Privatised transit in combination with road-based congestion charging: An inter-modal equilibrium model

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Abstract

An inter-modal equilibrium model links an urban road network subject to a congestion charge to a parallel urban transit market, with a view to finding the optimum congestion charge consistent with the commercial decisions of the transit operator(s). The objective is the sum of consumer and producer surplus, which is maximised with respect to the congestion charge. Monopoly and monopolistic transit markets are considered. The prices and supply of transit services are treated as endogenous variables. The problem has been formulated as a bi-level programme. We demonstrate the results of the model using a small example giving insights into the problem.

Keywords: Congestion charge, Equilibrium model, Transit market

1. INTRODUCTION

Road-based congestion charging has received much attention during the last four decades since Walters (1961) established what is now the standard way economists think about congestion. Congestion charging has long been a subject of significant importance from both theoretical and practical standpoints. Interest in this subject has grown due to irresistable traffic growth contrasting with limited road space in modern cities.

The concept of an optimal congestion charge rests on the notion of utility maximising travellers. This is reduced here to travel time minimising travellers with perfect information, leading to a Wardrop user equilibrium (Wardrop, 1952) whereby only minimum travel time paths are used. In previous work on this topic, a bilevel optimization problem has been formulated and solved as a Stackelberg game (Bard, 1998; Stackelberg, 1952). The cases where some or all links in the network are tolled have been considered, leading to the first-best and second-best problems respectively. The first-best problem can be solved by a simpler form of optimization, namely by minimising travel costs for the fixed demand case or maximising social surplus for the elastic demand case (Sheffi, 1985; Yang & Bell, 1997; Yang & Huang, 1998). For the second-best problem, researchers have investigated the optimal pricing locations and charges (May et al., 2002; May & Milne, 2000).

Although generally the road network is provided by the government free of charge, congestion charging has been implemented in the central area of a few cities where traffic congestion is a problem. This is the case in Singapore and London (Tisato, 1998; Verhoef & Small, 2004).

In such congested cities there will be transit services provided by either public or private sectors (or perhaps more commonly a combination of the two). This paper considers the case of a deregulated transit market with a limited number of operators making commercial decisions regarding fares and services. In this context, transportation planning needs to consider not only the decisions of travellers but also those of transit service providers. The government, travellers and transit operators interact, responding to decisions made by one another, thereby jointly determining the outcome.

Considerable research has been directed toward the analysis of market participation in urban transportation planning (Evans, 1987; Fernandes & Marcotte, 1992; Ferrari, 1999; Glaister, 1985; Harker, 1986; Zubieta, 1998). Kinds of urban transport tax for demand suppression have been considered without explicitly introducing a road-based congestion charge. Although many studies looked at the context of congestion charging and transit participation, none has attempted to link the congestion charge with the transit market despite the evident interrelationship.

The UK government's White Paper of July 1998 gave local authorities powers to introduce road congestion charging. It also emphasized the importance of improving local bus services by bus priority

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measures, integrated ticketing and quality partnerships (Mackie, 1998). However, it said nothing about the likely response of the transit market to a congestion charge.

The aim of this paper is to formulate a simple Cournot-Nash intermodal equilibrium transport model that incorporates both traveller mode choices as well as fare and service supply choices by profit maximising monopoly or monopolistic transit operators. The demand for travel is assumed to be elastic. The model is then used to seek a socially optimal congestion charge. Two public transport modes are considered, namely bus and train (and/or metro). Road congestion is represented and is assumed to impinge on both car and bus travel. In the case of a single public transport operator, fares and services are set to maximise profits. In the case of separate bus and train operators, fares and services are set to maximise operator profits non-cooperatively.

A simple example shows how the interests of travellers and transit operators diverge and how transit operators are able to exploit their monopoly or monopolistic positions to the detriment of travellers. A possible illustration of this effect in the real world is offered by the large increase in London Transport bus and underground fares following the introduction of the London congestion charge.

In the next section, the inter-modal equilibrium transport model is set out. Section 3 explores the results of the model. Finally, conclusions and policy implications are summarised in Section 4.

2. THE MODEL

The transportation system is assumed to consist of three types of agent (the government, the transit operators and the travellers). The government sets the congestion charge, the transit operator(s) set bus and train fares, and the travellers choose whether to travel and if so by which mode. Route choice is not considered. The implicit underlying network has two links, one for road and one for rail, connecting a single origin to a single destination. Bus and private transport share the road link and so experience the same congestion. It is assumed that bus and train systems have sufficient capacity to carry any demand that might arise. Bus flows adjust to accommodate the demand, but no account is taken of the effect of bus frequency on waiting times. Train schedules are assumed to be fixed. The model is strategic and designed for studying the consequences of policies, like whether or not to separate bus and rail operators or whether or not to introduce a congestion charge.

