RAILWAY TRAFFIC VIBRATIONS: GENERATION AND PROPAGATION - USE OF COMPUTATIONAL MODELS

Bonin G..
Research fellow – University of Rome "La Sapienza" – guido.bonin@uniroma1.it
Cantisani G..
Researcher – University of Rome "La Sapienza" – giuseppe.cantisani@uniroma1.it
Carbonari M..
Civil Engineer – mat.carbonari@libero.it
Loprencipe G..
Researcher – University of Rome "La Sapienza" – giuseppe.loprencipe@uniroma1.it
Pancotto A..
Civil Engineer – alessandro_pancotto@tele2.it

ABSTRACT

In the analysis of the railway traffic vibrations, calculation and simulation models can be useful for studying generation and propagation of waves. Following the analysis of scientific literature, presented in *"Railway traffic vibrations: generation and propagation - theoretical aspects"*, two different models are presented and applied in this paper.

The first is a mathematical theoretical model, implemented in MATLAB®, which allows to determine dynamic loads generated in the contact zone, between wheel and rail; the second one is a F.E. simulation models, developed with ADINA® code, that is able to reproduce the propagation of undulatory effects in the railway track and formation soil.

These models have been preliminarily calibrated by means of a comparison with literature and experimental data. Then, the models are applied to simulate different operational conditions, varying speeds of trains and level of irregularities.

The results are compared and discussed, and they demonstrate that models are able to investigate problems of vibrations in railway infrastructure engineering.

Keywords: railway, vibrations, analytical model, FE model.

1. INTRODUCTION

Vibrations are the effects produced by one or more oscillatory composed motions, which are generated and transmitted in physical bodies; the characteristics of motions can be constant or change during an observational period, and only in few cases they can be represented by means of simple mathematical forms or comprehensive models.

The phenomenon of vibrations apparently could seem easy, but it shows many complex facets when we try to quantify, analyze or control it. The measure of vibrations can be made by means of the observation of different variables, which are accelerations, velocities or displacements, and it can be represented in the time-domain or in the frequency-domain.

The railway vibrations, in particular, are periodic or non-periodic motions generated by the dynamic interactions between the different components of the complex railway system: the wheels, the rail line, the track and the formation soil. These elements, characterized by masses and physical features, produce various oscillatory motions when the trains pass, and the effects of these motions propagate in form of dynamic effects toward structures, buildings and people in the surrounding areas. In particular, the loads applied on the rail by the wheels, at the contact interfaces, cause the generation of the vibrations; then, these effects are transmitted in the track and in the soil: this is the phenomenon of the propagation.

There are many studies regarding vibrations, but the most of scientific literature generally is focused on the typical problems of mechanical engineering; for example, the control and the monitoring of rotating machines, the effects on the people of vibrations transmitted inside the cars, trains or plains, etc.. The passage from these problems to those typical of the civil engineering, like the railway vibrations, is quite difficult because the analytical schemes are very different.

Anyway, there are various studies which explain the theoretical aspects of railway vibration problems and the most recent research results. In the paper "*Railway traffic vibrations: generation and propagation - theoretical aspects*" [2], various kinds of approaches and models have been presented, as a literature review.

In this paper, two computational models will be presented: the first one is a mathematical theoretical model, able to reproduce the phenomenon of the generation of vibrations. The model can calculate the displacements of the involved bodies (wheel, rail, track, etc.) due to unevenness and the loads exchanged in the contact surfaces; it can analyze the phenomenon under various test conditions. This model can be implemented by means of a suitable software, like that here presented, which has been developed in MATLAB®.

The second tool used in the present study, indeed, is a FE model. These are models usually adopted in the structural analyses or to solve other problems, in the fields of the engineering or physics. As regards to the railway vibrations, the FE models are especially useful to study the phenomenon of propagation: this process, in fact, is caused by the transmission of the waves through the solids (the track and the formation soil), in which stresses and deformations are produced. The propagation can be measured by means of the monitoring of accelerations (or speeds) in various points of these bodies. In the present study, a common computational software (ADINA®) was used to evaluate the propagation modalities.

Described models (the theoretical-mathematical model and the FE model) can be used in sequence, to evaluate the problem of railway vibrations comprehensively. In the present paper, the models are improved and applied to particular conditions.

The results confirm that this approach can be used to study the importance of various parameters (like the train speed, the track stiffness, the unevenness levels, the material properties, etc.) over the problems caused by the railway vibrations, because the models are sensitive to all these parameters.

2. MODELLING OF THE GENERATION OF RAILWAY TRAFFIC VIBRATION

The modelling technique developed aims to evaluate the dynamic phenomena produced by the contact between wheel and rail, due to the rolling motion and to the superficial and geometric irregularities.

In a first phase the behaviour of the elements (track and vehicle) has been evaluated by means of suitable models and approximations. In particular, the behaviour of the track has been studied through a MDOF system (Multi Degree Of Freedom), while, as it regards the vehicle, it was considered a single wheel model, having mass equal to the mass of the entire wheel set, loaded by the weight of the vehicle components over it (bogie and box). This approximation, that considers only the unsuspended mass in the dynamics of the vehicle, is justified by the fact that the eigenfrequencies of the suspensions system are significantly lower than the vibrations induced by the wheel-rail contact.

2.1 Wheel set – track interaction

The study has been conducted separately analyzing the vehicle and the track, modelling the interaction through a linear Hertzian contact.

The behaviour of the track is represented, in the frequency domain, through the frequency response function (FRF). Preliminarily, this function has been calculated, through the harmonic analysis of the track, schematized with a bidimensional lumped mass model. Such calculation has been done using the ADINA® (Automatic Dynamic Incremental Not Linear Analysis) Finite Element code. The model is constituted by elementary mechanical components: masses, springs and dampers.

The frequency analysis consists of applying a sine wave force to the structure and calculating the response in terms of amplitude, varying the frequency of the action. Applying a unitary force the result of the analysis is a diagram (amplitude-frequency) that represents the frequency response function of the model (see Fig. 1, Fig. 6).

2.2 MDOF System Equivalent Model

The numerical model of the track was developed passing from the frequency domain to a time domain analysis, and approximating the continuous structure (infinite degrees of freedom) by means of a MDOF (multi degree of freedom) system. This task, realized through a "harmonic transfer function" able to best approximate the frequency response function, as described in the previous paragraph, is performed expressing such transfer function in the Laplace domain, as a polynomial form:

$$H(s) = \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)} = \frac{B(s)}{A(s)} = \frac{b_1 s^m + b_2 s^{m-1} + \ldots + b_m s + b_{m+1}}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} \qquad \text{(eq. 1)}$$

The numerator and the denominator are the Laplace transform, respectively, of the output signal and of the input signal of the system.

In the (1), s represents the Laplace variable, z_i denotes the zeros and p_i denotes the poles of the function; the number of poles represent the number of degrees of freedom considered in the MDOF system.

The transfer function has been therefore determined fixing the values of m and n (respectively 3 and 4) and calculating the real coefficients $a_1 \ldots a_n$ and $b_1 \ldots b_{m+1}$, using the *"invfreqs"* function in the "*Signal Processing"* MATLAB® Toolbox

Once represented the behaviour of the superstructure in the Laplace domain, it is possible to calculate the time history of track displacements, through the antitrasform of the transfer function *H(s)*, determining therefore the differential equation of the motion.

$$(D^n + a_1 D^{n-1} + \ldots + a_{n-1} D + a_n) y(t) = (b_1 D^m + b_2 D^{m-1} + \ldots + b_m D + b_{m+1}) f(t) \qquad \text{(eq. 2)}$$

In the (2) *D* represents the differential operator d/dt, *y(t)* and *f(t)* are respectively the output and the input of the system that, for this model, are the displacements of the rail and the interaction forces in the wheel-rail contact.

