Reliability of individual points of the road

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Synopsis

We propose the use of the structural engineering reliability model for evaluating the reliability of the individual points of a road infrastructure.

The specific processing for the following cases are reported:

1. Stability on curves.
2. Presence of a fixed obstacle located in the traffic stream.
3. Overtaking manoeuvre, in the hypothesis that it is not possible to return to the normal driving lane.
4. Overtaking manoeuvre without binding circumstances.

This method is essential for evaluating the ex post economic convenience of the improvements carried out on an existing road and the ex ante one for planned roads, using the data regarding similar situations.

FOREWORD

The first definition formulated in 1952 by R. Lusser (San Diego California) of the measurement universally known as *reliability* is well known:

"it is the ability of an item to perform a required function under given environmental and operational conditions for a stated period of time".

With this formulation the concept of reliability in numerical terms, enters the engineering world. Basically, a service is reliable if the users' expectations are almost always respected and the reliability increases if the frequency and/or the consequences of the impossibility to satisfy users decrease.

If the item is the road, the users' expectations are mainly to go along the road without any problems (under safe conditions) and in a reasonably short time. This study examines the first aspect, the one regarding safety in terms of accidents. The analyses will be carried out in "individual points", which mean uniform geometrical characteristics along their course (horizontal curves, vertical curves, straightaways) or specific to their function (intersections, straight line sections where overtaking is permitted for instance). The aim is that of designing the geometry of the road by taking account on the one hand of the uncertainty regarding the variables to be entered in the model and on the other the inherent randomness of the users' behaviour.

1. THE METHODS FOR ANALYSING RELIABILITY

It is well known that the study of the structural reliability evaluates the probability of physical structures withstanding external stress and the degradation of the material resistance characteristics, taking account of the intrinsic uncertainty in the two classes of variables. In this field effective evaluation methods have been established to help designers who find themselves up against the problem of designing the structures, looking for optimal values for the construction costs to be reduced and for the degree of safety from collapse (reliability) to be increased.

Normally, in the theory of structural reliability, the fundamental variables are split into load variables (or stress) and resistance variables (which express the response of the structure). Similarly, for road safety, the main variables can be divided into stress variables and resistance variables. The former are represented by the forces which tend to cause the accident, for instance the force of inertia which favours the impact against an obstacle, in spite of the braking action, or the centrifugal force which makes the vehicle leave the road. The latter express the response of the vehicle to manage to avoid an accident, for instance longitudinal or transversal adherence force. Both refer to a given road conformation and defined environmental conditions.

By indicating the resistance of the system with R and the stress induced on the system with S, both expressed by random variables, the objective of the analysis is to guarantee that $R > S$.

It is well known that structural reliability analysis methods may normally be divided into three main classes [3]. The same classification may be adopted in evaluating the road reliability:

- *Level 3:* methods in which calculations are made to determine the exact probability of an accident for a given geometrical road conformation, if the distribution of the speeds of the vehicles and the stochastic relations between the variables which regulate movement are known.
- *Level 2:* methods sometimes using iterative calculation procedures to obtain an approximation of the probability of an accident. They normally require a definition of the field of instability and are often associated to a simplified representation of the distribution of joint probability of the variables.
- *Level 1:* control methods of the existence of safety conditions. In these methods frequent use is made of safety coefficients, sometimes partial ones, assigned prior to control on the basis of technical-economic considerations, without further control.

From a practical viewpoint, it is necessary to have a reliability analysis method which is quick to calculate and efficient and which produces results at the required safety levels. Generally the methods which satisfy these requirements are level 2 methods.

2. EVALUATING RELIABILITY

As we know [3], given the distribution F_R of the ultimate strength of a structure and knowing the stress force S, the probability of collapse P_f is given by:

$$P_f = P(R-S \leq 0) = F_R(S) = P(R/S \leq 1) \qquad (2.1)$$

In particular weather conditions, for a given geometrical conformation and for a road surface whose distribution F_R of the stabilising forces of ultimate adherence R is known, the probability of a given vehicle sliding P_f if the stress force S is known, is similarly expressed by *(2.1)* [6,11].

If the stress force measurement S is also a random variable, with a distribution function F_S, the equation (2.1) is replaced by

$$P_f = P\,(R\text{-}S \leq 0) = \int_{-\infty}^{+\infty} F_R(x)\, f_s(x)\, dx \qquad (2.2)$$

with the condition that R and S are statistically independent.
Equation *(2.2)* gives the probability of sliding P_f, as a product of the probability of two independent events, integrated on all the possible values. In particular there is a probability P_1 that S will come in the x, $x+dx$ range and there is a probability P_2 that R will be less than or equal to x. In fig. 1 the coloured areas represent the two probabilities.

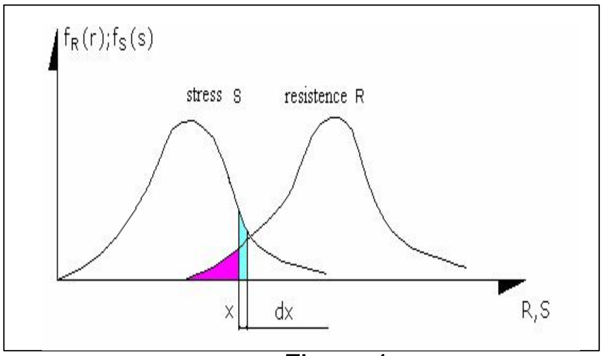

Figure 1

Under these conditions, the reliability A is the probability that the vehicle will adhere and is given by:

$$A = 1\text{-}\ P_f = \ 1\ -\int_{-\infty}^{+\infty} F_R(x)\, f_s(x)\, dx \qquad (2.3)$$

As is known [3], if the two independent variables R and S are distributed normally, P_f may be expressed as $P_f = P(M \leq 0)$, in which $M = R - S$. It follows that:

$\mu_M = E(M) = \mu_R - \mu_S$ and

$$\sigma_M^2 = Var(M) = \sigma_R^2 + \sigma_S^2 \qquad (2.4)$$

Since R and S are normal, M, which is a linear function of R and S, also has normal distribution. Moreover $(M - \mu_M)/\sigma_M$ is normal in standard units. It follows that:

$$P_f = \Phi\left(\frac{-\mu_M}{\sigma_M}\right) = \Phi\left(\frac{\mu_S - \mu_R}{(\sigma_R^2 + \sigma_S^2)^{1/2}}\right) \qquad (2.5)$$

in which Φ is the normal standard distribution function, and μ_S, μ_R, σ_S, σ_R are respectively the mean and standard deviations of the variables S and R.
The *reliability index* β [3] is defined as the relation

$$\beta = \mu_M / \sigma_M \qquad (2.6)$$

or as the number of standard deviations with which μ_M exceeds zero (figure 2). Therefore:

$$P_f = \Phi\,(-\,\mu_M / \sigma_M) = \Phi\,(-\,\beta) \qquad (2.7)$$

For the more general case in which R and S are normally and jointly distributed, with correlation coefficient ρ, equation *(2.7)* is still valid, but σ_M is given by:

$$\sigma_M = (\,\sigma_R^2 + \sigma_S^2 + 2\,\rho\,\sigma_S\,\sigma_R\,)^{1/2} \qquad (2.8)$$