For clarity, the model formulation can be described as three blocks corresponding to the three types of agent involved in the analysis (see Fig.1). The first block representing the government deals with social surplus maximisation by setting the congestion charge. The second block representing the trip makers deals with the congested network equilibrium mode choice. Finally, the third block representing the transit operator(s) deals with profit maximisation by setting bus and train fares.

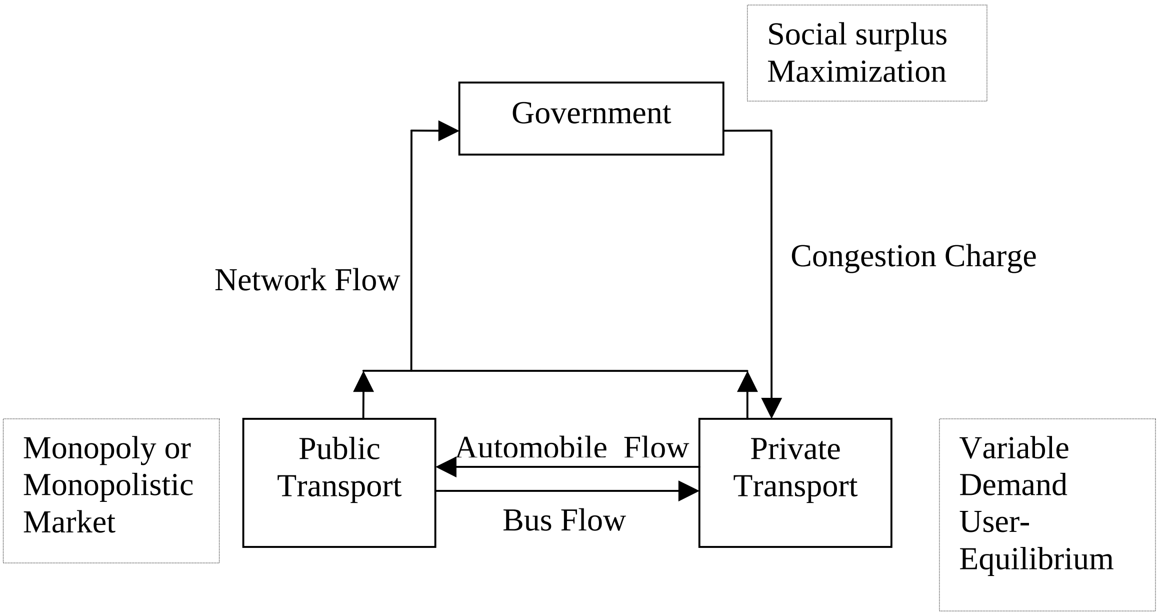

Fig. 1: Framework of the model

The notation used in this paper is defined as follows:

m : mode (*a* = auto, *t1* = bus, *t2* = train)

v_m : flow by mode *m*

$c_m(v_a, v_{t1})$: cost by mode *m* (*a* = auto, *t1* = bus)

t_{0a} : auto free-flow travel time

t_{0t1} : bus free-flow travel time

t_{0t2} : train travel time

p_m : fare for each mode (p_a = congestion charge or toll)

ω : value of time

C : generalize cost for the singular O-D pair

C^o : observed minimum generalized cost for the singular O-D pair

q : travel demand between origin and destination

q^o : observed demand between origin and destination

θ : positive dispersion parameter related to auto and transit mode
i.e. parameter for log sum function (estimated from data)

μ : positive dispersion parameter related to bus and train mode
i.e. parameter for log sum function (estimated from data)

γ : sensitivity parameter for the travel demand function

We consider a simple example of a congested road and a railway between one origin and one destination. Demand is elastic. For the demand function, we adopted the exponential form which is widely used in urban transport models. Instead of a potential demand q^o occurring when travel cost is zero (Evans, 1992), we represent the demand function q as:

$$q = q^o \exp\left(-\frac{C - C^o}{\gamma}\right) \quad (q^o > 0,\ C^0 > 0,\ \gamma > 0) \tag{1}$$

where q^o is an observed demand corresponding to an observed minimum travel cost C^0.