The representation of the (2) by means of MATLAB® "*state space*" approach is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ldots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 & 0 & 0 \\ -a_2 & 0 & 1 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -a_{n-1} & 0 & 0 & 0 & 1 \\ -a_n & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_m \\ b_{m+1} \end{bmatrix} \cdot f(t) \qquad \text{(eq. 3)}$$

$$\text{where} \quad x_1(t) = y(t) \qquad \text{(eq. 4)}$$

In the (3) only f and x_1 have a physical meaning: they represents the interaction forces and the displacements of the rail in the contact point.

2.3 Wheel - Rail Interaction Equation

Using the approximation to represent only one of the vehicle wheels, as previously discussed, it is possible to express the differential equation that describes the motion of the wheel. The input data used in the model are:

- W (load over a wheel)
- M_R (unsuspended mass representing half wheel set)

Therefore the differential equation can be expressed in the form:

$$M_R \cdot \ddot{x}_R = (W - f) \tag{eq. 5}$$

The wheel-rail interaction can be represented using the differential equations of wheel and track, as expressed before, with a nonlinear Hertzian contact. Therefore it is:

$$f = \begin{cases} C_H (x_R - x_1 - r)^{3/2} & x_R - x_1 - r > 0 \\ 0 & x_R - x_1 - r < 0 \end{cases} \tag{eq. 6}$$

Where x_R is the wheel displacement, x_1 is the rail displacement, f is the nonlinear interaction force, C_H is the Hertzian constant and r it is the excitation caused by the irregularity of the contact surfaces (the conventional sign for this parameter is positive in the case of a depression, negative for an asperity).

2.4 Input data of Irregularities

The proposed analysis model attributes the generation of the vibrations to various factors, and in particular to the presence of irregularities on the contact surfaces; these generally include the defectiveness of the rail head (undulated wear and surface roughness) and the "facets" of the circular profile of the wheel (*wheel flats*), usually due to the wear produced by the braking process, when the wheel blocks.

The distribution of these irregularities can be represented as a random process and, in this research, it was calculated using the theories of the stochastic analysis.

Analyzing the problem, it needs to consider that irregularity is not applied to a single system, as in the case of a simple random force applied on the track, but it takes place between two systems (wheel and rail) that are connected, in the time domain, through a nonlinear Hertzian contact.

For this motive it is not easy to apply a procedure frequently used for the analysis of mechanical systems, that determines the power spectrum of the forces applied to the track $F(\omega)$ multiplying the power spectrum of the irregularities and the square of the transfer function:

$$F(\omega) = |H(\omega)|^2 \cdot S(\omega) \tag{eq. 7}$$

Consequently, in the developed analysis the choice of the input of the irregularity and the evaluation of its effects has been done with a "ad hoc" procedure.

The defectiveness of the contact surfaces is expressed through the spectral density function $S(\omega)$: this can be obtained using experimental data, available from the measurements and data processing performed by the instrumented vehicles used to check the track geometry. Irregularity therefore can be represented overlapping sine wave functions, varying the amplitude, spatial frequency, phase, function of the adopted curvilinear abscissa (railway chainage). For this analysis two expressions have been assumed for $S(\omega)$, with different values of amplitude, to consider the cases corresponding to two different real conditions of “high” and “low” level of irregularity.

Given the particular interaction between wheel and rail, where the irregularity is on both the contact surfaces, the expression of $r(t)$ adopted in the differential equations has been determined first calculating the abscissas where defectiveness are distributed, and subsequently assigning to each point the relative values of irregularity. Overall, irregularity is obtained by adding the values assumed for the two sine wave components, related to the wheel and to the rail.

In the system of differential equations that represents the motion, the function *r(t)* is represented therefore by the given values of the defectiveness, expressed as a function of time, assuming a constant speed for the motion of the train.

2.5 Numerical simulation of the interaction

The wheel-rail interaction has been studied using two separate models, that differ for the schematization used for the sleepers and the elements that connect they to the rail and the ballast.

2.5.1 Continuous Support Model

In this model the sleepers are represented considering their mass uniformly distributed along the longitudinal abscissa; on the top these elements are connected to the rail and on the bottom to the ballast, through linear elasto-viscous elements. In Fig. 1, the schematization of the model and the frequency response function (FRF) obtained through harmonic analysis are shown.

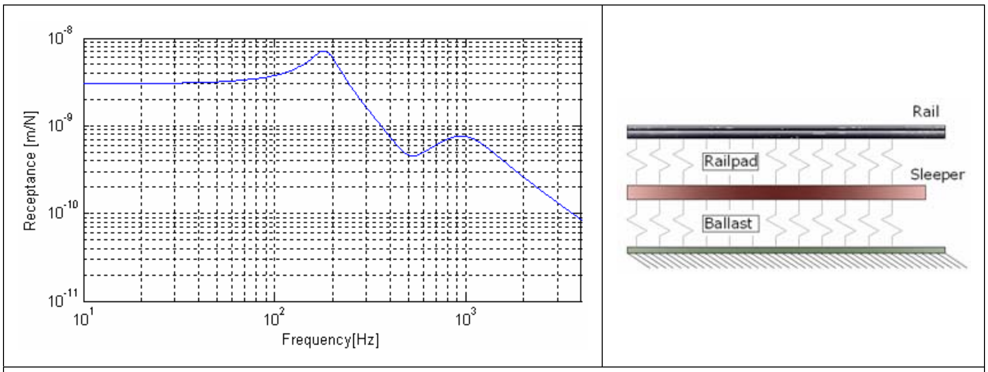

Fig. 1 - Continuous Support Model and its FRF

The MDOF model used for the track on continuous support has 4 d.o.f. (*n* = 4); the parameters obtained by MATLAB® calculation are shown in Tab. 1.

	1	***2***	***3***	***4***
a	$79.89\cdot10$	$12.31\cdot10^{5}$	$11.25\cdot10^{7}$	$40.77\cdot10^{9}$
b	$29.2\cdot10^{-8}$	$65.79\cdot10^{-5}$	$27.77\cdot10^{-2}$	$11.84\cdot10$

Tab. 1 - Parameters of the MDOF model (n=4) – continuous support.

The displacements and the contact forces were evaluated through the resolution of the *state-space* equations (3) and (4). The results are represented in Fig. 2, as a function of time and in spectral form, for the two considered levels of irregularity and three different train speeds (100, 140 and 200 km/h); in the figure, downwards displacements are positive. The deformations oscillate around the positions of equilibrium with peaks that grow with the train speed and the amplitude of the defectiveness.

low level irregularities	*high level irregularities*

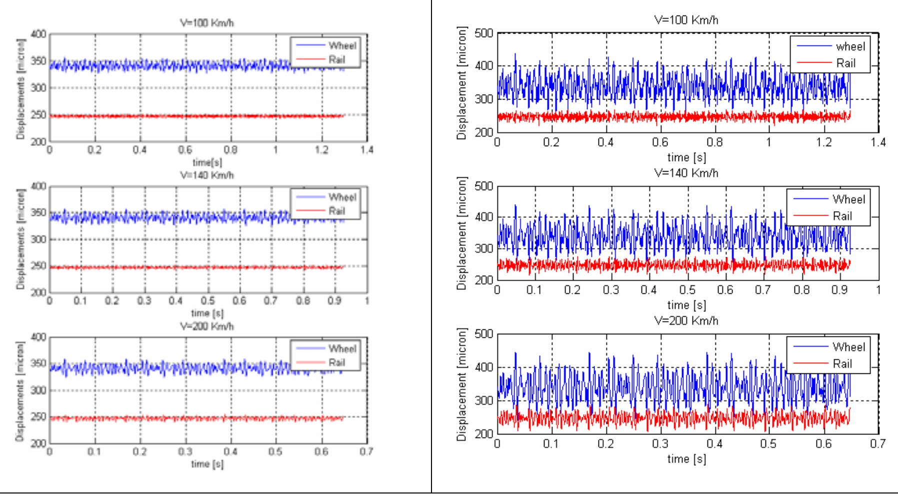

Fig. 2 – Time histories of displacements in the wheel-rail contact point at 3 different speeds (100, 140 and 200 km/h).

