Evaluation Of Roundabout Entries Reliability

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Synopsis

This paper presents criteria to evaluate roundabout performance reliability. After introducing and justifying the adoption of reserve of capacity and rate of capacity as performance functions, the discussion is developed using a general calculation criterion in which the values that are involved in the limit state service condition – traffic demand and entry capacity – are random variables described by their probability density functions, that is to say by their distribution functions.
A lower level criteria is then identified with which, on the basis of the estimation of suitable statistics of the performance function, a reliability index is calculated that can be compared to a prefixed reference value.
Using a set of numerical applications performed on the basis of the adoption of some of the methods for capacity calculation that are more largely used in the technical practice, the criteria elaborated are concretely exemplified.

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INTRODUCTION

A roundabout is a particular at grade intersection that consists of a central area (known as central island) surrounded by a circle (known as circulatory roadway) that accommodates one-way counter clockwise traffic coming from some legs.
Although adopted as a measure to improve circulation for the first time in France at the beginning of the 20^{th} century, roundabouts have become largely and rapidly popular in Great Britain where left driving with priority-to-the-right (that is, to the traffic in the circulatory roadway) has favored their good performance even with high traffic volumes.
In the 1980s, an innovative decision was taken in Europe: the circulation rule was changed and priority was given to the flow in the circulatory roadway as regards to entering vehicles (known as "priority-to-the-circle" rule or as "off-side" priority).
Thus, it was possible to obtain an increase in the total capacity of the intersection in spite of the reduction of its dimensions, with an increase in the safety level due to speed reduction. For these reasons, since the 1980s a great deal of roundabouts have been constructed in numerous Countries (besides England, in France, Germany, Switzerland, Holland, Scandinavian Countries, and, a little later, in Italy and in the USA).
Overlooking the geometric dimensions that are currently easily available (FEDERAL HIGHWAY ADMINISTRATION, 2000), the evolution of the criteria related to performance aspects, which are of interest in this paper, is briefly reported.
At the beginning, roundabouts were designed as a series of exchange areas situated along the sections of the circle between two consecutive legs that were assimilated to typical exchange areas: this procedure led to very big roundabouts and it hindered their diffusion.
The change of the circulation rule made it possible to interpret the performance, in spite of the presence of exchange maneuvers, as a series of particular T intersections with “off-side” priority.
The evaluation of the maximum number of vehicles at a given period of time (generally, one hour) that can enter from a certain leg (simple capacity) has been the object of numerous theoretical, experimental and simulation studies.
A total capacity can also be defined: it is the sum of the entering flow values from each leg, when these flows are simultaneously the capacity ones.
Capacity formulations that are available (see next paragraph “Reliability evaluation procedures”) today can be classified into three types as follows:
(1) the configuration is characterized only by the number of lanes at the entries and on the circulatory roadway (SETRA/CSTR,1987);
(2) the roundabout geometric layout is taken into account in a more or less detailed way (KIMBER R.M., 1980);
(3) the users’ behavior through the psychotechnical times T_c (critical gap) e T_s (follow-up time) is taken into account (T.R.B., 2000) (WU N., 1997).
Nowadays these procedures are also largely adopted in textbook (ESPOSITO T., MAURO R., 2003).
Currently, in the field of current design procedures, the capacity formulations are implemented using as input the traffic flows related to suitable levels of traffic demand.
These flows, that is, coincide with a pre-established rush hour.
This procedure does not take into account random variations in these flows due to the nature of the circulatory phenomenon or to the lack of information about the flows.
It follows that the determination of the entry capacities – which represent the output of the calculation procedures – that are obtained are only mean values of probability distributions.
With the procedure presented in this paper it is instead possible to take into account random variations in traffic demand and to obtain the probabilistic characterization of the capacities of the roundabout entries, which are calculated through the capacity formulations.
This makes it possible to evaluate roundabout performance reliability.

RELIABILITY AND PERFORMANCE FUNCTIONS

Consider a roundabout entry of capacity C, interested by an entering traffic demand Q_e.
To evaluate the reliability – intended as the value “Z” of the suitability of a system to perform its function – it is natural to compare the values C and Q_e.
The “Z” value, that is to say “performance function”, can be measured both with the difference $C-Q_e$, and with the ratio C/Q_e, which in the reliability theory terminology (KOTTEGODA N., ROSSO R., 1997) are indicated

as reliability margin and reliability factor respectively:

$$Z = C - Q_e \tag{1}$$

$$Z = C/Q_e \tag{2}$$

Eq.(1) and Eq.(2) are connected to the two most widely adopted roundabout capacity indexes used to characterize service conditions: in fact, Eq.(1) coincides with the reserve of capacity (RC), whereas Eq.(2) is the reciprocal of the rate of capacity when it is expressed in absolute terms.

Therefore, once prefixed two given minimum values z_{min} and z'_{min} for the reliability margin and reliability factor, the reliability condition of the system can be written:

$$Z = C - Q_e > z_{min} \tag{3}$$

$$Z = C/Q_e > z'_{min} \tag{4}$$

In particular, in the former value the limit $z_{min} = 0$ can be taken and in the latter $z'_{min} = 1$, so that the success condition is represented by

$$C - Q_e > 0 \tag{5}$$

$$C/Q_e > 1 \tag{6}$$

The complementary relations of Eq.(5) and Eq.(6) represent failure conditions:

$$C - Q_e < 0 \tag{7}$$

$$C/Q_e < 1 \tag{8}$$

What has been expounded so far is the deterministic position of the problem.

In reality, because of the random nature of the factors and relations on which traffic capacity and traffic demand values depend, the previous values C and Q_e can vary randomly.

Therefore, considering C and Q_e as random variables described by given probability laws or by adequate statistics, the above-mentioned relations must be suitably modified to take into account this circumstance.

In particular, the values "Z" that are yielded by Eq.(1) and by Eq.(2) are also to be considered, insofar as they are random variable functions, as random values.

Also, it should be considered that each entering traffic demand value Q_e at an entry of capacity C corresponds to a reliability value determination.

Assume, then, in the more general case, that the probability law of the random variable "Z" (see Eq.(1) and Eq.(2)) can be represented by the probability density function, p.d.f.,) f_Z (see Figure 1).