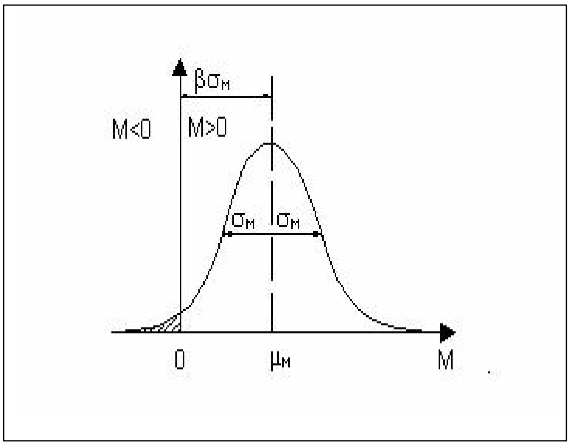

Figure 2

In most limit balance cases R and S depend on many other variables. In the case of adherence limit R and stress force S:

R = function (properties of the materials, their surfaces, the atmospheric conditions, the geometrical conformation of the road and the speed of the vehicles)

S = function (vehicle mass, initial speed and like for resistances, the geometrical conformation of the road)

R and S may not be statistically independent: for instance the speed of the vehicles influence (negatively) both the adherence coefficients and the stressing forces of inertia. In this case, the best solution to the problem consists of expressing each balance limit state or function in terms of a series of n fundamental variables X_i which influence the behaviour of the system, so that:

$$M = f(X_1, X_2, \ldots, X_n) \leq 0 \qquad (2.9)$$

corresponds to the negative state to be avoided. M is called the *safety margin* [3]. For simple problems there is no difficulty in finding a suitable form for f, in which some variables are stress ones and others resistance ones.

The function

$$f(X_1, X_2, \ldots, X_n) = 0 \qquad (2.10)$$

defines a surface with $(n-1)$ dimensions in a space with n dimensions of the fundamental variables. This is known as *limit balance surface* [3] and separates all the possible combinations of the variables X which cause the instability from all the possible combinations which do not cause it.

In the general case, the reliability of the road conformation may be expressed as:

$$A_T = 1 - \int\int_{\{f(X)\leq 0\}} \ldots \int f_{X1, X2 \ldots, Xn}(X_1, X_2 \ldots X_n)\, dx_1 dx_2 \ldots dx_n \qquad (2.11)$$

in which $f_{X1, X2 \ldots, Xn}(X_1, X_2 \ldots X_n)$ is the function of density of joint probability for the n X_i variables.

Two practical problems appear immediately. First of all the data available is almost never sufficient to define the function of density of joint probability for the n fundamental variables. There is usually not enough information to be sure about the marginal distributions and the covariances. Secondly, even if the joint density function was known, the several dimension integration necessary may require difficult calculations. There are no strict analytical solutions (level 3 methods) for most of the practical problems. These difficulties may be overcome in practice by using the level 2 methods.

3. LEVEL 2 METHODS

The level 2 methods have some simplifications compared to the “precise” level 3 methods. In particular, it is assumed that the limit balance surface may be likened to a tangent plan in the point nearest the origin, once the surface has been determined in a normal standard space. A level 2 method, in its simplest form, thus only includes a control in a point of the limit balance surface, unlike the level 3 methods in which the whole region of instability is examined.

In a level 3 method, knowledge of the function of joint probability density $f_{X1,X2 \ldots, Xn}(X_1, X_2 \ldots X_n)$ is required, whereas in the level 2 methods only the means and the covariances are necessary.

Hereafter, we assume to choose all the fundamental variables $X = f(X_1, X_2, \ldots X_n)$ in order to define a limit surface in the space $\boldsymbol{\Omega}$ with n dimensions of the fundamental variables. Figure 3 shows a two-dimensional case.

The function $f: \Omega \rightarrow R$ in the road case may be called *function of accident.*

It has been said that in the case of a linear safety margin M, and normal fundamental variables, the reliability index β is defined as:

$$\beta = \mu_M / \sigma_M \qquad (3.1)$$

where μ_M is the mean of M and σ_M is its mean square deviation.

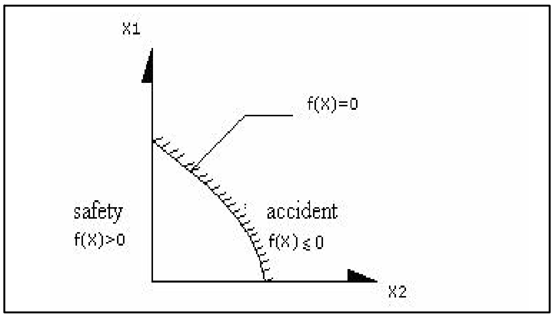

Figure 3

If the safety margin is not linear in $X = (X_1, X_2, \ldots X_n)$ approximate values of μ_M and σ_M can be obtained by using a linearised safety margin M, by developing in Taylor series [3]:

$$M = f(X) \approx f(\mu_1, \mu_2, \ldots \mu_n) + \sum_{i=1..n} \partial f/\partial X_i \; (X_i - \mu_i) \qquad (3.2)$$

in which $\partial f/\partial X_i$ is calculated for $(\mu_1, \mu_2, \ldots \mu_n)$.
From *(3.2)* the approximated values of μ_M and σ_M are ascertained to determine β through *(3.1)*:

$$\mu_M = f(\mu_1, \mu_2 \ldots, \mu_n) \qquad (3.3)$$

$$\sigma_M^2 = \sum_{i=1..n} \sum_{j=1..n} \partial f/\partial X_i \; \partial f/\partial X_J \; Cov(X_i, X_J) \qquad (3.4)$$

Clearly for non linear accident functions, the calculation of the reliability index $\beta = \mu_M/\sigma_M$ on the basis of a linearization like *(3.2)*, will depend on the choice of the linearization point. In *(3.2)* the so-called mean point $(\mu_1, \mu_2, \ldots \mu_n)$ was chosen, but as will be seen hereafter, to calculate the reliability with second level methods, a point on the accident surface will be chosen.
When the safety margin is linear in the variables X_i, for $i = 1, \ldots n$, and these variables have a normal distribution $N = (\mu_i, \sigma_i)$ then the following relation exists:

$$P_f = P(M \leq 0) = \Phi(-\beta) \Longleftrightarrow \beta = -\Phi^{-1}(P_f) \qquad (3.5)$$