The time histories of contact forces are represented in Fig. 3, these are proportional to the displacement of the contact point δ, through the stiffness parameter of the Hertzian spring. Equation (6) can be specialized using the values assumed by the Hertzian constant ($C_H=93.7\cdot10^9\ N/m^{3/2}$):

$$f = C_H \cdot \delta^{3/2} \text{ where } \delta = (X_{wheel} - x_{rail} - r) \quad \text{(eq. 8)}$$

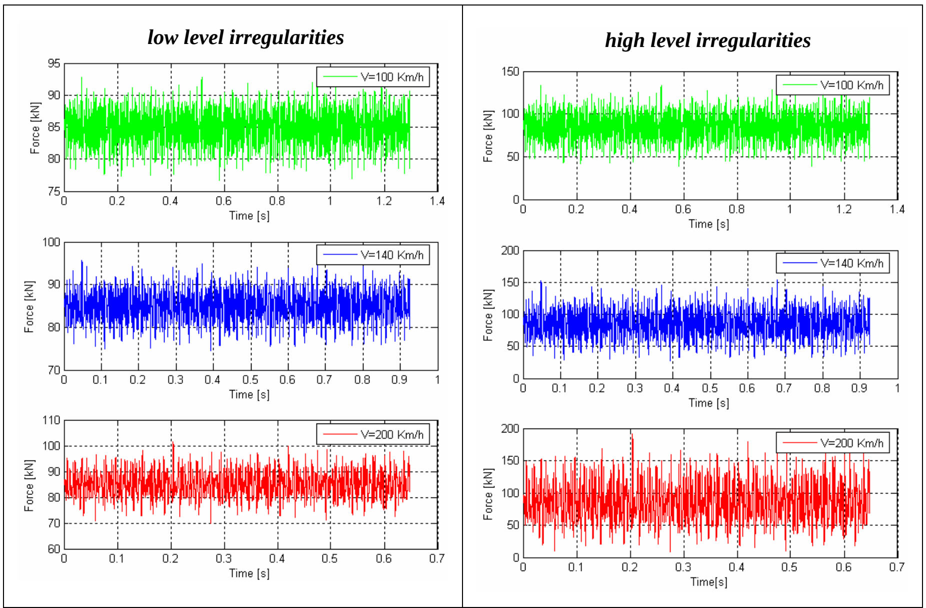

Fig. 3 – Time histories of the interaction forces in the wheel-rail contact point.

The forces oscillate around the static equilibrium values; the contribution due to high level irregularities is, as expected, greater than the corresponding value calculated with low irregularities; the peaks reached by the oscillations increase with the train speed.

To better understand the dynamic phenomena that occur between wheel and rail it is possible to represent the spectrum of contact forces in third octave band and to realize some comparisons between low and high level of irregularities, under various train speed (Fig. 4).

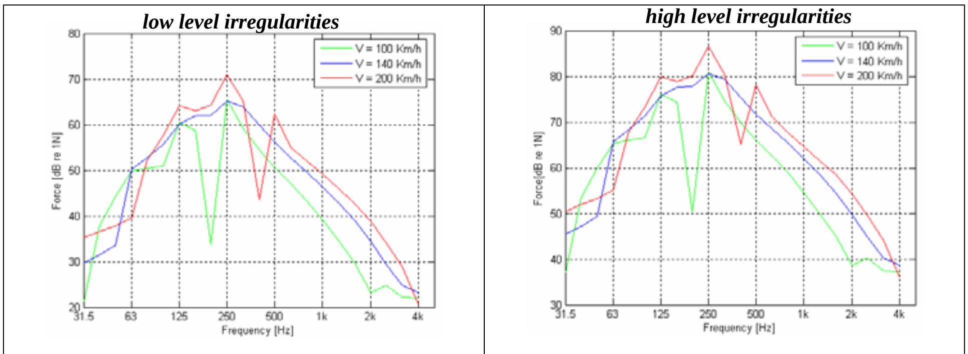

Fig. 4 – Spectrum of wheel-rail contact forces in 1/3 octave band

Varying the level of irregularity the spectrum shape does not change, while shifting the train speed and maintaining constant the wheels and track defectiveness, it is possible to distinguish frequencies where oscillations are damped and frequencies where there is an amplification. Changing the train speed from V=140 Km/h (speed that has the most regular spectrum), to V=100 Km/h, the average power of the signal decreases and the spectrum is damped around 200 Hz. The situation is different for a train speed V=200 Km/h: in this case there is an increase of the average power of the vibratory signal, which is damped around 500 Hz.

After these comparisons it is possible to point out these important aspects:

- The average power increases with the speed, as a consequence of the increase of kinetic energy of the body that cause the vibration (the wheel set).
- For every spectrum function, reported to different speeds, the peak is reached for the same value of frequency (250 Hz).

These results can be explained because the increase of train speed, that in the model corresponds to a variation of the input data (the function which expresses the irregularities as a function of time), does not influence the characteristics of the whole mechanical system, which maintains, also varying its kinetic energy, the same vibrational characteristics (expressed, for example, in terms of eigenfrequencies).

To better appraise this aspect it is possible to develop a more detailed analysis, representing the vibratory signal with spectrum bands narrower than those in third of octave, and showing the forces in linear scale for two levels of irregularity. The Fig. 5 shows that the peaks of the spectral density values are always reached around the same frequencies.

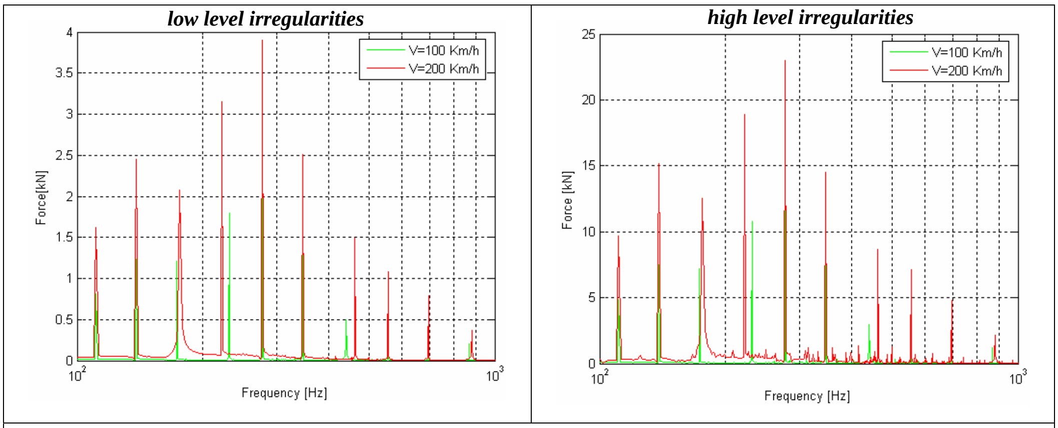

Fig. 5 – Spectrum of vibratory signal (linear scale) for two levels of irregularity.