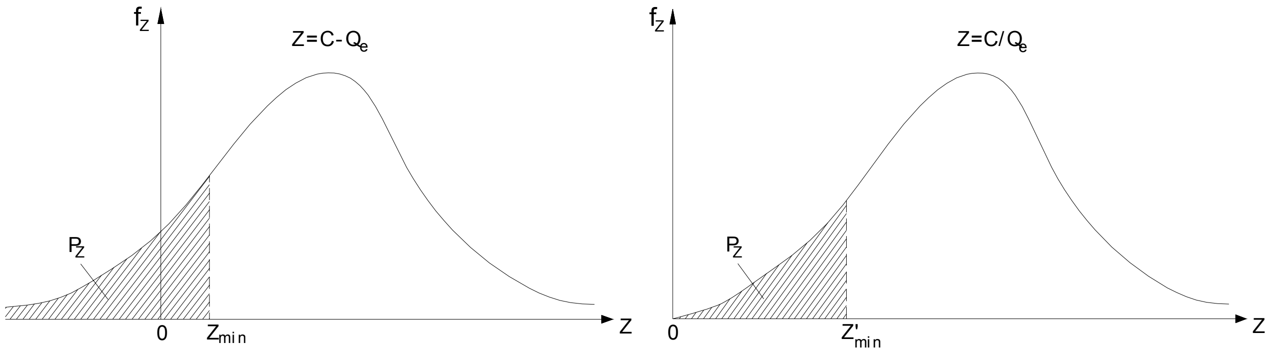


Figure 1: p.d.f. examples of the performance function (random variable Z)

On the basis of P_Z probability, the fractiles z_{min} and z'_{min} can be determined: this equals to let

$$P_Z = P\{Z = C - Q_e \le z_{min}\} = F_Z(z_{min}) \tag{9}$$

$$P_Z = P\{Z = C/Q_e \le z'_{min}\} = F_Z(z'_{min}) \tag{10}$$

where $F_Z(Z)$ are the distribution functions (c.d.f.) of the random variable "Z" in these two cases, $Z=C-Q_e$ e $Z= C/Q_e$.

P_Z is then the probability of the event {entry unreliability}, given that z_{min} and z'_{min} are minimum values prefixed by the reliability value.

The complimentary event {entry reliability} obviously results in

$$1\text{-}P_Z = P\{Z = C - Q_e > z_{min}\} = 1 - F_Z(z_{min}) \tag{11}$$

$$1\text{-}P_Z = P\{Z = C/Q_e > z'_{min}\} = 1 - F_Z(z'_{min}) \tag{12}$$

The "failure" event and the complementary "success" event are then obtained respectively (see Figure 2):

$$P_f = P\{Z = C - Q_e \le 0\} = F_Z(0) \quad \text{failure} \tag{13}$$
$$P_f = P\{Z = C/Q_e \le 1\} = F_Z(1) \tag{14}$$
$$A = 1\text{-}P_f = P\{Z = C - Q_e > 0\} = 1 - F_Z(0) \quad \text{success} \tag{15}$$
$$A = 1\text{-}P_f = P\{Z = C/Q_e > 1\} = 1 - F_Z(1) \tag{16}$$

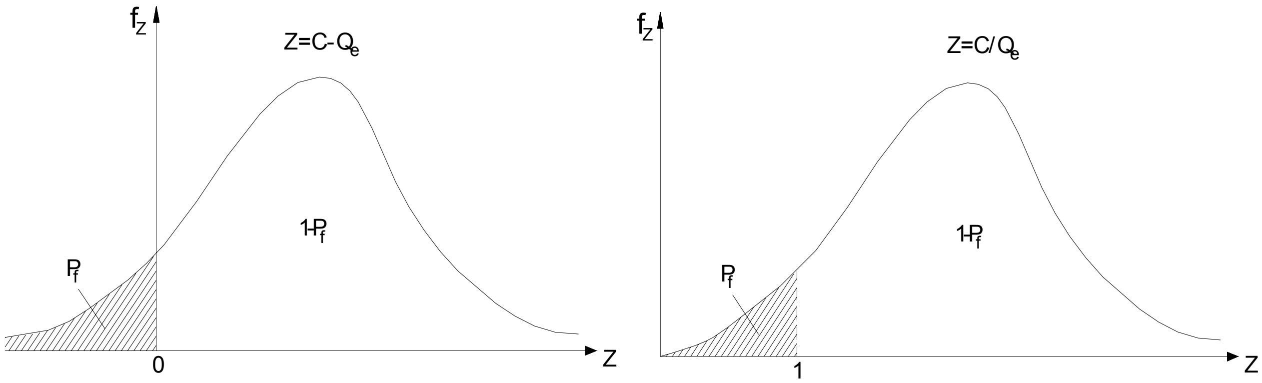


Figure 2: Examples of the identification of the probability of “failure” and “success” events

From now on, by reliability A it will be briefly indicated the probability $1\text{-}P_f$ associated with the “success” event. In particular, with reference to the performance variable $Z = C\text{-}Q_e$, if $f_{CQe}(c,q_e)$ is the combined p.d.f of C and Q_e, for Eq.(13) it results:

$$P_f = P(Z \le 0) = \int_D f_{CQ_e}(c, q_e)\,dc\,d\,q_e \tag{17}$$

In Eq.(17), D is the unsafe region, where, that is to say, the performance function $Z = C\text{-}Q_e$ takes values $Z \le 0$ (see Figure 3 for $(C, Q_e) \ge 0$). In other words, according to Eq.(17), the volume subtended by $f_{CQe}(c,q_e)$ in correspondence with the region D gives the value P_f.