In 1974 Hasofer and Lind proposed a calculation procedure for the reliability index β [3].
The first step consists of standardising the fundamental variables. The new set $Z = (Z_1, Z_2, \ldots, Z_n)$ is defined as:

$$Z_i = (X_i - \mu_{Xi}) / \sigma_{Xi} \qquad (3.6)$$

in which μ_{Xi} and σ_{Xi} are the mean and the mean square deviation of the random variable X_i. With the linear transformation defined by *(3.6)*, the accident surface of the system of coordinates Z, splits the space $\boldsymbol{\Omega_Z}$ into an accident region and a safety region, just like the space $\boldsymbol{\Omega_X}$. Thanks to the equation *(3.6)*, the system of Z coordinates has a characteristic importance: β represents the minimum distance of the accident surface from the origin in the new coordinates. The origin 0 of the Z coordinate system will normally be found in the safety region. Figure 4 shows a two-dimensional example.
The index β defined by *(3.1)* will coincide with the one just defined when the collapse surface is linear (hyper flat).
Therefore, in this case too, the important relation between β and the probability of collapse P_f can be formulated as long as the variables X_i with i = 1….n, are distributed normally.

$$P_f = \Phi(-\beta) \Longleftrightarrow \beta = -\Phi^{-1}(P_f) \qquad (3.7)$$

It has been said that an approximated calculation of β may be obtained when M is not linear, by developing M in Taylor series around point $(X_1, X_2 \ldots, X_n) = (\mu_1, \mu_2 \ldots, \mu_n)$.
It is now obvious, because of the previous observations, that the two definitions of β coincide if the development is made around the project A point, (shown in figure 4).

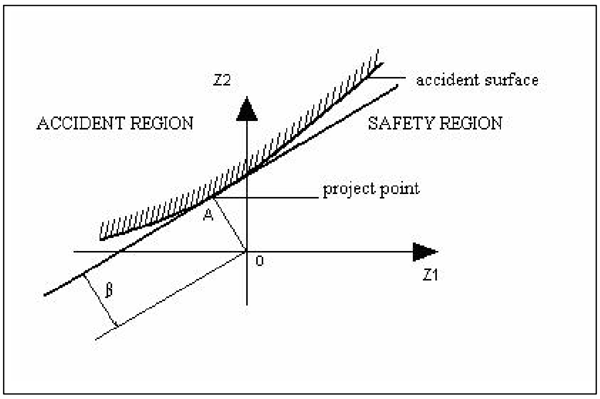

Figure 4

The definition of the Hasofer and Lind reliability index β may be formulated in the following manner [3]:

$$\beta = min \quad (\Sigma \quad Z_i^2)^{1/2} \qquad (3.8)$$

Where S_Ω is the collapse surface in the system of Z coordinates.
The calculation of β may be undertaken in different ways: in the general case of the collapse surface not being linear, a rapidly converging iterative method can be used [3].
The distance β and the unitary vector $\boldsymbol{\alpha} = (\alpha_1, \ldots \alpha_n)$ given by $\overline{OA} = \beta\, \boldsymbol{\alpha}$, in which A is the project point, can be determined by iteratively solving the following n+1 equations:

$$\begin{cases} \alpha_i = \dfrac{-\partial f/\partial Z_i (\beta\, \boldsymbol{\alpha})}{[\sum_{K=1..n} (\partial f/\partial Z_K (\beta\, \boldsymbol{\alpha}))^2]^{1/2}} \qquad i=1, 2, \ldots, n. \qquad (3.9') \\ f(\beta \alpha_1, \ \beta \alpha_2, \ldots \beta \alpha_n) = 0 \qquad (3.9'') \end{cases}$$

with the accident surface given by:

$$f(Z_1, Z_2, \ldots, Z_n) = 0 \qquad (3.10)$$

If the equation *(3.9")* were not easy to solve (equation a degree above the third for example), Taylor's series development can be used for the first member of this equation in the point of project A.
However, the point of project is generally not known beforehand. Indeed, it represents a problem unknown. In this case the gradient method can be used to direct the research of point A. We search the point on the surface S_Ω where the intensity of the vector β is minimum, that is:

$$\beta = \min_{Z \varepsilon S\Omega} (\sum_{i=1 \ldots n} Z_i^2)^{1/2} \qquad (3.12)$$

In application cases we report calculation examples of β.

4. EVALUATING THE RELIABILITY IN PARTICULAR POINTS OF THE ROAD

It is well known that there are no absolute safety conditions, because uncertainty remains. Because the values of reliability, like those of the probability of an accident, vary from zero to one without ever equalling these limit values, reliability represents only one degree of achievable safety. The probability of an accident represents the expected frequency of accidents, on the number of vehicles that cross that section.
Studies carried out have allowed us to ascertain that, by carrying out restructuring interventions (creating interchanges without any crossing for instance) and/or improving the geometry in some points of the road, the frequency of accidents drops significantly [8]. The calculation of the probability of accidents thus becomes essential for quantifying the economic viability of improving individual points of the road, particularly for intersections and sections on curves. By calculating the probability of an accident before and after carrying out the work, the economic benefits resulting from the reduction of the number of accidents can be quantified and compared with the cost of the work.
The calculation of the reliability of the individual points of the road may be carried out either for existing roads or for roads being designed. For existing roads the inputs necessary for the calculation will be given by the geometry of the road and by the statistical distribution of the speeds in that section. The experimental relationship between speed and adherence coefficients are well known in various weather

conditions, like those reported in technical literature. For the operations to be carried out, the geometry of the road will be the one in the project and the distribution of the speeds will be that determined in similar situations.
The following four cases have been considered:

1. Stability on curves.
2. Presence of a fixed obstacle located in the traffic stream.
3. Overtaking manoeuvre, in the hypothesis that it is not possible to return to the normal driving lane.
4. Overtaking manoeuvre without binding circumstances.

1. The first case is a case in itself and calculations for the individual points made up of bends on the road section must refer to it.
2. The second calculation model should refer to all those situations where the accident is avoided (or occurs), because of the existence (or absence) of a suitable visibility for the stopping. This model must, therefore, refer to those road sections in proximity of intersections, humps and bends with poor visibility. For instance, the free visual distance between a hump and an intersection will influence the value of reliability in that section.
3 and 4. The third and fourth cases both concern the reliability of a section of two-lane road, one in each direction, where it is possible to overtake. The two calculation models differ because of the additional assumption considered in the third case. The assumption consists of supposing that the vehicle which is overtaking does not return to the normal lane. The fourth case is the general one, that is without any additional assumption.
With reference to condition (2.1) on which the reliability calculation is based:

$$R > S \qquad (4.1)$$

Generally in road reliability, the stress variables S reflect the kinetic energy of the motor vehicle at various moments (for instance at the beginning of a skid) or in periods of extended time (during the braking phase for instance). The resistance variables R are generally represented by the wheel – road surface adherence force.
On some occasions, however, the active forces *"A"* may contribute to preventing the accident. This is the case of a completed overtaking in the fourth case. If motor vehicle "1" who overtakes manages to develop adequate acceleration, impact with vehicle "2", coming in the opposite direction and in the process of braking, may be avoided. In this case the accident is avoided if the following two conditions exist at the same time:

$$\begin{cases} A_1 > R_1 & \text{for motor vehicle 1} \\ R_2 > A_2 & \text{for motor vehicle 2} \end{cases} \qquad (4.2)$$