2.5.2 Discrete Support Model

When a wheel moves along a track, it suffers a parametric excitement due to the presence of the sleepers, that are discrete supports placed at regular interval.

Therefore, the rail has a variable stiffness along the span between two sleepers; as a result, the loads travelling on the track produce a periodic excitement, according to the frequency of their passage on the sleepers.

This analysis methodology is an evolution of that one previously described, referred to the track on continuous support; in this case the interaction train-track and the response of the system to this kind of dynamic excitement (that is called "parametric excitement"), are evaluated using a MDOF model, varying parameters in the time domain, according to the frequency response function of the track, that varies along each span between two sleepers.

Therefore, the railway track is represented through a FRF variable along the time, according to the train speed and to the longitudinal variation of the stiffness along the track. The definition of the FRF in various points of the rail has been developed through the harmonic analysis of the track on a lumped mass model, made by linear beam elements (rails) and sleepers (represented by discrete masses 60 cm spacing, connected on the top to the rails and on the bottom to the ballast through elasto-viscous elements).

The analysis of a single span has been done using a 12 node model (5 cm long elements), therefore 12 frequency response functions has been obtained, that represent the behaviour of the track for various positions of the load.

In Fig. 6 is shown the model of track and the results of the harmonic analysis (for ease of representation only 4 of the functions are represented: mid span, on sleeper, 0.10 and 0.20 m away from the sleeper).

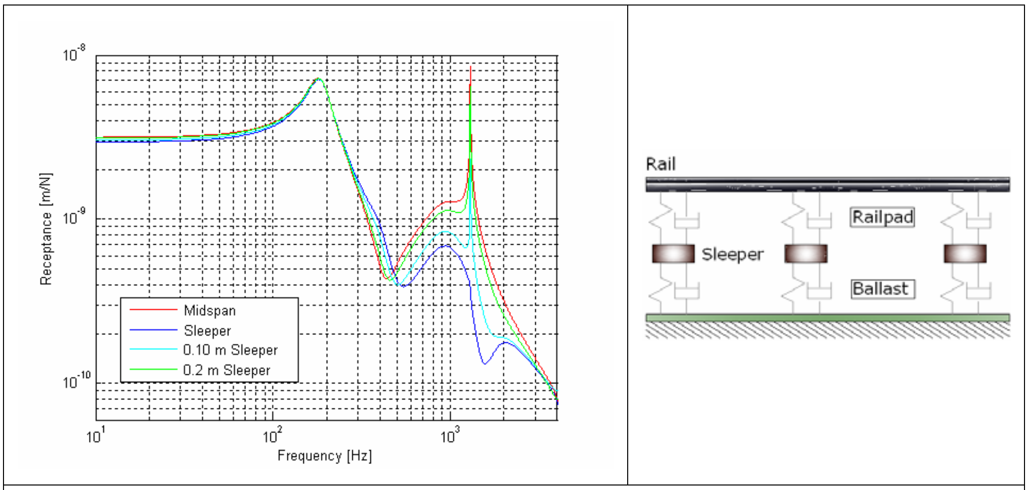

Fig. 6 – Discontinuous Support Model (lumped mass) and its FRF

Fig. 6 highlights the resonance phenomena of the model, that occurs at different frequencies; it is possible to observe that the first resonance frequency falls above the 100 Hz: in this situation the rail and the sleepers oscillate in phase. The second frequency, immediately after 800 Hz, corresponds to a phase displacement of the rail and sleepers vibration: this frequency mostly results influenced by the mechanical properties of the railpads.

Between these two resonance frequencies, there is an antiresonance one around 500 Hz, in correspondence of which the sleepers act as a vibration dampers for the rail, that remains static, while the sleepers oscillate in parallel on the railpad and ballast.

Around 1000 Hz the so-called *'pinned-pinned"* resonance was found: in this situations the rail oscillates with a wavelength 2 times the sleeper to sleeper distance; this means that the track, around this frequency, has a high dynamic stiffness in correspondence and next to the sleepers, while in the middle of the span between sleepers shows an opposite behaviour.

So, the behaviour of the track, in the time, can be represented by the FRF determined in various points, related to the train speed and the sequence of the positions of the wheel on the rail.

The track response is therefore resumed, in the Laplace domain, by a transfer function where the coefficients a_i, b_i, are functions of time and are established according to the 12 harmonic response functions previously determined. The transfer functions have been determined defining the values of *m* and *n* (8 and 9 respectively) and calculating, through the MATLAB® *"invfreqs"* function, the coefficients, $a_1 \dots a_n$ and $b_1 \dots b_{(m+1)},$ for each of the specified FRF.

The evaluation of the displacements and the contact forces was developed through the resolution of the *state-space* equations, using a procedure similar to that used previously (see equation (3) and (4)) in continuous track support.

2.6 Parametric excitement on discontinuous model

In a first approximation it was assumed that vibrations are caused only by the variable dynamic stiffness of the discrete supports. It is possible to represent this condition assuming equal to zero the term of the irregularity in the equation (6):

$$f = \begin{cases} C_H (x_R - x_1)^{3/2} & x_R - x_1 > 0 \\ 0 & x_R - x_1 < 0 \end{cases}$$

Although, in the reality, irregularities are always present on the wheel-rail contact surfaces, this hypothesis allows to appraise the effects of the parametric excitement without the trouble produced by other factors.

2.6.1 Speed effect

For this analysis, two train speeds have been selected: 100 and 200 Km/h; the corresponding passing frequencies on the sleepers are respectively 46 and 93 Hz, for a element-to-element distance of 0.6 ms.

In Fig. 7 and Fig. 8 the simulation results of the wheel-rail interaction are shown. The results are expressed in terms of displacements and interaction forces, respectively in the time domain and in spectral form.

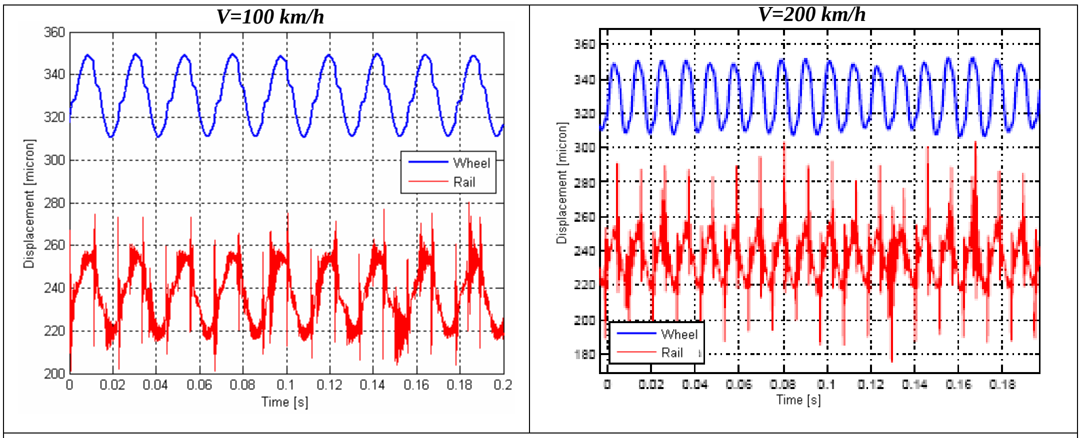

Fig. 7 - Wheel and rail vibrations in the contact point, only due to parametric excitement, at two different speeds (100 and 200 km/h)