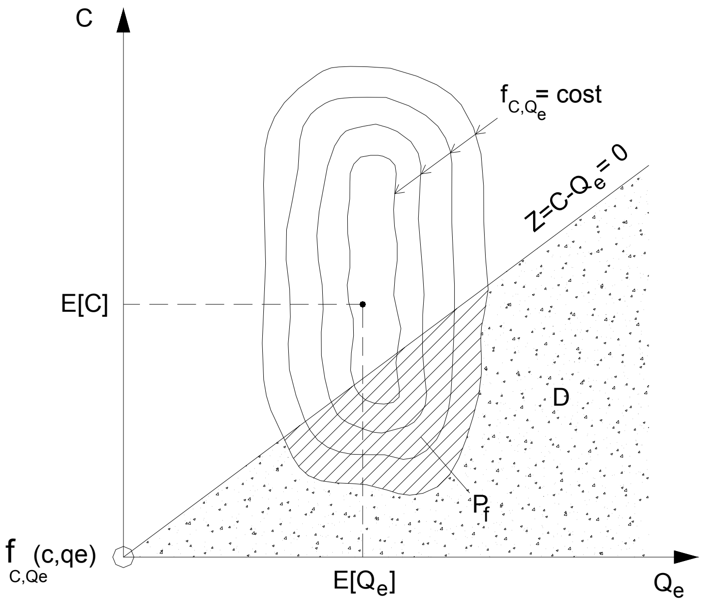


Figure 3: Graphic identification of the unsafe region

In conclusion, the problem of the evaluation of a roundabout reliability refers, in its general formulation, to the thorough probabilistic characterization of the values that the entry flows Q_e can take and to the entry capacity C.

However, as it will be demonstrated in the following discussion, when this thorough characterization is not available, Level 1 reliability methods can be performed on the basis of statistical estimations of the expected value and of the standard deviation of the random variables Q_e and C, that is to say using only one measurement of the random variability of the values in question (the standard deviations $s_{Q_e} = \sqrt{VAR[Q_e]}$ and $s_C = \sqrt{VAR[C]}$, once the mean values $E[Q_e]$ and E[C] are known).

RELIABILITY EVALUATION PROCEDURES

In general, the capacity C of a roundabout entry can be expressed as (BRILON W., 1988) (BRILON W, 1991)

$$C = C(\tilde{G}; Q_d; \tilde{\tau}; \tilde{S}) \tag{18}$$

where

$\tilde{G}$ is a set of variables representing the geometrical layout of the intersection (for instance, approach width, external roundabout diameter, etc.) or its configuration (number of lanes of roundabout carriageway, number of roundabout entry lanes, etc.);

Q_d is the impeding flow on the circulatory roadway that blocks the entering traffic; Q_d is a function of the entering flows $\tilde{Q}_e$ from the other entries;

$\tilde{\tau}$ represents psychotechnical times, relating to users' behavior at the intersection (i.e. critical gap, follow-up time);

$\tilde{S}$ are numerical constants resulting from the model calibration, the analytical development of the formulation, the relations among the units of measurement of the variables adopted, etc.

In the capacity equations available not all the above-mentioned variables, except for impeding flow, are present, but according to the model type, some of them can be present instead of others.

As we shall see later, in the reliability calculation procedures, it is necessary to determine the probability law or some moments of the entry capacity.

On the basis of Eq.(18), C can be expressed as a function of random variables, and therefore, the determination of C distribution or of the above-mentioned moments is carried out with the results of Calculus Probability once the above-mentioned random variables are suitably characterized probabilistically.

The solution to some problems of this type is reported in the numerical examples given below.

Finally, as it is easily understandable, since for a given intersection the uncertainties connected to a given geometrical layout can be neglected with respect to the ones associated with traffic demand or users' psychotechnical features, from now on, C randomness will be exclusively considered as a function of the random variables of the sets $\tilde{Q}_e$ and $\tilde{\tau}$.

General Calculation Procedure

Adopting for a common entry as performance variable Eq.(1), $Z = C - Q_e$, if C and Q_e are random variables that are statistically independent, so with the calculation rules for double integrals from Eq.(17) (see Appendix 1) for P_f the two following equivalent expressions are obtained

$$P_f = \int_{-\infty}^{+\infty} f_C(c)[1 - F_{Qe}(c)]dc$$
$$P_f = \int_{-\infty}^{+\infty} f_{Qe}(q) F_C(q)dq \tag{19}$$

where $f_c(\cdot)$ and $f_{Qe}(\cdot)$ are the p.d.f. respectively of C and Q_e , and $F_c(\cdot)$ and $F_{Qe}(\cdot)$ are the c.d.f. respectively of C and Q_e.

From Eq.(19), if C and Q_e are normally distributed, for P_f it results (KOTTEGODA N., ROSSO R., 1997)

$$P_f = \frac{1}{2\pi} \int_{-\infty}^{-\beta} e^{-t^2/2} dt = 0{,}5 - \text{erf}(\beta) \tag{20}$$

where erf(·) is the error function and β is the safety index (inverse of the coefficient of variation of the performance function Z)

$$\beta = E[Z]/\sqrt{VAR[Z]} \tag{21}$$

with E[Z] and $\sqrt{VAR[Z]}$ as the expected value and the standard deviation of the performance function Z respectively (see Eq.(1)).

If C is normally distributed with mean $\overline{C}$ and variance σ_c^2 and Q_e is distributed, for example, exponentially with parameter α, for P_f it results

$$P_f = F\left(-\frac{\overline{C}}{\sigma_c}\right) + \exp\{0{,}5\alpha^2\sigma_c^2 - \alpha\overline{C}\}\left[1 - F\left(-\frac{\overline{C}}{\sigma_c} + \alpha\sigma_c\right)\right] \tag{22}$$

where F(·) is the c.d.f. of the standardized normal distribution.

A Numerical Example

For this example, the capacity formulation of SETRA (SETRA/CSTR, 1987) is adopted.

In Figure 4 it is schematically reported a roundabout with flow indications and geometrical features for the calculation of C capacity.