As for user choices, it is assumed that travellers minimize their generalized costs of travel with perfect travel information. For the road link, link cost is flow-dependent. Assuming that there is interaction between the bus and car modes, in-vehicle travel time on the road link is a non-negative, increasing function of link flow. The relationship between the flow and the travel time is represented by a travel time function (see Eq.(2)) adopted from the Bureau of Public Roads (BPR). Travel time is converted into generalised cost to which is added the direct cost of travel (the congestion charge in the case of the car and the fare for the bus). Buses do not pay the congestion charge (see Eq.(3)). Out-of-vehicle travel cost associated with time for walking to the car, bus or train and from the car, bus or train to the final destination is omitted.

$$c_a(v_a, v_{t1}) = p_a + \omega t_{0a}\left\{1 + 0.15\left(\frac{v_a + bv_{t1}}{k}\right)^4\right\} \tag{2}$$

$$c_{t1}(v_a, v_{t1}) = p_{t1} + \omega t_{0t1}\left\{1 + 0.15\left(\frac{v_a + bv_{t1}}{k}\right)^4\right\} \tag{3}$$

where *k* represents road capacity measured in equivalent trips by private transport and *b* converts trips by bus into equivalent trips by private transport. Note that 0 < *b* < 1, to reflect that *ceteris paribus* a transfer of trips from car to bus would reduce congestion and therefore travel time, but not by as much as if the trips were not made, as increasing bus use would lead on average to more bus flows.

Trains are assumed to run to a fixed schedule, so the generalised cost of travel by train is

$$c_{t2} = p_{t2} + \omega t_{0t2} \tag{4}$$

The expected minimum cost of travel by public transport is

$$c_t = -\frac{1}{\mu}\ln\left\{\sum_{m\in(t1,t2)}\exp(-\mu c_m)\right\} \tag{5}$$

while the expected cost of travel assuming the least cost mode is chosen is

$$C = -\frac{1}{\theta}\ln\left\{\sum_{m\in(a,t)}\exp(-\theta c_m)\right\} \tag{6}$$

Flow conservation is assumed, hence:

$$v_a + v_t = q \tag{7}$$

Car flow is given by:

$$v_a = q\ \frac{1}{1+e^{\theta(c_a - c_t)}} \tag{8}$$

and bus passenger flow is given by:

$$v_{t1} = v_t\ \frac{1}{1+e^{\mu(c_{t1}-c_{t2})}} \tag{9}$$

The governmental objective function is social surplus which consists of consumer surplus plus producer surplus. Consumer surplus, equal to γq, expresses the perceived benefits experienced by potential travellers. As γ is the elasticity of demand, this expression comes from the result of integration of the exponential demand function minus the travel cost function over the demand q (Evans, 1992). Finally, the producer surplus reflects the operator gain in addition to the government revenue from the congestion charge.

A single transit operator in a monopoly market chooses train and bus fares to maximise profit. In a monopolistic market, the Cournot-Nash (simultaneous) game (Nash, 1951) governs bus and train fares. Suppose that U_{t1}, U_{t2} are the profits of the two operators and that R_{t1}, R_{t2} are their best response fare functions. According to Nash's concept of equilibrium, we obtain Eq.(10) and Eq.(11):

$$U_{t1}(R_{t1}(p_{t2}), p_{t2}) \geq U_{t1}(p_{t1}, p_{t2}) \tag{10}$$

$$U_{t2}(p_{t1}, R_{t2}(p_{t1})) \geq U_{t2}(p_{t1}, p_{t2}) \tag{11}$$

By the definition of the Nash equilibrium solution, the optimal fare p^*_{t1} and p^*_{t2} satisfy Eq.(12) and Eq.(13):

$$p^*_{t1} = R_{t1}(p^*_{t2}) \tag{12}$$

$$p^*_{t2} = R_{t2}(p^*_{t1}) \tag{13}$$

3. RESULTS

We consider a single road and rail connection for a given origin and destination with the methodology described in Section 2. Fig. 2 shows the changes in social surplus with the toll charge. The optimal toll occurs at the point where social surplus reaches its maximum. The optimal toll is slightly lower in monopolistic market producing a slightly higher social surplus at the optimum. Where the toll is higher than the optimum, monopoly market produces a higher social surplus and vice versa when the toll is less than the optimum.

The main reason for a lower social surplus for monopolistic market when the toll is higher than optimal is revealed in Fig. 3, Fig. 4 and Fig. 5. Consumers (car, bus and train travellers) gain more in a monopolistic market (Fig. 3) but at the expense of government revenue and operator profit (Fig. 4 and Fig. 5). Fig. 4 shows that the total profits of monopolistic operators are less than the profit of a monopoly operator due to some degree of competition.

Figs. 6 and 7 show changes in car and transit mode share with respect to the toll. When the toll is high, people shift to the transit mode, so there are less people paying the toll (this also because of elastic demand). This happens to a higher degree in a monopolistic market because transit fare tends to be lower. Note that the share of car is around 90% in the absence of a toll, because the model looks only at non-captive travellers. In reality, a certain proportion of travellers will not have access to a car.