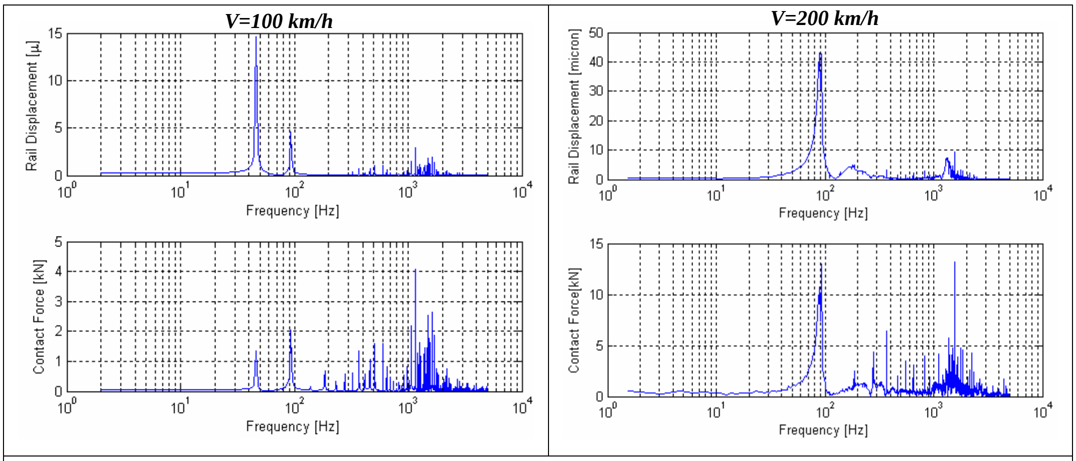

Fig. 8 - Rail vibrations and contact forces in spectral form, at two different speeds

All simulations were developed considering the starting point in correspondence of a support, therefore the origin of the times corresponds, in all cases, to the wheel put above a sleeper.

The wheel and rail vibrations in the contact point are represented in Fig. 7 in terms of displacements (expressed in μm, positive downward).

Comparing the curve that represents the wheel response with the curve referred to the rail, it is possible to notice that first one has a smoother shape because of the greater inertia.

Analyzing the spectrums in Fig. 8 is possible to see that the contact forces are proportional to speed. The main components in the force spectrum are those corresponding to the frequencies of passing over the sleepers, although in the considered range it is possible to observe higher level harmonic frequencies. At around 1000 Hz, near to *"pinned pinned"* resonance frequency, the spectrum of contact forces has high values; in fact a track with discrete supports shows the higher differences, in terms of spectrum response, just because of the "*pinned pinned*" resonance phenomenon (for instance the response reaches a maximum in the centre line and a minimum on the sleeper). As a result, the high parametric excitement in correspondence of this frequency, involves an increase of the contact forces.

Following these considerations it is also expected that the contact force components, around the "*pinned pinned*" resonance frequency can progressively contribute to the wearing of the track.

2.6.2 Level of irregularity effect

Increasing the analysis complexity, both the factors that cause vibrations are considered: the dynamic stiffness (due to discrete supports) and the irregularity of the contact surfaces.

Two train speeds have been selected: 100 and 200 km/h: (Fig. 9 shows the vibrations of wheel and rail in the contact point (displacements are expressed in μm, positive downward)

Comparing the response of the wheel to that of the rail, it is possible to notice that in this case too, the first one is smoother, because of the greater inertia.

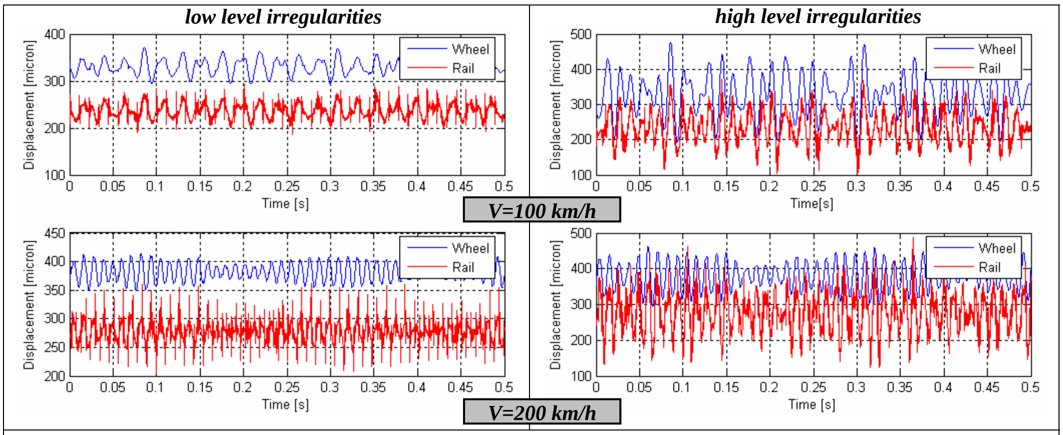

Fig. 9 – Rail vibrations due to low and high level irregularities, at two different speeds

In Fig. 10 the interaction forces in the time domain are represented; they result proportional to the contact deflection (δ) through the Hertzian stiffness constant.

From the graphs it is possible to assume that the forces oscillate around the positions of static equilibrium, and the oscillations increase proportionally to both the level of the irregularities and the train speed.

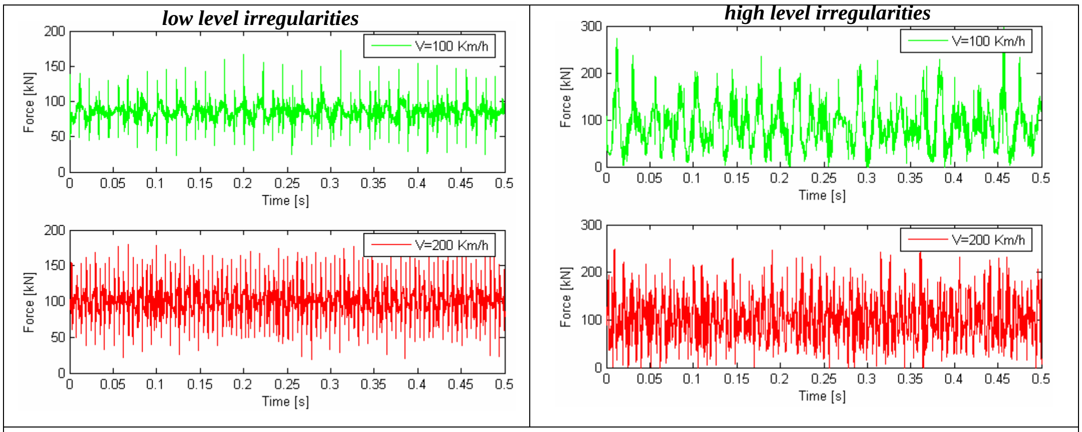

Fig. 10 - Contact forces induced by low and high level irregularities, at two different speeds

It is possible to make a comparison using the contact forces response spectrum in third of octave (Fig. 11).