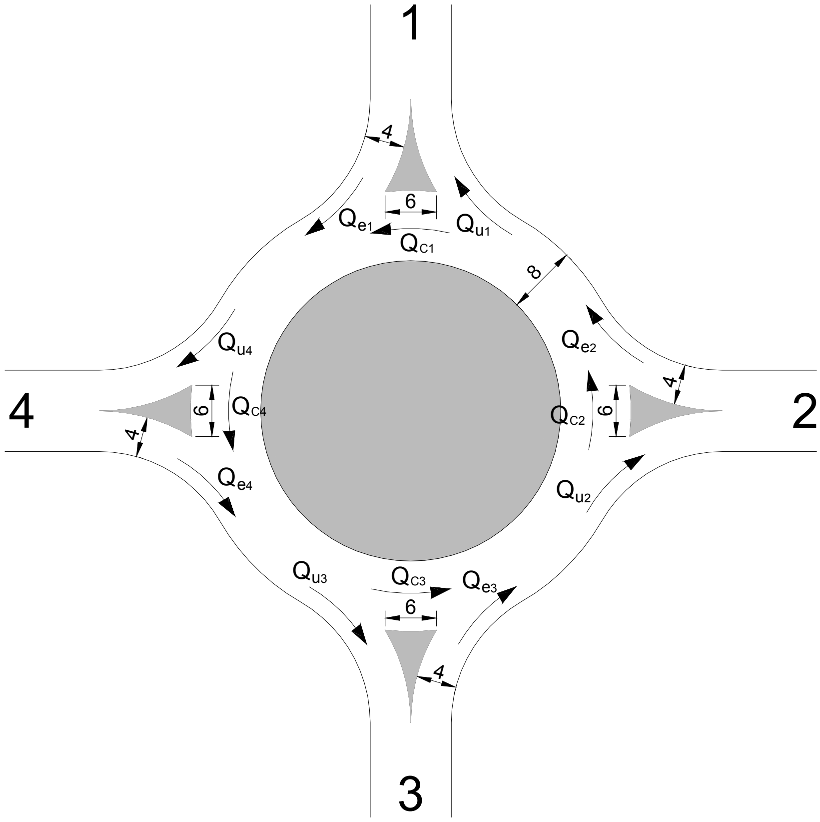


Figure 4: Geometrical layout of the roundabout

We have, for all branches:
- entry width ENT = 4.00 m
- splitter island SEP = 6.00 m

The circulatory roadway width is ANN = 8.00 m.

For an entry capacity "i" the formulation selected gives

$$C = (1330 - 0.7 \cdot Q_{di}) \cdot [1 + 0.1 \cdot (ENT - 3.50)] \tag{23}$$

where the impeding flow Q_{di} is

$$Q_{di} = \left(Q_{ci} + \frac{2}{3} Q'_{ui} \right) [1 - 0.085 \cdot (ANN - 8)] \tag{24}$$

with Q_c circulating traffic on the circulatory roadway at the level of the branch considered and where Q_{ui} is the exiting traffic at the level of the branch i selected.

$$Q'_{ui} = Q_{ui} \frac{15 - SEP}{15} \tag{25}$$

With the given values of ENT, SEP and ANN, Eqs. (23), (24) and (25) yield

$$C = 1397 - 0.735 \cdot Q_{di} \tag{26}$$

with

$$Q_{di} = Q_{ci} + 0.4 \cdot Q_{ui} \tag{27}$$

supposing that, on the basis of experimental observations, mean flow $E[Q_{ij}]$ and variance $VAR[Q_{ij}]$ of all of the turnings have been estimated and that these flows result in being statistically independent.

These data are reported in the matrix O/D of Table 1. In Table 1 in each square ij the top value is the mean, and the bottom one is the variance associated with the turning from branch i into branch j. Table 1 also shows the estimated values of the mean and variance for the overall flows for each branch at entry Q_{ei} and at exit Q_{ui}. The flows are expressed in pcu/h.

Table 1: Matrix O/D of the mean values $E[Q_{ij}]$ in (pcu/h) and variance $VAR[Q_{ij}]$ in (pcu/h)^2

C \ D	1	2	3	4	Q_{ei}
1	-	200	150	250	600
	-	2500	2000	2200	6700
2	300	-	100	150	550
	6400	-	1800	900	9100
3	300	200	-	100	600
	3000	2500	-	1800	7300
4	150	350	200	-	700
	1500	9000	2500	-	13000
Q_{ui}	750	750	450	500	-
	10900	14000	6300	4900	-

Mean and Variance of the circulating flows Q_{ci}

The circulating flows Q_c are linear functions of traffic demand expressed as turning flows. For example, for entry 1 it results in $Q_{c1} = Q_{24} + Q_{23} + Q_{34}$.

On the basis of the properties of mean E[·] and variance VAR[·] operators when applied to random variables statistically independent and with the data of Table 1, it gives

$$E[Q_{c1}] = E[Q_{24}] + E[Q_{23}] + E[Q_{34}] = 150 + 100 + 100 = 350 \text{ uvp/h} \quad (28)$$

$$VAR[Q_{c1}] = VAR[Q_{24}] + VAR[Q_{23}] + VAR[Q_{34}] = 900 + 1800 + 1800 = 4500 \text{ (uvp/h)}^2 \quad (29)$$

The coefficient of variation $(cv)_1$ is

$$(cv)_1 = \sqrt{VAR[Q_{c1}]}/E[Q_{c1}] = 67/350 = 0.192 \quad (30)$$

Doing the same for the other entries, the values of the mean and variance for the circulating flows in Table 2 can be obtained.

Mean and variance of impeding flows Q_{di}

For entry 1 Eq.(27) gives $Q_{d1} = Q_{c1} + 0{,}4 \cdot Q_{u1}$ where $Q_{u1} = Q_{21} + Q_{31} + Q_{41}$.

On the basis of the above-mentioned properties of the mean and variance operators and with the values of Table 1, it results in:

$$E[Q_{d1}] = E[Q_{c1}] + 0{,}4 \cdot (E[Q_{21}] + E[Q_{31}] + E[Q_{41}]) = 350 + 0{,}4 \cdot (300 + 300 + 150) = 650 \text{ uvp/h} \quad (31)$$

$$\begin{aligned} VAR[Q_{d1}] &= VAR[Q_{c1}] + \overline{0{,}4}^2 \cdot (VAR[Q_{21}] + VAR[Q_{31}] + VAR[Q_{41}]) = \\ &= 4500 + 0{,}16 \cdot (6400 + 3000 + 1500) = 6244 \text{ (uvp/h)}^2 \end{aligned} \quad (32)$$

In this case, the coefficient of variation equals to

$$(cv)_1 = \sqrt{VAR[Q_{d1}]}/E[Q_{d1}] = 79/650 = 0.122 \quad (33)$$

Repeating the calculation for the other entries, the impeding flow statistics reported in Table 2 can be determined.

Table 2: Statistics of circulating flows, impeding flows, and entry capacities. Safety index and reliability index values.