Fig. 8 shows now the optimal bus and train fares change with the toll. As expected, the fare moves in the same direction as the toll charge. The optimal train fare is greater than the optimal bus fare in both markets. The fares in the monopoly market tend to be larger and the difference with respect to the monopolistic market becomes greater for higher tolls.

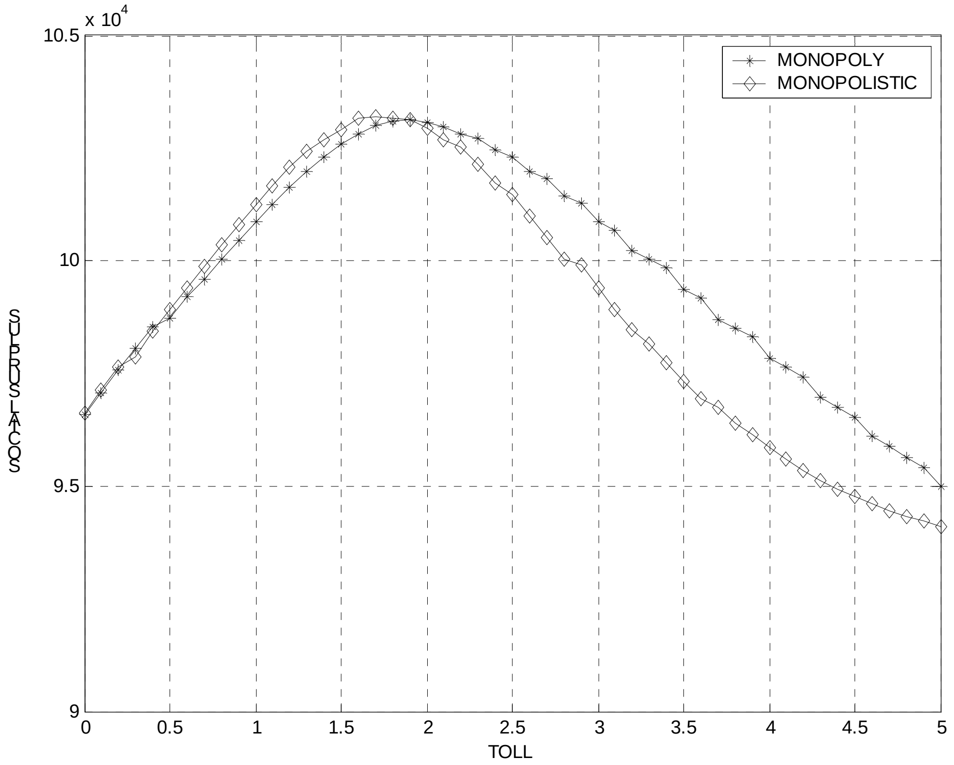

Fig. 2: Changes in social surplus with toll

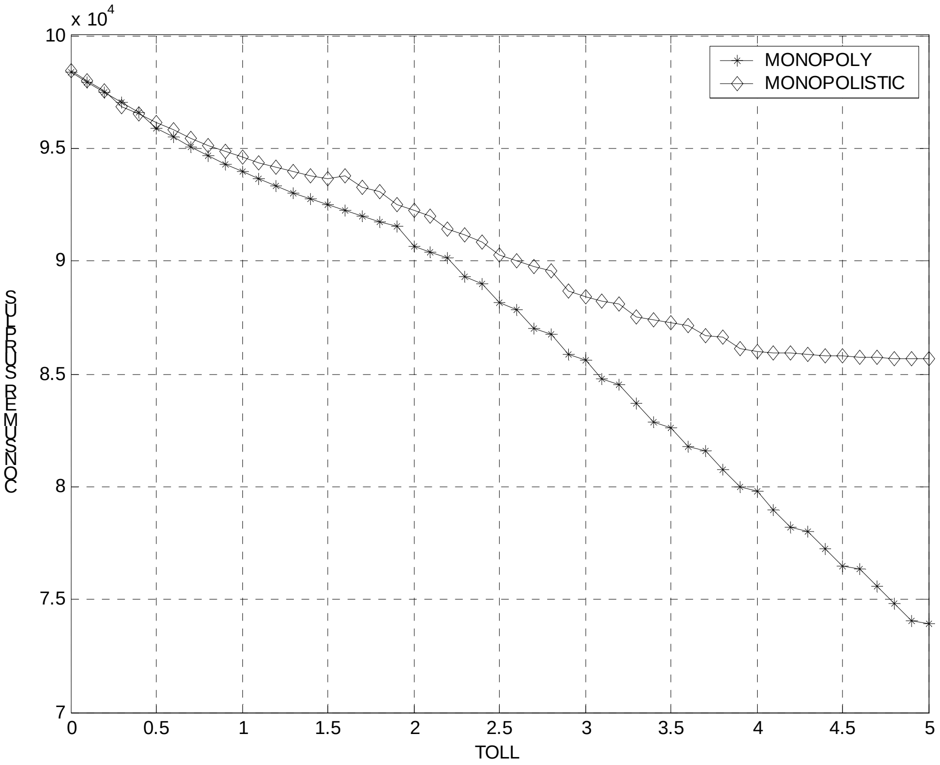

Fig. 3: Changes in consumer surplus with toll

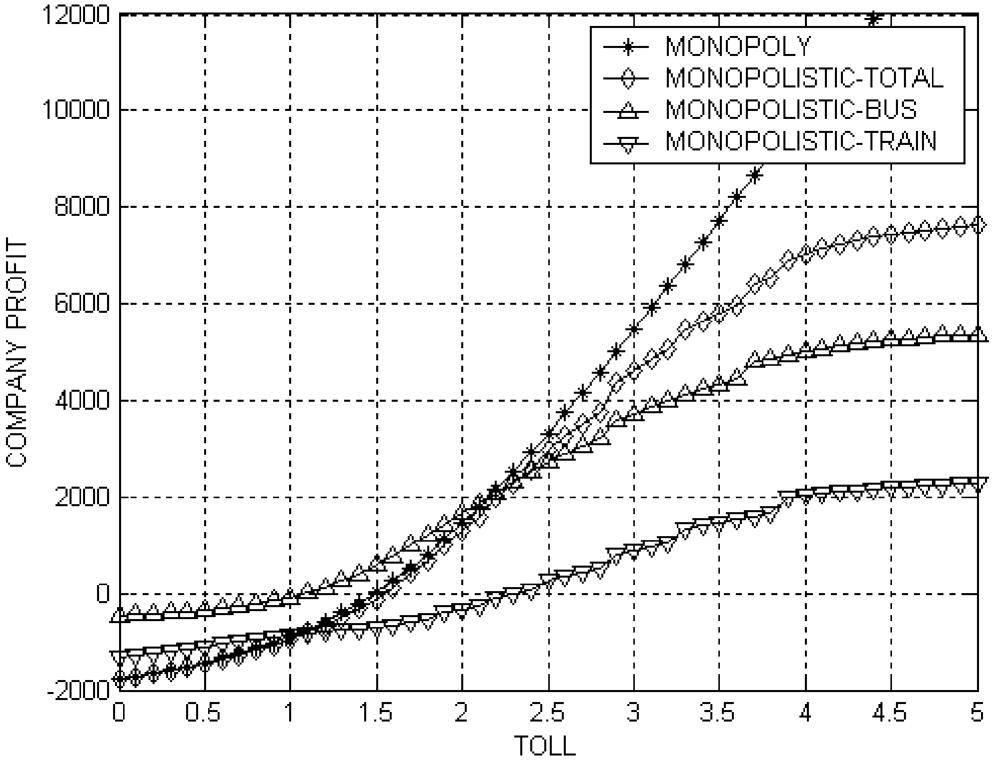

Fig. 4: Changes in companies' profits with toll

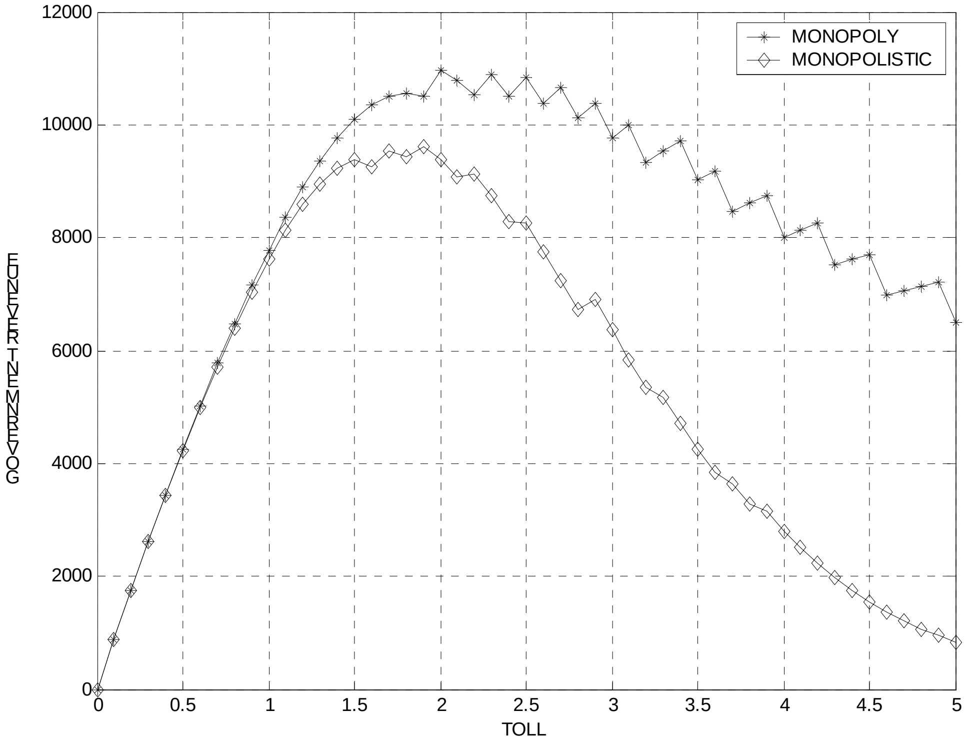