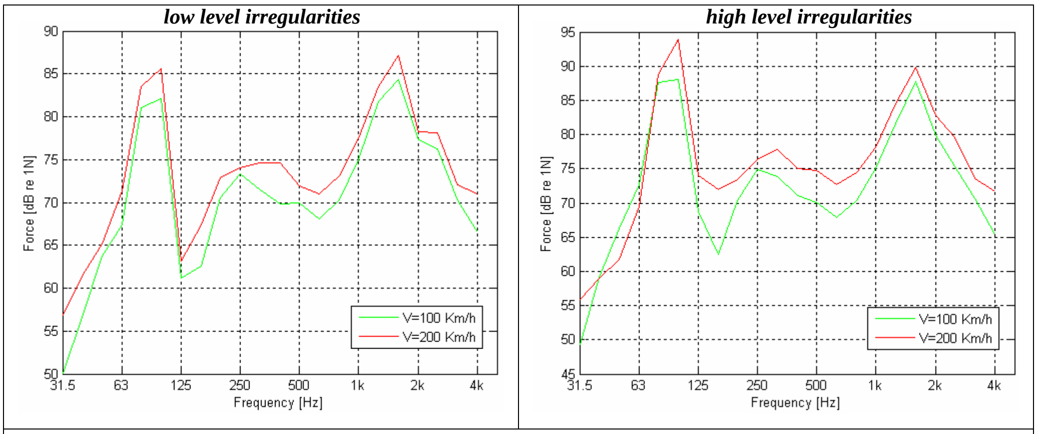

Fig. 11 - Contact forces response spectrum in 1/3 band of octave of Fig. 10 signals

For every speed there is a zone of resonance over 1000 Hz (correspondent to the value of the "*pinned - pinned*" frequency); this, despite the presence of an irregularity input that produces excitement component in a wide range of frequencies, shows that there is a concentration and an increase of the contact forces around those frequencies.

Following these considerations it is possible to deduce that, for an accurate analysis of the train-track system, it is necessary to adopt a model representing the track on discrete support, more similar to the real condition.

3. MODELLING OF THE PROPAGATION OF RAILWAY TRAFFIC VIBRATION

The literature analysis shows many physical-geometric schematizations, useful to model the propagation in the track of railway vibrations; some of these privilege longitudinal sections and are particularly detailed in the track area, where generally it is easier to recognize the static and dynamic function of the elements. The models that include also the lower layers of the railway track (starting from the subballast) are not frequent and, more important, usually less detailed.

The "longitudinal" schematization of track used for the calculation of the transfer function, based on a lumped masses model, is valid only for the evaluation of energy balance between the track components. Indeed, to better represent the continuity of the track components and to evaluate the condition of stress, deformation and vibration at different distances from the moving wheel set, the structure has been schematized using a three-dimensional finite element model.

3.1 Model Characteristics

In this case the rail track is the conventional one, and the elements that compose it, as shown in Fig. 12, lean on a 1 meter tall embankment through:

- a 30 cm thick protective layer of sand/gravel compacted subgrade (called, in the italian railways, *"supercompattato"*),

- a 12 cm subballast layer made of bituminous (hot-mix asphalt) concrete, and

- a traditional ballast layer 35 cm thick.

The characteristics of the track elements (sleepers, rails and fastenings) are those requested by the italian standard (RFI), corresponding to the "high speed" lines.

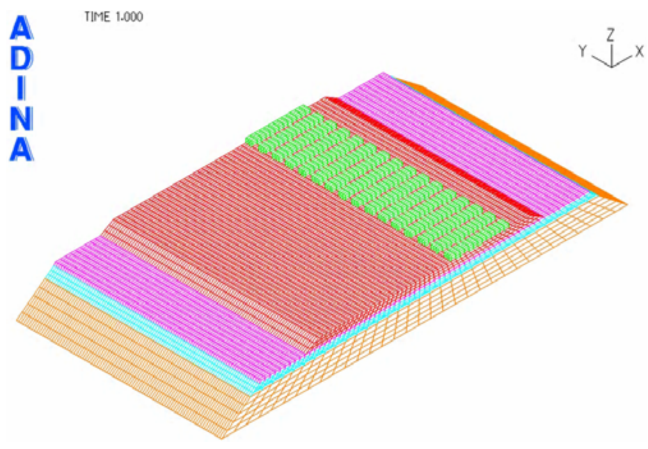

Fig. 12 – FEM of whole rail track

The physical-mechanical characteristics of the materials, used in the model, derive from tests and available experimentations in literature. The adopted values are shown in Tab. 2.

Mechanical characteristics	Rail (UIC60) (*)	Sleeper (concrete)	Traditional Ballast	HMA Subballast	Protective layer	Embankment
Density ρ (kg/m3)	7850	2400	1250	2200	2000	1000
Young's modulus E (MPa)	210000	30000	130	6000	160	80
Poisson's Ratio ν	0.3	0.15	0.3	0.4	0.45	0.5

(*) Moment of inertia J_X =3055 cm^4;Weight G=3055 kg/m

Tab. 2 - Characteristics of the materials

The track has been modelled using the ADINA® finite element code. To avoid the boundary problems due to a limited size, the model has been developed over a length of 100 meters. The characteristics of the model are summarized in Tab. 3.

Track Element	FE type	Total Dimensions (b×h×l) [cm^3]	Note
Rail	1 cm long beam elements (2 nodes each element); 10000 elements in total	17.2 × 15 × 10000	This type of discretization was necessary due to the way the load is applied to the rail
Rail Pad	parallel discrete spring and damper		Spring constant K=11.2·10^8 N/m; Damper constant c=12·10^4 Ns/m
Sleeper	3Dsolid with 27 nodes each (8 principals 19 secondary)	30×20×300	Span is 60 cm
Ballast	3Dsolid (8 nodes)	900×35×10000	Continuous and homogeneous material hypothesis
HMA Subballast	3Dsolid (8 nodes)	1400×12×10000	Continuous and homogeneous material hypothesis
Protective layer	3Dsolid (8 nodes)	1400×30×10000	Continuous and homogeneous material hypothesis
Embankment	3Dsolid (8 nodes)	1400×35×10000	We adopted wider spacing between nodes to minimize computational time

Tab. 3 - Characteristics of the model

In the commonly accepted hypothesis (for these elements) of a visco-elastic behaviour, the input parameters, for the materials characterization, are the modulus of elasticity (E), Poisson's coefficient (ν), density (ρ), (α) and (β) Rayleigh's coefficients for the definition of the ***C*** damping matrix.

3.2 Static analysis

To assess the validity of the model, by means of the evaluation of stress and deformation state induced by static loads, two 100 kN vertical forces, 3 m (one each other) spaced, were applied, in two different configurations: 1) loads put in the mid of spans between sleepers, and 2) loads put above of sleepers (Fig. 13). Because the analysis is static, in the calculation of vertical displacements the inertial and damping components have no influence.

For these conditions, the vertical stresses induced in the track and embankment are shown in Fig. 13; it is possible to observe that the compression stresses zone (yellow and green) is limited to a longitudinal distance of about 1.2 m. The forces induced are higher in the case of loads put above the sleepers, while, putting the static loads in the mid of spans, there is a more uniform distribution of stresses in the underlying layers.

Further considerations can be made considering the vertical deformations shown in Fig. 14, in particular as regards to the structural behaviour of the protective layer that, being a layer more rigid than the subgrade, is the less deformable one.