Ingresso	$E[Q_{ci}]$ (uvp/h) / $VAR[Q_{ci}]$ (uvp/h)^2	$E[Q_{di}]$ (uvp/h) / $VAR[Q_{di}]$ (uvp/h)^2	$E[C_i]$ (uvp/h) / $VAR[C_i]$ (uvp/h)^2	Average Capacity Reserve $E[C]-E[Q_e]$ (uvp/h)	β_i	A
1	350	650	920	320	3.19	0.999
	4500	6244	3373			
2	550	850	772	222	1.90	0.971
	6300	8540	4613			
3	700	880	750	150	1.23	0.897
	13000	14008	7567			
4	450	650	919	219	1.55	0.939
	6300	7084	7084			

Mean and variance of flow capacities C_i

For entry 1, with Eq.(26), since the relation between C_i and Q_{di} is linear and with the above-mentioned values calculated for Q_{di} moments, it is obtained

$$E[C_1] = 1397 - 0.735 E[Q_{d1}] = 1397 - 0.735 \cdot 650 = 920 \text{ uvp/h} \quad (34)$$

$$VAR[C_1] = \overline{0.735}^2 \cdot 6244 = 3373 \text{ (uvp/h)}^2 \quad (35)$$

$$(cv)_1 = 58/920 = 0.063 \quad (36)$$

For the other entries, the capacity moments reported in Table 2 can be determined in the same way.

Reliability calculation

For entry 1, the performance function (1) Z_1 is on average equal to

$$E[Z_1] = E[C_1] - E[Q_{e1}] = 920 - 600 = 320 \text{ uvp/h}$$

$$VAR[Z_1] = VAR[C_1] + VAR[Q_{e1}] = 3373 + 6700 = 10073 \text{ (uvp/h)}^2$$

It follows that $\beta_1 = \dfrac{E[Z_1]}{\sqrt{VAR[Z_1]}} = \dfrac{320}{\sqrt{10073}} = 3.19$

and thus $\text{erf}(\beta_1) = \text{erf}(3.19) = 0.499$.

With Eq.(20) reliability A for entry 1 on the basis of Eq.(15) equals to

A = 0.5 + erf(3.19) = 0.999.

Table 1 shows the reliability values calculated for the remaining entries to the roundabout considered. The present example has been carried out using the hypotheses of mutual statistical independence among entering flows. If this circumstance does not occur, for statistically dependent flows it must also be taken into account their covariance cov[(·);(·)], as shown below.

With reference to, for example, entry 1, suppose that the elaboration of experimental data traffic shows the mutual statistical dependence among the turning flows Q_{24}; Q_{23}; Q_{34} and among Q_{21}; Q_{31}; Q_{41}, so that to have, for the covariances, the values of Table 3 (the flows Q_{ij} are expressed in pcu/h).

Table 3: Values of the cov[(Q_i);(Q_j)] in (pcu/h)^2

Q_i \ Q_j	Q_{23}	Q_{31}	Q_{34}	Q_{41}
Q_{21}		3500		2000
Q_{31}				1800
Q_{23}			1800	
Q_{24}	1000		1100	

While $E[Q_{c1}]$ remains equal to Eq.(28) the $VAR[Q_{c1}]$ is, with the values of Tables 1 and 3, with respect to Eq.(29), thus modified:

$$\begin{aligned} VAR[Q_{c1}] &= VAR[Q_{24}] + VAR[Q_{23}] + VAR[Q_{34}] + 2(\text{cov}[Q_{24};Q_{23}] + \text{cov}[Q_{24};Q_{34}] + \text{cov}[Q_{23};Q_{34}]) = \\ &= 900 + 1800 + 1800 + 2 \cdot (1000 + 1100 + 1800) = 12300 \text{ (uvp/h)}^2 \end{aligned} \quad (29')$$

For the impeding flow Q_{d1} it is obtained, for the mean $E[Q_{d1}]$, the same value yielded by Eq.(31), while for the calculation of $VAR[Q_{d1}]$ it is necessary, together with the covariances of Table 3, to know the cov[(Q_{c1});(Q_{u1})].

Suppose that for cov[(Q_{c1});(Q_{u1})] it results, on the basis of traffic measurement treatment, cov[(Q_{c1});(Q_{u1})]=13000 (pcu/h)^2. With the values of Table 1 and Table 3, it is obtained:

$$\begin{aligned} VAR[Q_{u1}] &= VAR[Q_{21}] + VAR[Q_{31}] + VAR[Q_{41}] + 2(\text{cov}[Q_{21};Q_{31}] + \text{cov}[Q_{21};Q_{41}] + \text{cov}[Q_{31};Q_{41}]) = \\ &= 6400 + 3000 + 1500 + 2(3500 + 2000 + 1800) = 25500 \text{ (uvp/h)}^2 \end{aligned} \quad (37)$$

$$\begin{aligned} VAR[Q_{d1}] &= VAR[Q_{c1}] + \overline{0{,}4}^2 \cdot VAR[Q_{u1}] + 2 \cdot 0{,}4 \cdot \text{cov}[Q_{c1};Q_{d1}] = \\ &= 12300 + 0{,}16 \cdot 25500 + 2 \cdot 0{,}4 \cdot 13000 = 26780 \text{ (uvp/h)}^2 \end{aligned} \quad (38)$$

With Eq.(38) and with Eq.(31), it is obtained, on the basis of Eq.(23), for entry 1 capacity:

$$E[Q_{c1}] = 1397 - 0{,}735 E[Q_{d1}] = 1397 - 0{,}735 \cdot 650 = 920 \text{ uvp/h} \quad (39)$$

$$VAR[Q_{c1}] = \overline{0{,}735}^2 \cdot VAR[Q_{d1}] = \overline{0{,}735}^2 \cdot 26780 = 14467 \text{ (uvp/h)}^2 \quad (40)$$

In conclusion, with $E[Q_{e1}]$= 600 pcu/h (see Table 1) for the performance function Z_1 it results:

$$E[Z_1] = E[C_1] - E[Q_{e1}] = 920 - 600 = 320 \text{ uvp/h} \quad (41)$$

$$VAR[Z_1] = VAR[C_1] + VAR[Q_1] = 14467 + 6700 = 21167 \text{ (uvp/h)}^2 \quad (42)$$

Thus

$$\beta_1 = \frac{E[Z_1]}{\sqrt{VAR[Z_1]}} = \frac{320}{145.5} = 2.20 \quad (43)$$

and for reliability A it results (see Eqs.(15) and (20))

A = 0.5 + erf(β) = 0.5 +erf (2.20) = 0.986. (44)

