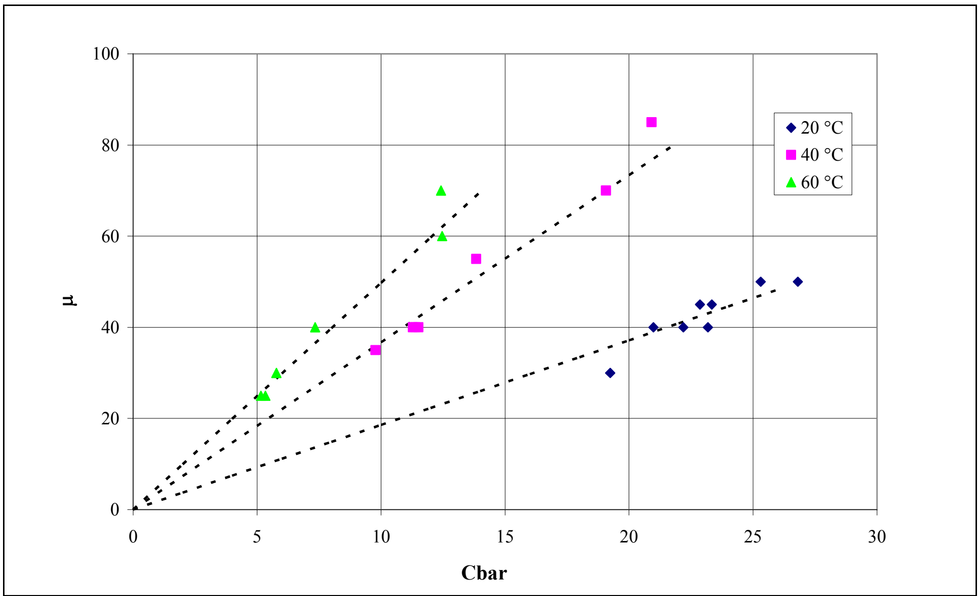

Figure 10 Parameter μ as a function of $\widetilde{C}$

The final step of the fitting procedure proposed in this work consisted in using Equation 15, with σ_0 assumed to be equal to 100 kPa as in Collop *et al.* (Collop, 2003) (see Figure 11) to calculate the equivalent uniaxial viscosity as a function of temperature giving:

$$Log\lambda_{uni} = -0.1021\cdot T + 17.91 \qquad \text{(Eq. 28)}$$

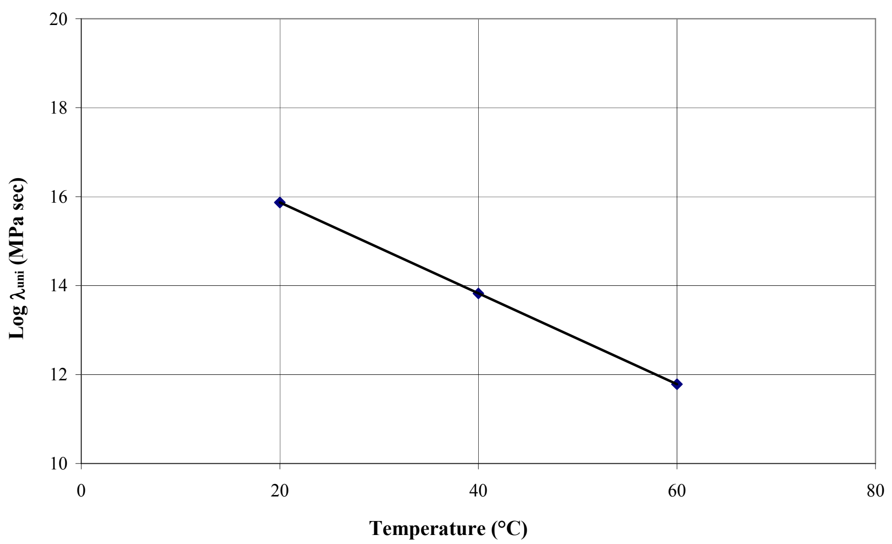

Figure 11 Variation of λ_{uni} with the temperature.

As a first stage verification, one of the creep tests curves was simulated in CAPA-3D using the constitutive model and parameters determined above. A single-element mesh representing the specimen was subjected to the uniform vertical contact stress used in the test. Figure 12 shows the simulated and measured responses. It can be seen from this figure that agreement is generally good. The main difference between the predicted and measured curves occurs early in the test (before 10 seconds) where the predicted curve is greater than the measured curve. This is due to difference in the way the load is applied. In the simulation the load is applied instantaneously whereas in the test the load was applied over a finite number of seconds resulting in lower strains.

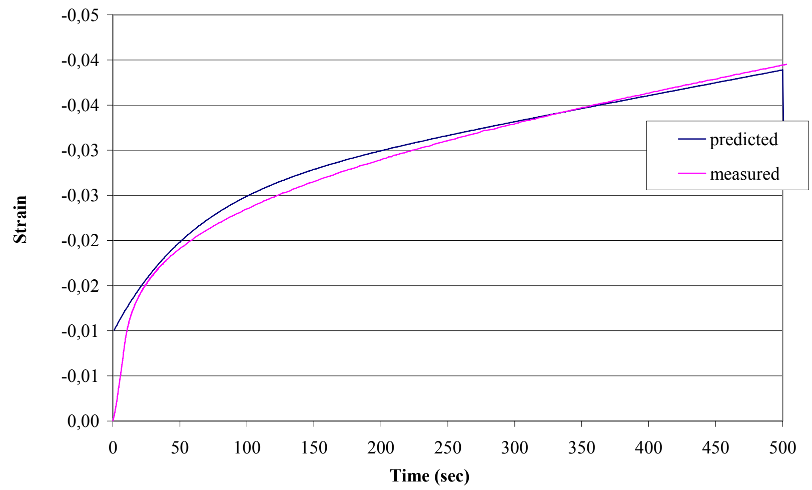

Figure 12 Simulation of a creep test with CAPA-3D

CONCLUSIONS AND FUTURE WORK

A range of uniaxial creep tests has been undertaken for a polymer modified asphalt over a range of temperatures and stress conditions, in order to determine the parameters required for an elasto-visco-plastic constitutive model. Particular attention has been given to the determination of the parameters related to visco-plastic flow and damage accumulation at high strain levels. In the framework of the simple approach to damage of Continuum Damage Mechanics (CDM) Rabotnov's theory has been used to describe damage growth. The elasto-visco-plastic constitutive model has been implemented into a Finite Element (FE) program and preliminary verification has been undertaken.

Based on the fitting procedure described in this paper, it has been shown that starting from a typical laboratory testing (creep/recovery tests) on asphalt mixtures, it is possible to determine the parameters of a relatively simple constitutive model (Burger's model with one Voigt element, combined with Rabotnov's damage law) that allows to account for the complex mechanical response (elastic, viscous, plastic and with damage) of an asphalt pavement when subject to a quasi-static load.

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