In most cases, as can be seen in the case of the completed overtaking, a *(4.2)* type system may be reduced to a single disparity like the *(4.1)* type. Moreover, generalities are not taken from the procedure based on disparity, if other measurements expressing speed or distance for instance appear in place of measurements expressing force. In all cases a limit measurement X_L that the variable X must not exceed in order for the accident to be avoided would always be present.

$$X_L > X \qquad (4.3)$$

This relation is at the basis of the reliability calculation in place of *(4.1)*.
In the road sections where the presence of the conditions foreseen by the third case is probable (impeded return), for instance on uphill sections and with a great deal of heavy traffic, both cases of overtaking are treated and the probability of an accident is determined for each one of them. If p_3 and p_4 indicate the probability of impact respectively for circumstances 3 and 4 and p_a the frequency (determined or calculated) of the occurrence of circumstance 3, the probability of an accident for the considered section will be given by:

$$P_l = p_3\, p_a + p_4\,(1 - p_a) \qquad (4.4)$$

For all the considered cases, the calculation may be carried out by supposing different environmental or weather conditions, obtaining values of probability of accidents p_i for each i^{th} circumstance. If the frequency of the environmental conditions is called f_i, the probability of accidents in the individual point of the road will be given by:

$$p_l = \sum_{i=1\ldots n} p_i\, f_i \qquad (4.5)$$

The reliability a_l is defined as:

$a_l = 1 - p_l$ (4.6)

This measurement may be interpreted as the expected frequency of the motor vehicles not having accidents, on the total number passing along the road section concerning the specific point considered.
It is then possible to check whether the calculated value corresponds to what was found in the individual points of an infrastructure existing where the number of accidents is statistically higher.
It is, finally, well known that, once the probability p_{li} of an accident (or the corresponding reliability a_{li}) have been determined by homogeneous individual points or sections, the reliability A for the whole route will be given by [3,5,6]:

$$A = \prod_{i=1....n} a_{li} = \prod_{i=1....n} (1- p_{li}) \quad (4.7)$$

It is, therefore, possible to express the quality of the whole route by means of its reliability, calculated as a product of the values of reliability of the individual points. Moreover, the improvements to be made can be planned by means of cost-benefit analysis procedures. Indeed, the reliability of the design solution and the reliability of the road at its current state will be calculated using the above mentioned methods. The difference between the two values will represent the expected reduction of accidents to be considered amongst the benefits.
This method may be also applied to designing a new road. The expected benefits will be calculated by referring to similar situations, above all as regards the distribution of the speeds of the vehicles.

5. APPLIED EXAMPLES

5.1 Stability on bends

For a bend, given the following: Curve radius R = 250 m, transversal slope $tg\,\beta$ = 0.045
The reliability of the bend is to be examined under wet conditions. For a vehicle taking the curve, the balance equation will be the following:

$$f_{at} - 1/gR\ v^2 + tg\,\beta = f_{at} - 0.00040816\ v^2 + 0.045 = 0 \quad (1)$$

The experimental relationship considered for the transversal adherence coefficient f_{at} as a function of the speed, for wet surfaces, is the following (Bird and Scott):

$f_{at} = 0.000003906\ V^2 - 0.001331084\ V + 0.346779947$ with V expressed in Km/h
$f_{at} = 0.0005062\ v^2 - 0.00479190\ v + 0.34677995$ with v expressed in m/sec (2)

So by inserting the expression (2) of f_{at} in the relation (1), we obtain:

$$- 0.00035758\ v^2 - 0.0047919\ v + f_{ao} + 0.045 = 0 \quad (3)$$

where f_{ao} represents the intercept on the ordinate axis of the curve expressing the relationship (2).
The mean values and the mean square deviation of the variables v and f_{ao} are the following:

$\underline{f_{ao}} = 0.346780$ $\sigma_f = 0.05$
$\underline{v} = 60$ Km/h = 16.66 m/sec $\sigma_v = 8$ Km /h = 2.22 m/sec

Considering the standardised values:

$z_v = (v - \underline{v})/\sigma_v$ $z_f = (f_{ao} - \underline{f_{ao}})/\sigma_f$

and the cosine directors α_v and α_f of the vector β, *(3)* becomes:

$$f(\beta\alpha_v, \beta\alpha_f) = - 0.001762\ \alpha_v^2 \beta^2 - 0.0370884\ \alpha_v\beta - 0.05\ \alpha_f\beta + 0.2126986 = 0 \quad (4)$$

The solution is obtained by considering the relationships of the director cosines α_f and α_v, calculated with (*3.9'*) and with $\sum_k \alpha_k^2 = 1$ together with *(4)*.
The following system of three equations in the three unknowns α_v, α_f and β is obtained:

$$\begin{cases} \beta = \dfrac{0,0370884\ \alpha_v - 0.05\ \alpha_f +/-(0.0370884^2\ \alpha_v^2 + 0.05^2\ \alpha_f^2 - 0.00370884\ \alpha_v\ \alpha_f + 0.001499352\alpha_v^2)^{1/2}}{- 0.003524\ \alpha_v^2} \\ \alpha_f = \dfrac{- 0.05}{((- 2 * 0.001762\ \alpha_v\beta - 0.0370884)^2 + 0.05^2)^{½}} \\ \alpha_v = (1- \alpha_f^2)^{½} \end{cases} \quad (5)$$

Because the absolute value of β must be minimum to calculate the reliability, the expression *(5)* must present the negative sign before the square root of the discriminate, because α_v must be positive (acting force) and α_f negative (resisting force). The system is resolved with the iterative method of substitution. Having initially assigned $\alpha_f = -0.7071068$ and $\alpha_v = 0.7071068$, the following values are obtained at each iteration:

Iterat:	1	2	3	4	5	6	7
β =	3.301520	3.297077	3.297037	3.297037	3.297037	3.297037	3.297037
α_f =	-0.741280	-0.744441	-0.744743	-0.744772	-0.744775	-0.744775	-0.744775
α_v =	0.671195	0.667687	0.667350	0.667318	0.667315	0.667314	0.667314

The final values will be given by:

$\beta = 3.297037,\ \alpha_f = -0.744775,\ \alpha_v = 0.667314$

From the value of β found, we obtain the value of the probability of skidding equal to $P_f = \Phi(-\beta) = 0.000489$ and of reliability to skidding in the given curve, under wet conditions, amounting to:

$A_f = 1 - P_f = 0.999511.$

The speed and adherence coefficient limit values to skidding will be given by:

$v_L = 2.22\,\alpha_v \beta + 16.66 = 21.544\ m/sec = 77.560\ Km/h$
$f_{aL} = 0.05\,\alpha_f \beta + 0.346778 + 0.0005062\,v_L^{\,2} - 0.00479190\,v_L = 0.144$

Fig. 5 shows the limit skidding surface for the bend concerned under wet conditions. In the two-dimension space it is represented by a line.