APPROXIMATED METHOD

This procedure is exclusively based on the knowledge of the mean value and on only one measurement of the random variability (generally, variance or standard deviation) of the values involved: it can thus be considered an approximated approach to evaluate reliability compared to the criterion illustrated in the previous paragraph.
Keeping in mind the meaning and the definition of mean value and variance, this method could be called the two-moment method. The use of this approximated method requires the introduction of a safety index β which can be defined using the performance function Z. This index is the number of standard deviations that separate the mean value Z from the value Z = 0 which – by definition – corresponds to the failure limit. This method is structured in this way.
Calculate the mean and variance statistics E[Z] and VAR[Z] of the performance variable (1)

$$Z = C - Q_e \tag{1}$$

starting from the known homologues of Q_e and of C ($E[Q_e]$; $VAR[Q_e]$; [E[C];VAR[C]) with the relation (see Figure 5)

$$E[Z] - \beta s_Z = 0 \tag{45}$$

index β is calculated, and it provides the number of standard deviations $s_Z = \sqrt{VAR[Z]}$ of Z that separate the mean value E[Z] from the value Z=0, corresponding by definition to the limit that marks the failure condition $(Z = 0 \leftrightarrow C = Q_e \ \ \forall Q_e; C \neq 0)$.

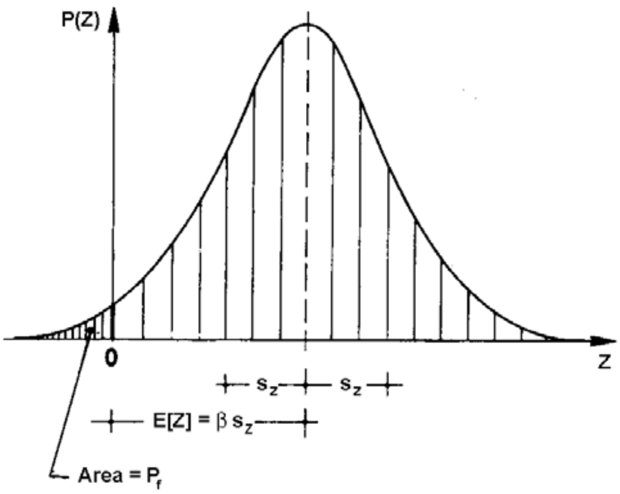


Figure 5: Reliability index β and p.d.f. of the performance function Z

By introducing the normalized performance function as

$$\xi = \frac{Z - E[Z]}{s_Z} \tag{46}$$

the “success” and “failure” events are, respectively, equivalent to the occurrence of the inequalities:
- <u>success</u>

$$\xi \geq -\beta \tag{47}$$

- <u>failure</u>

$$\xi < -\beta \tag{48}$$

In fact, putting in Eq.(13) a value of $\xi \geq -\beta$ yields Z, $Z \geq 0$, that is to say that $C \geq Q_e$, while, substituting $\xi < -\beta$ yields Z < 0, which equals to $C < Q_e$.
If Z probability law, that is ξ, is known, each limit of β corresponds to a well-determined value of the failure probability

$$P_f = P(\xi < -\beta) = F(-\beta) \tag{49}$$

$$A = 1 - P_f = 1 - F(-\beta) \tag{50}$$

Even though the law distribution of the performance function Z is not known or easily determinable, β can be considered as a coherent reliability value. In fact, the tail of the most common probability density functions can be adequately approximated with an exponential function (suitably identified with two parameters A and b), (A>0, b>0),

$$F(\xi) = A \cdot \exp(b \cdot \xi) \quad \text{se} \quad F(\xi) =<< 1 \tag{51}$$

For example, if Q_e and C are both lognormally distributed, it can be demonstrated that Eq.(51) becomes

$$F(-\xi) = 460 \cdot \exp(-4{,}3\beta) \tag{52}$$

and, thus,

$$A = 1 - F(-\xi) = 1 - 460 \cdot \exp(-4.3 \cdot \beta) \tag{53}$$

It follows that as β increases, reliability also increases. With the most frequent f.d.p. the values of β at least equal to 2 always indicate high probabilities that Q_e is systematically smaller than C, that is to say that the entry does not become saturated.

A numerical example

Suppose one adopts as a capacity formulation the one presented in the 2000 edition of the H.C.M. (T.R.B., 2000) on the basis of which

$$C = \frac{Q_c \exp\{-Q_c T_c/3600\}}{1-\exp\{-Q_c T_f/3600\}} \text{ (uvp/h)} \tag{54}$$

For an assigned value of Q_c it results

$$C = C(T_c; T_f) \tag{55}$$

where T_c is the critical gap (sec) and T_f is the follow-up time (sec).
Assume that Q_c is known without doubt and that T_c and T_f are instead random variables.
Using the linearization of Eq.(54) it can be demonstrated (BENJAMIN R., CORNELL C.A.,1970) that an approximated evaluation of the first order of E[C] and VAR[C] is yielded by

$$E[C] = C(E[T_c]; E[T_f]) = Q_c \cdot \frac{\exp\{-Q_c E[T_c]/3600\}}{1-\exp\{-Q_c E[T_f]/3600\}} \text{ (uvp/h)} \tag{56}$$

$$VAR[C] = k_1^2 VAR[T_c] + k_2^2 VAR[T_f] + 2 \cdot k_1 \cdot k_2 \cdot cov[T_c; T_f] \tag{57}$$

where $E[T_c]$; $E[T_f]$; $VAR[T_c]$; $VAR[T_f]$; $cov[T_c;T_f]$ have the usual meaning and for k_1 and k_2 it results

$$k_1 = \left(\frac{\partial C}{\partial T_c}\right)_{E[T_c];E[T_f]} = -\frac{Q_c^2}{3600} \cdot \frac{\exp\{-Q_c E[T_c]/3600\}}{1-\exp\{-Q_c E[T_f]/3600\}} \tag{58}$$

$$k_2 = \left(\frac{\partial C}{\partial T_f}\right)_{E[T_c];E[T_f]} = -\frac{Q_c^2}{3600} \cdot \frac{\exp\{-Q_c (E[T_c] - E[T_f])/3600\}}{(1-\exp\{-Q_c E[T_f]/3600\})^2} \tag{59}$$

Assume therefore, for an entry for which Q_c = 250 pcu/h, the following values of the statistics of the random variables T_c and T_f :