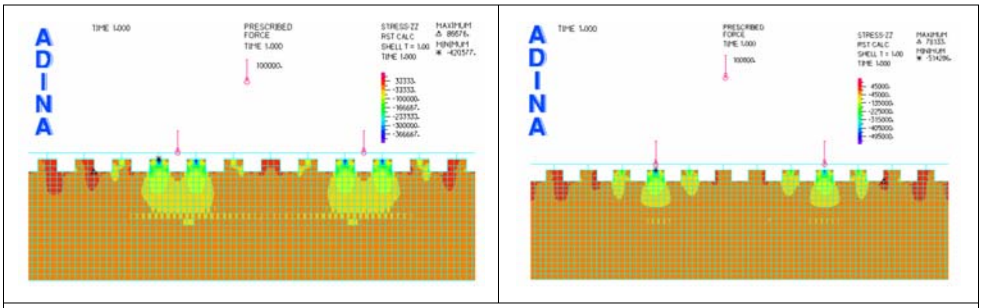

Fig. 13 - Vertical stress distribution, load on mid span and on sleepers

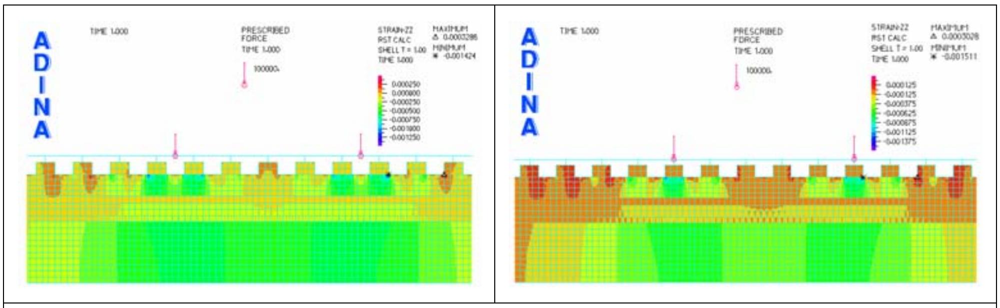

Fig. 14 - Vertical deformations: load on mid span and on sleepers

3.3 Structural characterization and dynamic modelling

The vibrational response of the track, under travelling loads, depends on the characteristics of its components. Since the oscillatory motions are considered in the vertical plane of the track, the considered loads are those acting in that plane.

With the proposed schematization, the components can be separated in two different categories:

- Massive elements, represented by the rail, the sleepers and by the lower layers of the structure;
- Resistive elements, represented by the fastenings (elasto-viscous connections between rail and sleepers), the ballast and the lower layers up to the embankment, through their viscous characteristics.

The acting forces are a sequence of axial loads, moving like the train (the loads are similar to those of a ETR 500). The excitement forces, applied to the model, derive from the simulation of the wheel-rail interaction, previously made in MATLAB®, as discussed in the previous paragraphs. The excitement, in the time domain, is applied to a section in the center of the model.

The forces are applied in each node using a time function, that represents the time history of the force in that node. During the simulation, 3600 nodes (1 cm spaced, total length of the rail 36 m) are excited, every node will be loaded four times, reproducing the passing of four train axles.

Three speeds have been considered: 100, 140 and 200 Km/h: the duration of the simulation is respectively 1.82 s, 1.30 s and 0.91 s; considering the 5049 time intervals (steps) of the proposed model, the single time step duration is respectively of 0.00036 s, 0.0002571 s, 0.00018 s.

3.4 First results of the dynamic simulations

The dynamic analysis implemented through the ADINA® finite element code, has been solved by means of Newmark step by step implicit integration in the time domain. The results, expressed in terms of vertical accelerations, have been evaluated in the middle section of the model, to avoid signal bounce due to boundary effects.

In Fig. 15, Fig. 16 and Fig. 17 are represented, for the 3 selected speeds and for a high level of irregularity, the accelerations values in three nodes of the model (corresponding to rail, sleeper and subballast), in the time and in the frequency domain.

In particular, Fig. 15 shows the accelerations in the rail node: it is possible to deduce that, maintaining constant the level of irregularity and increasing the train speed, higher peak values are found.

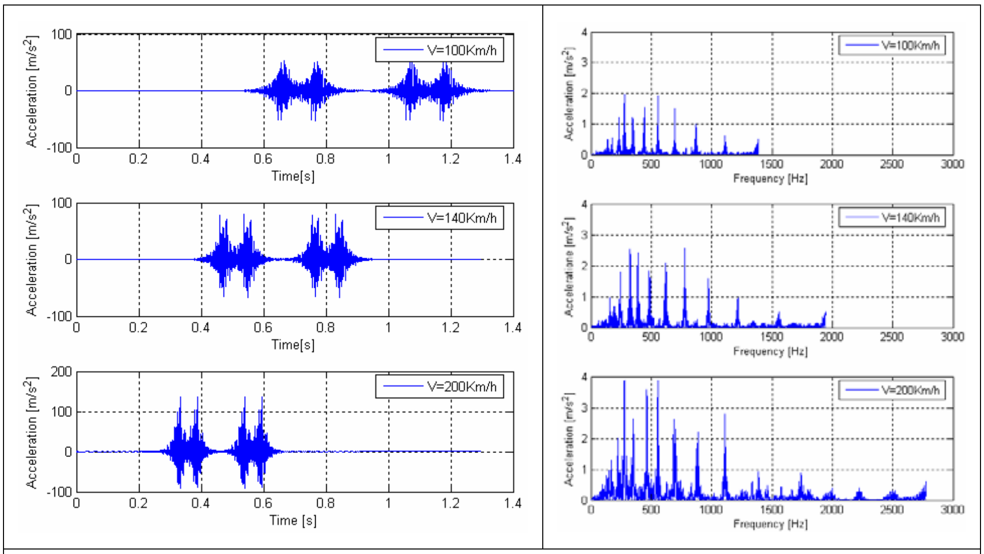

Fig. 15 – Signals and spectrums of accelerations: 3 speeds, rail node and high-level of irregularity

Analyzing the results that represent the propagation of the vibratory phenomenon in the elements and in the layers of the track, it is possible to represent the accelerations in correspondence of the sleeper and the subballast. In Fig. 16 the accelerations in the sleeper nodes are shown; the graphs underline as the amplitudes, in absolute terms, are significantly lower, compared to the values of the rail, and increase with train speed too.

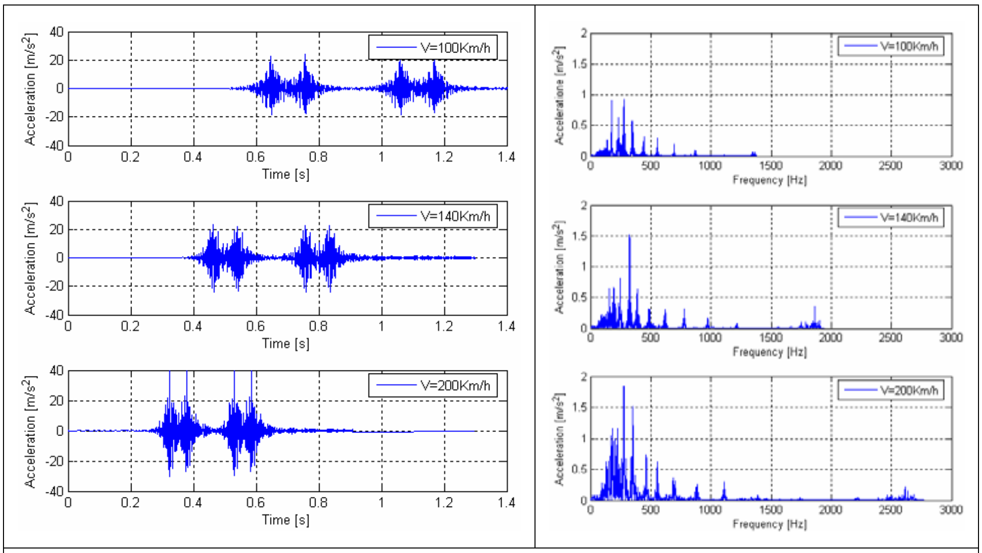

Fig. 16 - Signals and spectrums of accelerations: 3 speeds, sleeper node and high-level of irregularity

The spectrum graphs of accelerations in the subballast nodes, shown in Fig. 17, underline that an increase of speed involve a moderate variation in terms of acceleration amplitude and that the signal components move toward lower frequencies. These considerations can be explained by the presence of the ballast, which has the characteristic to filter the higher frequencies.