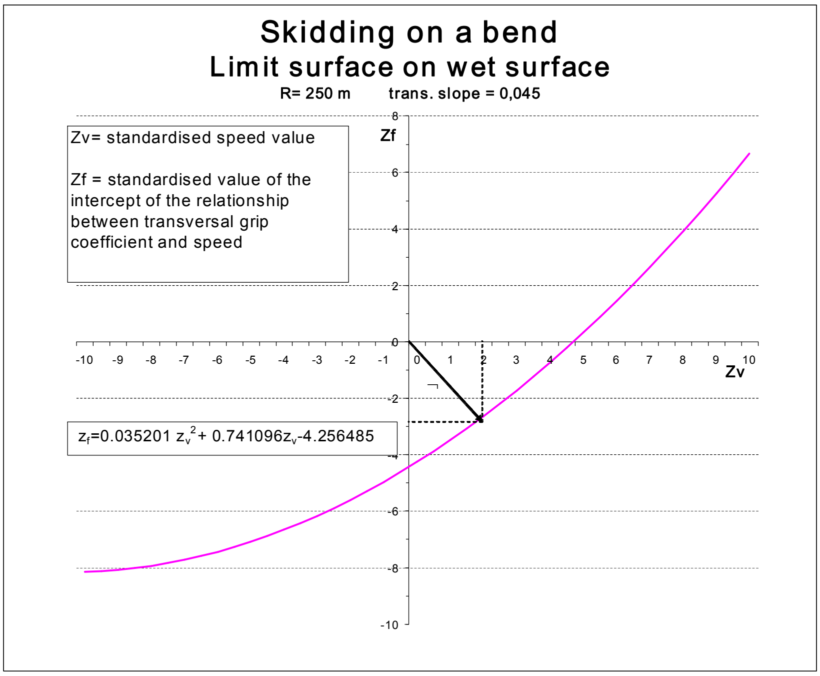

Fig. n. 5. Limit surface for the bend concerned

5.2. Presence of an obstacle at visibility distance D

There is a road conformation which foresees a free visual distance, in the middle of the lane, $D = 70\ m$. From the equation which provides the stopping visibility distance, the limit equation is obtained:

$$D - v - v^2/2gf_e = 0 \qquad (1)$$

The experimental relationship binding the adherence coefficient equivalent to speed (Ferrari-Giannini) is the following:

$f_e = 0.0000055804\,V^2 - 0.0020290179\,V + 0.7068805804$ with V expressed in Km/h
$f_e = av^2 + b\,v + c = 0.00007232v^2 - 0.00730446\,v + 0.7068805804$ with v expressed in m/sec *(2)*

So by inserting the expression (2) f_e in the relation (1), we obtain:

$$-(2\,g\,a)\,v^3 - (2\,g\,b + 1 - 2\,g\,D\,a)\,v^2 - (2\,g\,c - 2\,g\,D\,b)\,v + 2\,g\,D\,c = 0 \qquad (3)$$

where c represents the intercept on the ordinate axis of the curve expressing the relationship (2).
The mean values and the mean square deviation of the variables v and c are the following:

$\underline{c} = 0.7068805804$ $\qquad \sigma_c = 0.05$
$\underline{v} = 60\ Km/h = 16.66\ m/sec$ $\qquad \sigma_v = 8\ Km/h = 2.22\ m/sec$

Considering the standardised values and the cosine directors α_v and α_f of the vector β, the relation *(3)* becomes:

$$f(\beta\alpha_v, \beta\alpha_f) = -0.01552488\,\alpha_v^3\beta^3 - 4.082089642\,\alpha_v^2\beta^2 - 111.70525\,\alpha_v\beta - 2.17782\,\alpha_f\,\alpha_v\beta^2 + 52.32654\,\alpha_f\,\beta + 355.86928 = 0 \qquad (4)$$

In resolving the third degree equation in β reported above, we obtained two imaginary solutions and a real solution for each iteration. The imaginary solutions have been rejected, whereas the real root was determined as described below.
The triad solution of the problem $(\alpha_v, \alpha_f, \beta)$ is determined by considering along with (4), the relations of the director cosines α_f and α_v with $\sum_k \alpha_k^2 = 1$.
The following system of three equations is thus obtained in the three unknowns α_v, α_f and β:

$$\begin{cases} f(\beta\alpha_v, \beta\alpha_f) = 0 \\ \alpha_f = \dfrac{-(-2.17782\,\alpha_v\,\beta + 52.32654)}{[(-2.17782\alpha_v\beta+52.32654)^2+(-3{*}0.1555248{*}\alpha_v^2\beta^2-2{*}4.0820896\alpha_v\beta-2.17782\alpha_f\beta-111.705254)^2]^{1/2}} \\ \alpha_v = (1-\alpha_f^2)^{1/2} \end{cases}$$

The system is resolved with the iterative method of substitution already adopted for the previous case.
The final values will be given by:

$\beta = 2.7194236,\ \alpha_f = -0.3362677,\ \alpha_v = 0.9417664$

The limit speed v will be given by:

$v = 2.22\,\alpha_v\beta + 16.66 = 22.346\ m/sec = 80.44\ Km/h$

Therefore, we obtain the value of probability of impact against the obstacle as $P_f = \Phi(-\beta) = 0.003269$ and of reliability to impact for given visibility distance and in dry conditions of:

$A_f = 1 - P_f = 0.996730.$

5.3. Overtaking manoeuvre with impossibility of returning to the normal driving lane

Let us suppose that a vehicle is travelling on a road with a designed speed of 100 Km/h, on an upwards gradient of $i_1 = 2{,}5\%$ and in conditions of a wet road. Consider the case in which the vehicle is already overtaking with a speed of v_1 and let us assume that it cannot return to the normal driving lane because there is a line of lorries which prevent the manoeuvre. The driver sees a second vehicle coming in the opposite direction at a visibility distance of 550 m. This vehicle is driving along a downhill slope of $i_2 = 1{,}8\%$ at an initial speed of v_2.
The limit equation for the impact between the two vehicles is the following:

$$D - (v_1 + v_1^2/(2g(f_e+i_1)) + (v_2 + v_2^2/(2g(f_e-i_2)) = 0 \qquad (1)$$

with: $D = 550\ m$, $\quad i_1 = 2.5\%$, $i_2 = 1.8\%$.
Constructing the regression line $f_e = f_e(V)$ in wet surface conditions, on the basis of the values reported in the Ferrari-Giannini text, limited to the speed range $V\ [80,112]\ Km/h$, we obtain:

$f_e = -0.000625\ V + 0.36$ $\qquad$ with V expressed in Km/h
$f_e = a\,v + b = -0.00225\,v + 0.36$ $\qquad$ with v expressed in m/sec $\qquad (2)$

So by inserting the expression (2) of f_e in the relation (1), we obtain:

$$(2gDav_1 + 2gDb + 2gDi_1)\,(av_2 + b - i_2) - [((2ga+1)\,v_1^2 + 2g\,v_1\,(b + i_1))\,(a\,v_2 + b - i_2)] - [((2ga+1)\,v_2^2 + 2gv_2\,(b - i_2))\,(av_1 + b + i_1)] = 0 \qquad (3)$$

The mean values and the mean square deviation of the variables v and f_{ao} (the intercept in relation (2)) are the following:

$\underline{f_{ao}} = 0.36$ $\sigma_f = 0.05$

$\underline{v_1} = 80\ Km/h = 22.22\ m/sec$ $\sigma_{v1} = 8\ Km/h = 2.22\ m/sec$

$\underline{v_2} = 85\ Km/h = 23.61\ m/sec$ $\sigma_{v2} = 8\ Km/h = 2.22\ m/sec$

Considering the standardised values and the cosine directors α_v and α_f of the vector β: the relation (3) becomes:

$$f(\beta\alpha_{v1},\beta\alpha_{v2},\beta\alpha_f) = 0.0235\,\alpha_{v1}^2\,\alpha_{v2}\,\beta^3 + 0.0235\,\alpha_{v2}^2\,\alpha_{v1}\,\beta^3 - 0.2355\,\alpha_{v1}^2\,\alpha_f\,\beta^3 - 0.2355\,\alpha_{v2}^2\,\alpha_f\beta^3 - 0.1088\,\alpha_f^2\alpha_{v1}\beta^3 - 0.1088\,\alpha_f^2\,\alpha_{v2}\,\beta^3 + 0.021\,\alpha_f\,\alpha_{v1}\alpha_{v2}\,\beta^3 - 1.3608\,\alpha_{v1}^2\beta^2 - 1.5781\alpha_{v2}^2\,\beta^2 + 24.7295\,\alpha_f^2\,\beta^2 + 1.3989\,\alpha_{v1}\,\alpha_{v2}\,\beta^2 - 8.762\,\alpha_{v1}\,\alpha_f\,\beta^2 - 9.0706\,\alpha_{v2}\,\alpha_f\,\beta^2 - 44.2038\,\alpha_{v1}\,\beta - 53.41952\,\alpha_{v2}\beta + 256.0085\,\alpha_f\beta + 627.9148 = 0 \qquad (4)$$

From this third degree equation, three real solutions are obtained. The real root chosen is represented by the lower value of β.

The quaternion $(\alpha_{v1}, \alpha_{v2}, \alpha_f, \beta)$ solution of the problem is determined considering along with *(4)*, the relations of the cosine directors α_{v1}, α_{v2}, α_f expressed by *(3.9')* and with $\sum_k \alpha_k^2 = 1$. The system is solved with the iterative substitution method, already seen before. The final values will be given by:

$\beta = 3.7193199,\ \alpha_f = -0.9261294,\ \alpha_{v1} = 0.1924937,\ \alpha_{v2} = 0.3243923.$

The limit speeds v_1 and v_2 will be given by:

$v_1 = 2.22\,\alpha_{v1}\beta + 22.22 = 23.809\ m/sec = 85.71\ Km/h,\ v_2 = 2.22\,\alpha_{v2}\beta + 23.61 = 26.288\ m/sec = 94.63\ Km/h$

These values are in the speed values range *[80,112 Km/h]* for which the regression line of $f_e = f_e(v)$ has been constructed.

The value of probability of accident will be $P_f = \Phi(-\beta) = 0.0000998$; the value of reliability to impact for the given visibility distance in wet conditions and in the impossibility of returning will be:

$A_f = 1 - P_f = 0.9999002.$

Figure 7 shows the limit surface of impact between two vehicles in the given situations.

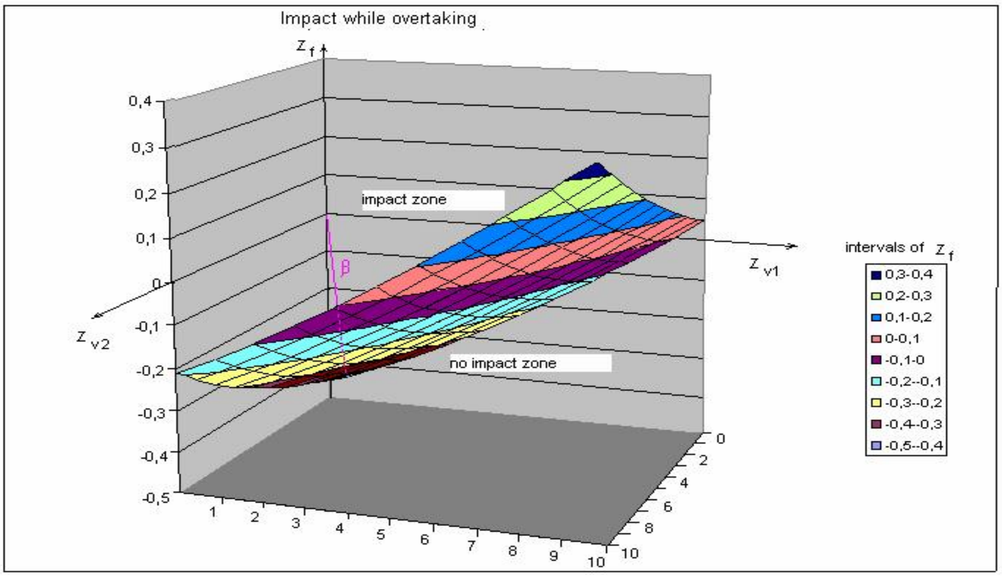

Figure 7. Limit surfaces for incomplete overtaking

5.4. Completed overtaking manoeuvre

Let us assume that a vehicle (vehicle 1) is travelling along a road with a designed speed of 100 Km/h under conditions of a wet road. At the initial instant *(t=0 sec)* vehicle *1* travels at a speed v_1, behind a vehicle (vehicle *3*), travelling at the same speed $v_3 = v_1$, and keeps a free visual distance of 550 m. After the perception and reaction time of 2 sec, vehicle *1* begins the overtaking manoeuvre with constant acceleration. At this instant *(t=2 sec)* the driver sees a motor vehicle appear (vehicle *2*) travelling in the opposite direction. This has an initial speed of v_2. After a further perception and reaction time of 1 sec (t=3 sec), vehicle *2* begins the slowing manoeuvre, with constant deceleration $a_2 = g\,f_e$.