Fig. 5: Changes in government revenue with toll

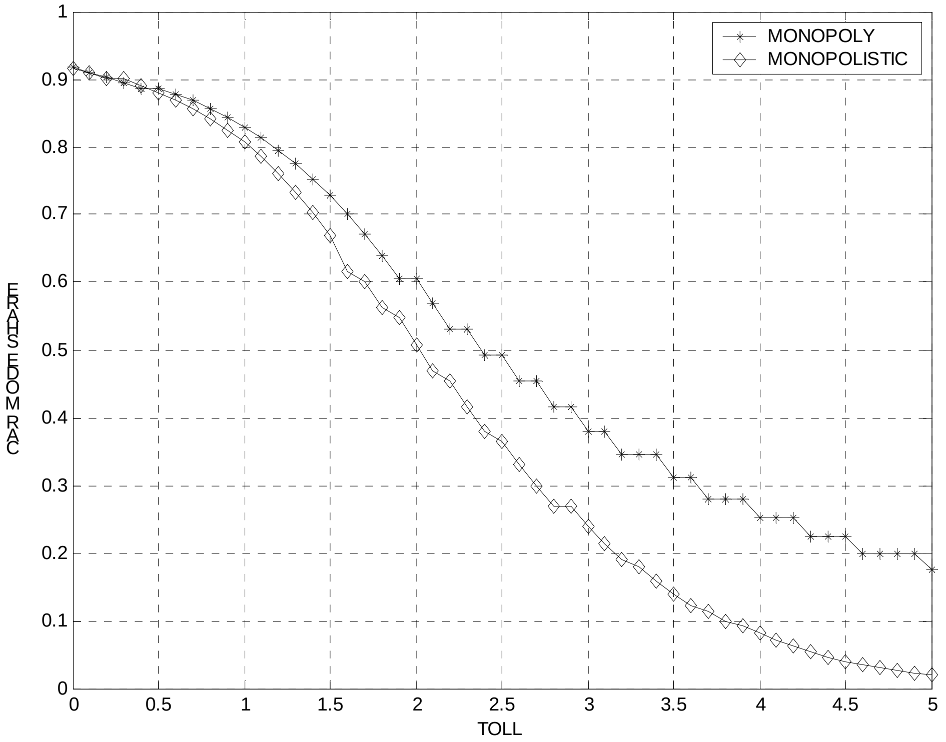

Fig. 6: Changes in car mode share with toll

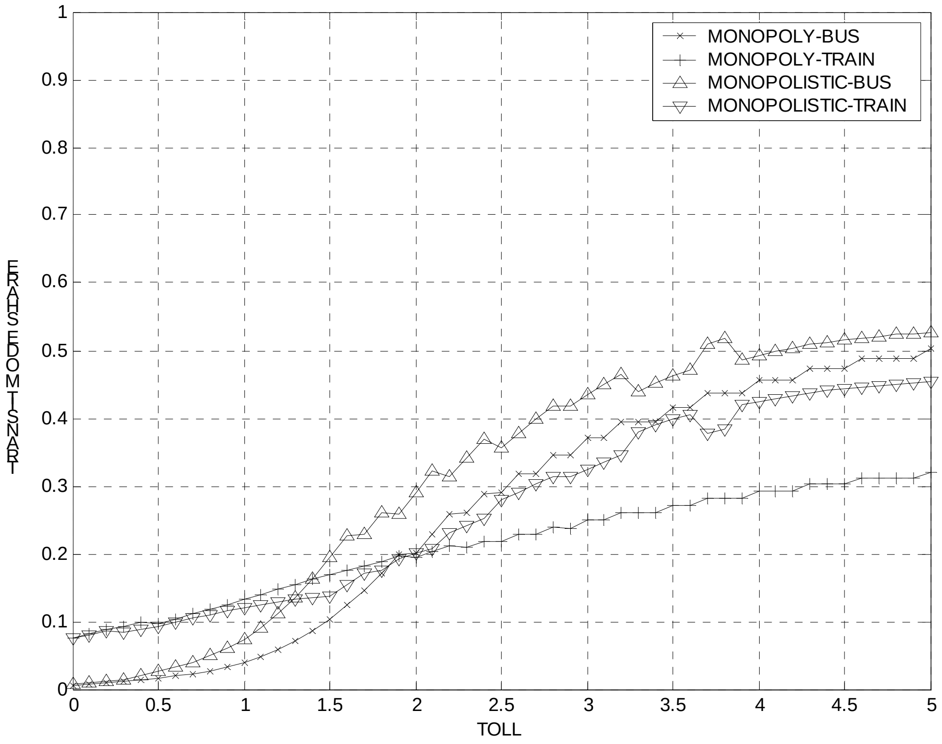

Fig. 7: Changes in transit mode share with toll

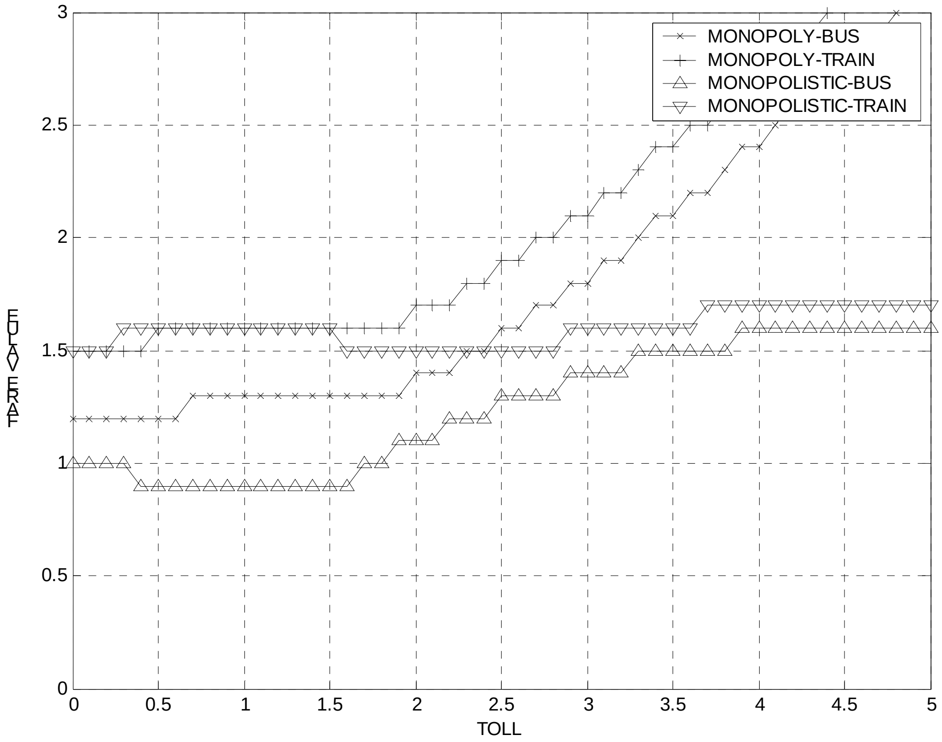

Fig. 8: Changes in transit fare with toll

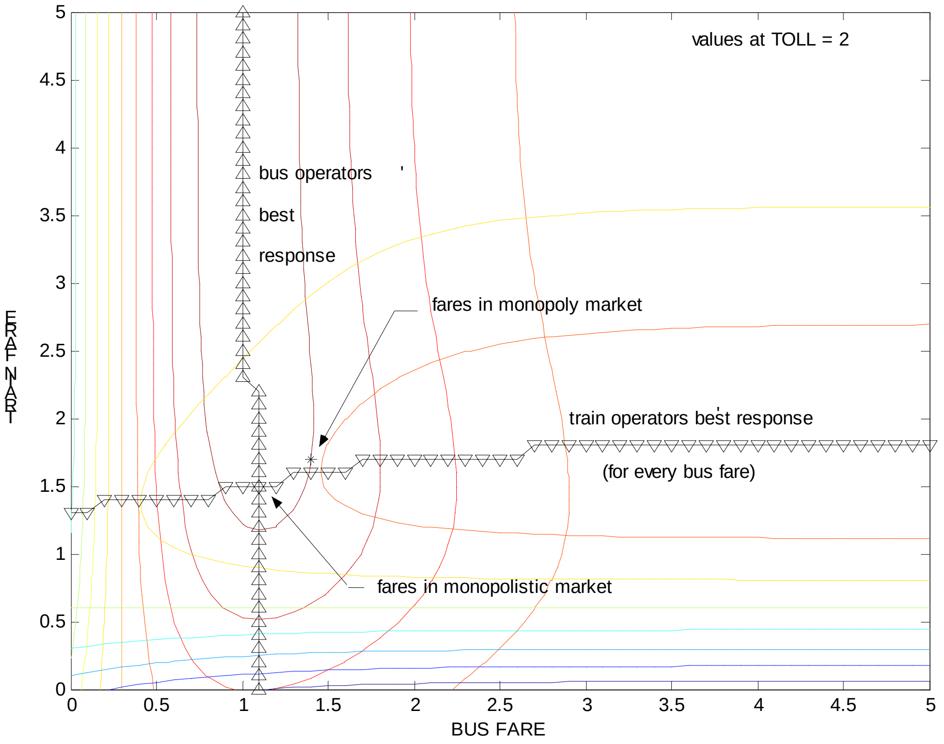

Fig. 9: Optimal fares and contour lines of bus and train operators' profit at toll =£2

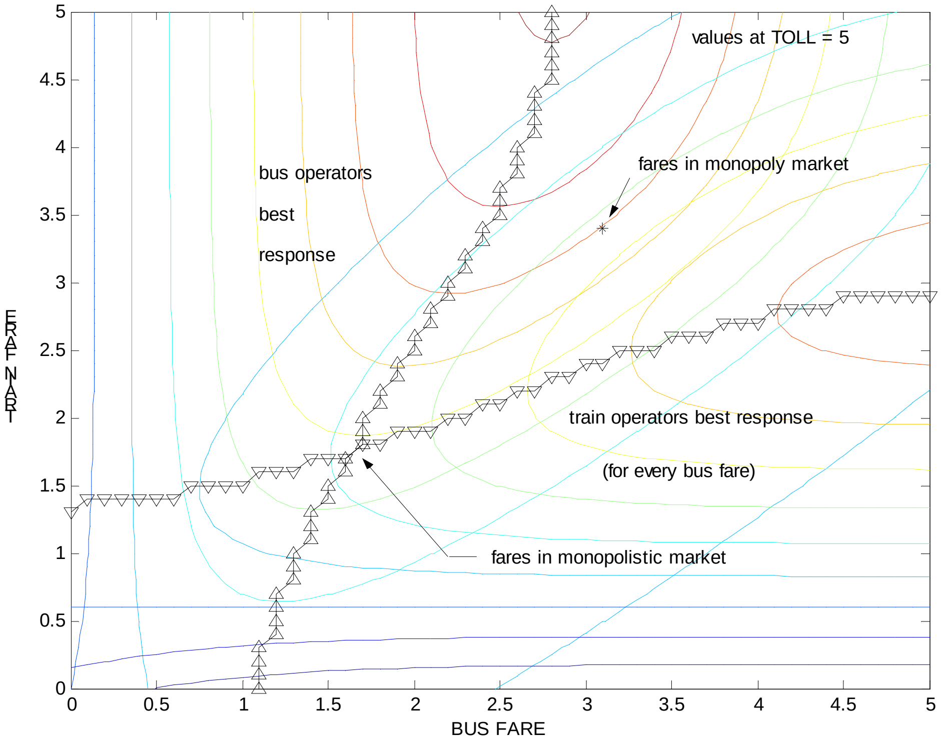

Fig. 10. Optimal fares and contour lines of bus and train operators' profit at toll =£5