$E[T_c]$ = 4.4 sec	$VAR[T_c]$ = 0.36 sec^2	$cov[T_c; T_f]$ = 0.30 sec^2
$E[T_f]$ = 2.9 sec	$VAR[T_f]$ = 0.25 sec^2	

With these values and with $C=C(T_c, T_f)$ given by Eq.(54), it is obtained for Eqs. (56), (57), (58) and (59)

$$k_1 = \left(\frac{\partial C}{\partial T_c}\right)_{E[T_c];E[T_f]} = -\frac{\overline{250}^2}{3600} \cdot \frac{\exp\{-250 \cdot 4.4/3600\}}{1-\exp\{-250 \cdot 2.9/3600\}} = -70$$

$$k_2 = \left(\frac{\partial C}{\partial T_f}\right)_{E[T_c];E[T_f]} = -\frac{\overline{250}^2}{3600} \cdot \frac{\exp\{-250 \cdot (4.4-2.9)/3600\}}{(1-\exp\{-250 \cdot 2.9/3600\})^2} = -470$$

$$E[C] = 250 \cdot \frac{\exp\{-250 \cdot 4.4/3600\}}{1-\exp\{-250 \cdot 2.9/3600\}} = 1010 \text{ uvp/h}$$

$$VAR[C] = (-70)^2 \cdot 0{,}36 + (-470)^2 \cdot 0.25 + 2 \cdot 0.30 \cdot 32900 = 76729 \text{ (uvp/h)}^2$$

$$\sqrt{VAR[C]} = 277 \text{ uvp/h}$$

The coefficient of variation (cv) is equal, in this case, to

$$(cv) = \sqrt{VAR[C]}/E[C] = 277/1010 = 0.27$$

If $E[Q_e]$ = 450 veic/h and VAR $[Q_c]$ = 1500 $(pcu/h)^2$ it is obtained for the statistics of the performance function (1)

$$E[Z] = E[C] - E[Q_e] = 1010 - 450 = 560 \text{ uvp/h}$$

$$\sqrt{VAR[Z]} = \sqrt{VAR[C] + VAR[Q_e]} = \sqrt{76729 + 1500} = 280 \text{ uvp/h}$$

and thus, for β it results

$$\beta = \frac{560}{280} = 2.00$$

This value of β means high reliability values.

When it seems right to apply to this case the approximation provided by Eq.(52), it is obtained for A, for example, the value (see Eq.(53))

A = 1-460 exp {-4.3 · 2.00} = 0.92

SOME REMARKS

Reliability depends, when the mean of the Capacity Reserve and/or of the Capacity Rate does not change, on the level of uncertainty that affects the values in question (dispersion around the mean values of the flows).

The role of the dispersions centered on the mean values $E[Q_e]$ and $E[C]$ is evident from the observation of the curves $P_f = P_f(\omega_o)$ in Figure 6. They can be obtained as follows.

Expressed β as (see Eq.(21))

$$\beta = \frac{E[C] - E[Q_e]}{\sqrt{VAR[C] + VAR[Q_e]}} = \frac{\omega_o - 1}{\sqrt{\omega_o^2 (cv)_c^2 + (cv)_{Qe}^2}}$$

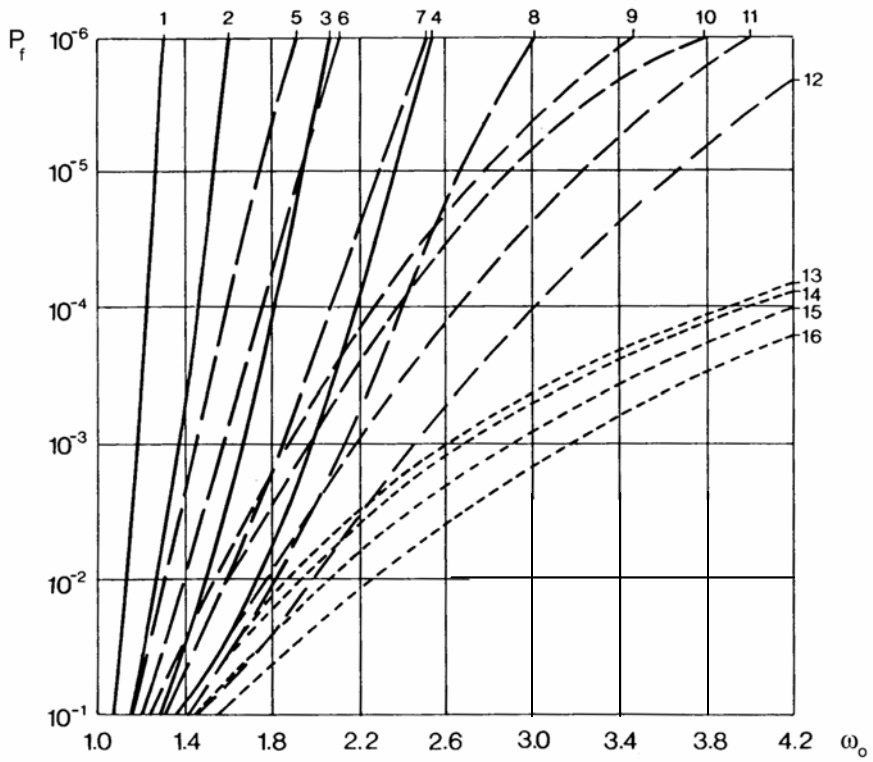
where $(cv)_c = \sqrt{VAR[C]}/E[C]$ and $(cv)_{Qe} = \sqrt{VAR[Q_e]}/E[Q_e]$ are, respectively, the capacity and demand coefficients of variation at an entry and ω_o= E[C]/ E[Q_e] the ratio among the means of the same ones, for Eq.(21) it results in

$P_f = P_f(\omega_o; (cv)_c; (cv)_{Qe})$

Once fixed the values that form the couples ($(cv)_c$; $(cv)_{Qe}$) of the table, with them, from Eq.(20), the 16 curves $P_f = P_f(\omega_o)$ of Figure 6 can be obtained.

Figure 6 shows that:

- for high values of $(cv)_c$, even increasing considerably ω_o, it is not possible to keep the failure probability within small values;
- for small values of $(cv)_c$, the variability of Q_e is significant (this is instead unimportant for big values of $(cv)_c$, that is to say with uncertain capacities).