The necessary time for the overtaking manoeuvre, assuming that lane changes are carried out in 1 second, is given by: $t_s = 2(v_1 / a_1)^{1/2}$ [G. Tesoriere].

The space covered by vehicle 1 from the initial moment (t = 0 sec) to the end of the overtaking manoeuvre (t = t_f) is given by:

$$d_1 = 2\,v_1 + v_1\,t_s + \tfrac{1}{2}\,a_1\,t_s^{\,2} = 2\,v_1 + 2\,v_1\,(v_1/a_1)^{1/2} + 2\,a_1\,(v_1/a_1) = 2\,v_1^{3/2}/a_1^{1/2} + 4\,v_1$$

The space covered by vehicle 2, from the moment it appears (t=2 sec) up to the time t_f, is given by:

$$d_2 = v_2 + v_2\,(t_s - 1) - \tfrac{1}{2}\,g\,f_e\,(t_s - 1)^2 = v_2 + v_2\,[2\,(v_1/a_1)^{1/2} - 1] - \tfrac{1}{2}\,g\,f_e\,[4v_1/a_1 - 4(v_1/a_1)^{1/2} + 1] =$$
$$= 2\,v_2\,(v_1/a_1)^{1/2} - 2\,v_1\,g\,f_e/a_1 + 2\,a_2\,(v_1/a_1)^{1/2} - 1/2\,g\,f_e$$

The limit equation for the impact between the two vehicles is the following:

$$D - d_1 - d_2 = D - 4\,v_1 - 2v_1^{3/2}/a_1^{1/2} - 2\,v_2\,(v_1/a_1)^{1/2} + 2\,v_1\,g\,f_e/a_1 - 2\,gf_e\,(v_1/a_1)^{1/2} + 1/2\,g\,f_e = 0 \qquad (1)$$

with $D = 550\ m.$
Constructing the regression line $f_e = f_e\,(V)$ in wet surface conditions, on the basis of the values reported in the Ferrari-Giannini text, limited to the speed range V [80,112] Km /h, we obtain:

$f_e = -0.000625\ V + 0.36$ with V expressed in Km/h
$f_e = p\ v + b = -0.00225\ v + 0.36$ with v expressed in m/sec (2)

So by inserting the expression (2) of f_e in the relation (1), we obtain:

$$D - 4\,v_1 - 2v_1^{3/2}/a_1^{1/2} - 2v_2\,(v_1/a_1)^{1/2} + 2v_1\,g\,(p\,v_2 + b)/a_1 - 2g(p\,v_2 + b)(v_1/a_1)^{1/2} + 1/2g\,(p\,v_2 + b) =$$
$$= D - 4\,v_1 - 2v_1^{3/2}/a_1^{1/2} - 2v_2(v_1/a_1)^{1/2} + 2gpv_1v_2/a_1 + 2gv_1b/a_1 - 2gpv_2\,(v_1/a_1)^{1/2} - 2\,gb(v_1/a_1)^{1/2} + 1/2\,gpv_2 + 1/2\,gb = 0 \qquad (3)$$

The mean values and the mean square deviation of the variables v_1, v_2, a_1 and b (the intercept in relation (2)) are the following:

$\underline{b} = 0.36$ $\sigma_b = 0.05$
$\underline{v_1} = 80\ Km/h = 22.22\ m/sec$ $\sigma_{v1} = 8\ Km/h = 2.22\ m/sec$
$\underline{v_2} = 85\ Km/h = 23.61\ m/sec$ $\sigma_{v2} = 8\ Km/h = 2.22\ m/sec$
$\underline{a} = 1\ m/sec^2$ $\sigma_a = 0.15\ m/sec^2$

Considering the standardised values, the relation (3) becomes:

$$D - 4\,(\underline{v_1} + \sigma_{v1}z_{v1}) - 2(\underline{v_1} + \sigma_{v1}z_{v1})^{3/2}\,(\underline{a} + \sigma_a z_a)^{-1/2} - 2\,(\underline{v_2} + \sigma_{v2}z_{v2})\,(\underline{v_1} + \sigma_{v1}z_{v1})^{1/2}\,(\underline{a} + \sigma_a z_a)^{-1/2} + 2gp(\underline{v_1} + \sigma_{v1}z_{v1})\,(\underline{v_2} + \sigma_{v2}z_{v2})\,(\underline{a} + \sigma_a z_a)^{-1} + 2g\,(\underline{v_1} + \sigma_{v1}z_{v1})\,(\underline{b} + \sigma_b z_b)\,(\underline{a} + \sigma_a z_a)^{-1} - 2\,gp\,(\underline{v_2} + \sigma_{v2}z_{v2})\,(\underline{v_1} + \sigma_{v1}z_{v1})^{1/2}\,(\underline{a} + \sigma_a z_a)^{-1/2} - 2g\,(\underline{b} + \sigma_b z_b)\,(\underline{v_1} + \sigma_{v1}z_{v1})^{1/2}\,(\underline{a} + \sigma_a z_a)^{-1/2} + 1/2\,g\,p\,(\underline{v_2} + \sigma_{v2}z_{v2}) + 1/2\,g(\underline{b} + \sigma_b z_b) = 0 \qquad (4)$$

Given the surface S_Ω representing the relation (4) in coordinates (z_{v1}, z_{v2}, z_b, z_a), the function to minimise is

$$\beta = \min\ (z_{v1}^{\,2} + z_{v2}^{\,2} + g_b(z_{v1}, z_{v2}, z_a)^2 + z_a^{\,2})^{1/2} \qquad (5)$$

with the expression of $z = g_b(z_{v1}, z_{v2}, z_a)$ obtained from *(4).*
The gradient method was used to find the minimum value of β. Table no. 1 reports the results of the various phases of the research. θ is the parameter which minimises the function $\beta(\mathbf{z} - \nabla\beta(\mathbf{z})\,\theta)$, in the direction of the gradient.
The values of the project point were found to be the following:

$V_1 = 93{,}55\ Km/h = 26{,}021\ m/sec$, $V_2 = 93{,}07\ Km/h = 25{,}838\ m/sec$, $a = 0{,}8546\ m/sec^2$, $\beta = 2{,}39276$.

The probability of impact was found to be equal to $p_{ij} = 0{,}836111\ \%$
The values of the project point were found to be the following:

$V_1 = 93.55\ Km/h = 26.021\ m/sec$, $V_2 = 93.07\ Km/h = 25.838\ m/sec$, $a = 0.8546\ m/sec^2$, $\beta = 2.39276$.