Figs. 9 and 10 show the optimal fares in monopoly and monopolistic markets. The contour lines indicate the profit for each operator. Inner contours represent higher profit. At a Nash equilibrium, the tangents to the intersecting contour lines are horizontal and vertical, and therefore at right-angles to each other. The best response curve of each player runs through the contours where the tangents are horizontal, in the case of the bus operator, and vertical, in the case of the rail operator. The intersection of the best response curves defines the Nash equilibrium. At the monopoly solution, the profit contours share the same tangent. As expected, the fares are higher in the monopoly market. Comparing Figs. 9 and 10 shows that monopoly fares increase with the toll. For the monopolistic market, the figures show that as the toll increases, the bus-train game which determines the optimal fares leads to higher fares, although these fares are not as high as those in a monopoly market.

4. CONCLUSIONS

The problem of determining the optimum congestion charge should take the response of the transit market into account. This paper has shown that the problem can be formulated as a bilevel programme. In the model, total demand, mode shares and transit fares are computed endogenously as the congestion charge that maximizes social surplus is sought. The results suggest the effect of the congestion charge on transit fares depends to some degree on the nature of the transit market. It can be concluded that the monopolistic transit market is more beneficial in general than a monopoly transit market as it leads to a higher consumer surplus, although the social surplus may be lower for tolls above the optimum. This benefit comes from competition within the transit market. Whenever a congestion charge scheme is to be introduced it is important to inject competition into the transit market to reduce the ability of transit operators to convert consumer surplus into profit.

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