$(cv)_c$	$(cv)_{Qe}$			
	0	0,1	0,2	0,3
0,05	1	2	3	4
0,10	5	6	7	8
0,15	9	10	11	12
0,20	13	14	15	16

Figure 6: $P_f = P_f(\omega_o)$ for the couples of values ($(cv)_c$; $(cv)_{Qe}$)

CONCLUSION

This paper has highlighted the concept that the indexes normally used to design roundabouts (Capacity Reserve or Capacity Rate at each branch) do not always ensure by themselves an adequate performance of the intersection.
This occurs because the flows of the various branches (and the capacities that depend on them) are random variables. They, in general, are non-statistically independent. Therefore to evaluate Reliability, that is to say the probability that the system does not fail (in the specific case, that demand does not exceed the single branch capacities) it is necessary to characterize the flows and their related values by means of their probability functions or when these laws are not available, by synthetic indexes such as means, variances, and covariances.
This paper presents a general criteria for the evaluation of Reliability in each branch based on the study of the performance function Z ($Z = C\text{-}Q_e$ o $Z = C/Q_e$) and it provides the analytical relations in the particular case in which capacity and demand (and thus also Z) are normally distributed with means and variances known.
An approximated criteria is also provided to be used in the cases where the probability laws of capacities and demands, and thus of performance function Z, are unknown or difficult to determine.
The numerical examples developed to illustrate the method (and the many others that have not been reported for the sake of brevity) show, as it was logical to expect, that the two only indexes normally used (Reserve of Capacity and/or Rate of Capacity) are not sufficient by themselves to ensure that the system does not fail.
Reliability depends, when the mean of the Capacity Reserve and/or of the Capacity Rate does not change, on the level of uncertainty that affects the values in question (dispersion around the mean values of the flows).
Regarding the threshold value to attribute to Reliability, it must be stated that it cannot be fixed in general terms, but it should be identified on a case-to case basis in relation to the damage (excessive mean and global waiting times, safety decrease, repercussions on the surrounding network) caused by the system failure.
We also wish to underline that, as far as we know, the methods developed in this paper for the study of functional performances of roundabouts have never been presented by other authors so far. This is an innovative element about roundabouts since it makes it possible to quantify the uncertainties relating to their performance estimations. Thus, the results presented in this paper can help to design roundabouts in a better and more rational way.
Finally, even others performance indexes for roundabouts, as simple and total capacity, can be expressed in a probabilistic way. The authors of this paper are deeply interested in this theme and are carrying out new research on these topics.

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APPENDIX 1

Figure 7 shows the tails of the p.d.f. of demand Q_e and of capacity C (generic p.d.f.).
The probability that demand Q_e is bigger than an assigned value c of Capacity C is equal to

$$P(Q_e > c) = 1 - F_{Q_e}(c)$$

The probability that capacity C falls in the neighborhood of c is equal to

$$P(c - dc < C \leq c) = f_c(c)dc$$

The "failure" probability when C is equal to a given c is

$$P_{f_c} = [1 - F_{Qe}(c)] \cdot f_c(c)dc$$

For all the possible c, it results

$$P_f = \int_{-\infty}^{+\infty} [1 - F_{Qe}(c)] \cdot f_c(c)dc$$

which is Eq.(19).
Dually (see Figure 8)

$$P(C \leq q) = F_c(q)$$

$$P(q < Q_e \leq q + dq) = f_{Q_e}(q)dq$$

$$P_{f_{Qe}} = f_{Q_e}(q)F_c(q)dq$$

For all the possible q it results

$$P_f = \int_{-\infty}^{+\infty} f_{Q_e}(q)F_c(q)dq$$

which is Eq.(19).

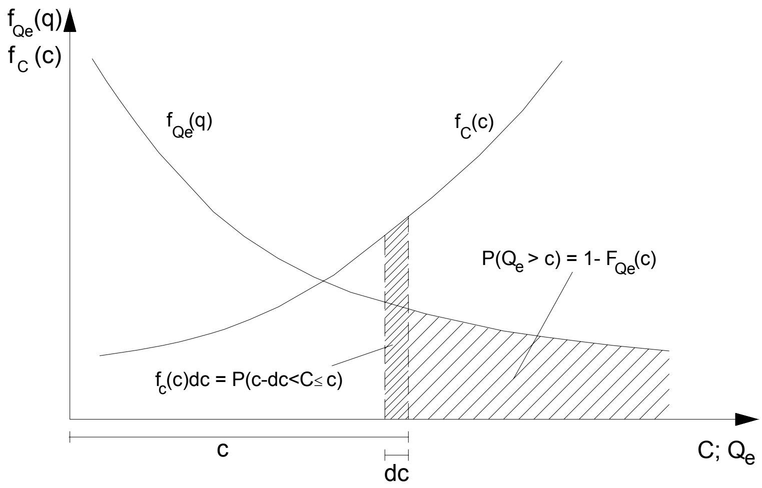

Figure 7: Tails of p.d.f. of demand Q_e and of capacity C (generic p.d.f.)

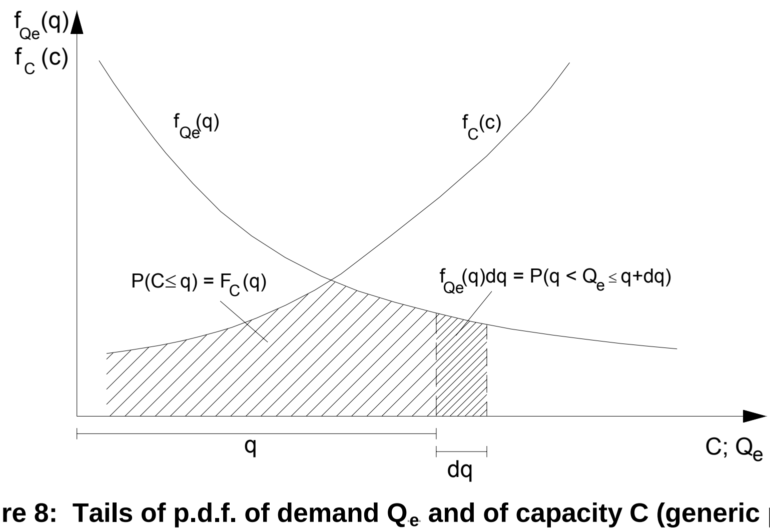

Figure 8: Tails of p.d.f. of demand Q_e and of capacity C (generic p.d.f.)