Tab. 1. Solution with the gradient method

			Project points				Gradient				
v_1 m/sec	v_2 m/sec	a m/sec^2	z_{v1}	z_{v2}	z_a	z_b	$d\beta/dz_{v1}$	$d\beta/z_{v2}$	$d\beta/d_{za}$	θ	β
22.220	23.610	1	0	0	0	-7.805414	-3.140196	-1.301434	2.145234	0.552	7.805413
26.068	25.205	0.822375	1.733388	0.718392	-1.18417	-0.967813	0.000455	-0.130273	-0.0800967	2.19	2.420669
26.066	25.838	0.848686	1.732391	1.00369	-1.00876	-0.841904	0.076213	0.0371174	-0.0577986	0.344	2.394779
26.008	25.810	0.851669	1.706174	0.990922	-0.98888	-0.924838	-0.004692	-0.007163	-0.0077015	1.78	2.392938
26.026	25.838	0.853725	1.714526	1.003672	-0.97517	-0.909749	0.009792	0.0046549	-0.0091069	0.38	2.392811
26.018	25.834	0.854244	1.710805	1.001904	-0.97170	-0.922208	-0.002415	-0.001977	-0.001301	0.71	2.392768
26.022	25.837	0.854383	1.712519	1.003307	-0.97078	-0.918453	0.001321	0.0003186	-0.0027844	0.79	2.392763
26.019	25.837	0.854713	1.711476	1.003056	-0.96858	-0.922969	-0.003072	-0.001955	0.0004654	0.275	2.392759
26.021	25.838	0.854694									

The probability of impact is equal to $P_{//} = 0.836111\%$ and the reliability to overtaking in that section amounts to:

$A_{//} = 99.16389\ \%$

Seeking the project point may be carried out, not only by moving in the direction of the gradient, but also by iteratively seeking the minimum value of β in the direction of the single variables z_i. In this way laborious calculations of the partial derivates are avoided.
The iterations have given the results reported in table no. 2. It shows the values of the variables which minimise the value of β at each step of the procedure.

Table no. 2: Research method according to the direction of the individual variable z_i

V_1 Km/h	V_2 Km/h	v_1 m/sec	v_2 m/sec	a m/sec^2	z_{v1}	z_{v2}	Z_b	z_a	β
100	80	27.7778	22.2222	0.9	2.5035	-0.6251	-1.546879	-0.66667	3.08149
100	88	27.7778	24.4444	0.9	2.5035	0.37588	-0.462091	-0.66667	2.65834
100	88	27.778	24.444	0.92	2.5035	0.37588	-0.583371	-0.53333	2.65209
98	88	27.222	24.444	0.92	2.25325	0.37588	-1.045648	-0.53333	2.56832
98	91	27.222	25.278	0.92	2.25325	0.75125	-0.629390	-0.53333	2.51438
98	91	27.222	25.278	0.915	2.25325	0.75125	-0.597176	-0.56667	2.51381
96	91	26.667	25.278	0.91	2.003	0.75125	-1.035476	-0.6	2.45125
96	92.5	26.667	25.694	0.91	2.003	0.93894	-0.826240	-0.6	2.43645
96	92.5	26.667	25.694	0.896	2.003	0.93894	-0.731439	-0.69333	2.43091
95.1	92.5	26.417	25.694	0.896	1.89039	0.93894	-0.944462	-0.69333	2.41410
95.1	92.9	26.417	25.806	0.896	1.89039	0.98899	-0.888851	-0.69333	2.41297
95.1	92.9	26.417	25.806	0.872	1.89039	0.98898	-0.724925	-0.85333	2.40943
94.2	92.9	26.167	25.806	0.872	1.777778	0.98898	-0.935020	-0.85333	2.39605
94.2	93	26.167	25.833	0.872	1.77778	1.0015	-0.921251	-0.85333	2.39591
94.2	93	26.167	25.833	0.864	1.77778	1.0015	-0.865710	-0.90667	2.39479
93.9	93	26.083	25.833	0.864	1.74024	1.0015	-0.935848	-0.90667	2.39359
93.9	93.09	26.083	25.858	0.864	1.74024	1.01276	-0.923496	-0.90666	2.39354
93.9	93.09	26.083	25.858	0.86	1.74024	1.012763	-0.895622	-0.93333	2.39319
93.8	93.09	26.055	25.858	0.86	1.72772	1.012763	-0.918967	-0.93333	2.39298
93.8	93.07	26.055	25.853	0.86	1.72772	1.01026	-0.921707	-0.93333	2.39297
93.8	93.07	26.055	25.853	0.858	1.727728	1.01026	-0.907763	-0.94667	2.39288
93.70	93.07	26.027	25.853	0.858	1.715215	1.01026	-0.931097	-0.94667	2.39285
93.70	93.10	26.027	25.8611	0.858	1.71522	1.01401	-0.926990	-0.94667	2.39283
93.7	93.1	26.028	25.8611	0.856	1.71522	1.01401	-0.913030	-0.96	2.39278
93.67	93.1	26.0194	25.8611	0.856	1.71146	1.01401	-0.920020	-0.96	2.39277
93.67	93.07	26.0194	25.8528	0.856	1.71146	1.01026	-0.924122	-0.96	2.39277

In the first phase, the values of two of the four variables were established, for example $V_2 = 80\ Km/h$ and $a = 0.9\ m/sec^2$. By varying the third variable, for example V_1, the values of z_{v1}, z_{v2}, z_a are calculated and the corresponding value of z_b on the accident surface is calculated by means of (5).
Then the distance of this point of the surface from the origin of the axes of z_i coordinates is calculated by means of:

$$\beta = (z_{v1}^2 + z_{v2}^2 + z_b^2 + z_a^2)^{1/2}$$

We seek the value of V_1 to which the minimum value of β corresponds. In the example reported, this value is $V_1 = 100\ Km/h = 27.7778\ m/sec$.
Starting from the values of the corresponding point in the four dimension space *(2.5035, -0.6251, -1.546879, -0.66667)*, V_2 is varied and we seek the value in the proximity of which there is the minimum value of β. This value is found to be $V_2 = 88\ Km/h$.
Starting from the values found in correspondence of $V_2 = 88\ Km/h$ the procedure is iterated, by varying the acceleration a. In correspondence of the minimum value of β, a new starting point for further exploration is obtained.
The last value of β found *(2.39277)*, along with the values of the other parameters, basically coincides with the value found with the gradient method *(2.39276).* Thus the values of the probability of accidents and of the reliability previously calculated are confirmed.

6. CONCLUSIONS

The transfer of calculation methods from the structural field to that of road safety has solved the problems which at first seemed to be difficult to solve, considering the number of variables at stake and the intricate relations between them.
Reliability calculated in an individual point of the road makes it possible:

- to check whether the calculated value corresponds to what was found in the critical points of an infrastructure existing where the number of accidents is statistically higher.
- to express the quality of the whole route in terms of its reliability, calculated as a product of the values of reliability of the individual points;
- starting from the determined data regarding the speed maintained by the users, to plan improvements by cost-benefit analysis, considering the reduction of accidents as one of the benefits. This coincides with the increase in the reliability of the project solution compared to the current state;
- in planning a road, to calculate the expected benefits, referring to similar situations, above all as regards the distribution of the speed of vehicles.

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