4. CONCLUSIONS

In this study two models have been developed, to analyze and simulate the process of generation and propagation of railway traffic vibrations. The complexity and the huge number of elements that influence these phenomena do not allow the development of a single and comprehensive model, able to entirely describe the problem. The proposed methodology, indeed, is articulated in two well defined phases that divide and differentiate the analysis.

The first part, that mainly involves the phenomenon of generation of the vibrations, is carried out with a analytical model implemented in MATLAB®, while the second part is developed by means of a finite element model realized with ADINA® code, and allows the analysis of the propagation through the rail track.

The choice to divide the analysis of the phenomenon in two parts, through two separate models, allowed to determine in a more accurate way the interaction forces produced at the wheel-rail contact, and to have a valid input for the dynamic simulation model of the track.

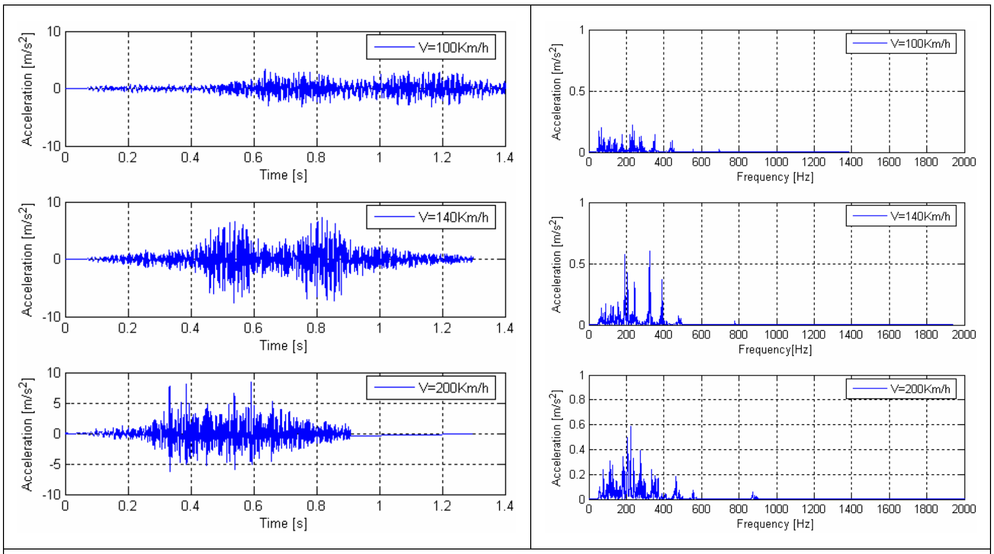

Fig. 17 - Signals and spectrums of accelerations: 3 speeds, subballast node and high-level of irregularity

The results of MATLAB® simulations show what are the elements that mainly influence the phenomenon of vibrations generation.

In particular, the development of a track model on discrete supports allowed to underline the behaviour of the track around the "*pinned-pinned*" resonance frequency, in correspondence of which the rail oscillates with a wavelength equal to 2 times the sleeper-to-sleeper distance. Despite the assumption of an input irregularity on the rolling surface (that produce component of excitement in a wide range of frequencies), the simulations show a concentration of forces in correspondence of the "*pinned-pinned*" resonance frequency, for all considered train speed. This results also remarks the importance of this parametric excitation mechanism for the models used to calculate the undulatory rail wearing, that is mainly caused by the rail stiffness discontinuity.

Regarding the phenomenon of vibration propagation, the analyses contemplate the evaluation of the railway track response to the train transit, the results show that increasing the speed the vibrations on the elements and layers of the track grow, with an increase of the high frequencies. More precisely it is possible to underline that, analyzing the response in the lower layers of the track (subballast, protective layer), the increase of acceleration is less evident and the high frequencies are less affected. This consideration confirms the properties of ballast to damp the high frequency vibrations.

The obtained results, that should be validated through field experimentation and measurements, carry to two important considerations.

- About the typical elements and layers material properties there is a confirmation that the model is performing well.
- Contrarily, the contact forces graph carries high values and the displacements, in particular those calculated through the dynamic analysis, are in the scale of microns: such results do not match the literature values. With a more accurate analysis, trying to look for the modelling defect, it was found that probably there is an overestimation of the fastening stiffness.

Therefore, the models developed in this study seem able and useful for a deep and accurate investigation on the vibration phenomena, and in particular on the characteristics of materials able to reduce the level of transmitted vibration.

REFERENCES

[1] AURESH, L. (2006) – "Ground vibration due to railway traffic – The calculation of the effects of moving static loads and their experimental verification" – *Journal of Sound and Vibration*, 293, pp. 599-610.

[2] BONIN G., CANTISANI G., CARBONARI M., LOPRENCIPE G., PANCOTTO A. (2007) – "Railway traffic vibrations: generation and propagation - theoretical aspects" – *4th SIIV Congress "Advances in transport infrastructures and stakeholders expectations"*, Palermo, 12-14 Sept 2007.

[3] DI MINO, G. (2003) – "Le vibrazioni in campo ferroviario: analisi F.E.M. dell'interazione convoglio-sovrastruttura-terreno" – *Proceedings of the XIII National Conference SIIV*, Padova, ITALY.

[4] DIANA, G., BOCCIOLONE, M., COLLINA, B., CAVAGNA, B. and ACQUATI, M. (2003) – "Armamento Milano Massivo: concezione, simulazione dinamica e sperimentazione su prototipo in laboratorio" – *Ingegneria Ferroviaria*, febbraio 2004, pp. 113-123.

[5] GRASSIE, S.L., GREGORY, R.W., HARRISON, D. and JOHNSON, K.L. (1982) – "The dynamic response of railway track to high frequency vertical excitation" – *Journal Mechanical Engineering Science*, 24(2), pp. 77-90.

[6] NIELSEN, J.C.O. and IGELAND, A. (1995) – "Vertical dynamic interaction between train an track – influence of wheel and track imperfections" – *Journal of Sound and Vibration*, 187(5), pp. 825-839.

[7] O'BRIEN, J. and RIZOS, D.C. (2005) – "A 3 BEM-FEM methodology for simulation of high speed train induced vibrations" – *Soil Dynamics and Earthquake Engineering*, 25, pp. 289-301.

[8] SHENG, X., JONES, C.J.C. and THOMPSON, D.J. (2003) – "A comparison of a theoretical model for quasi-statically and dynamically induced environmental vibration from trains with measurements" – *Journal of Sound and Vibration*, 267(3), pp. 621-635.

[9] URBANI, L. (2004) – "Prestazioni antivibranti di subballast ferroviari in conglomerato bituminoso" – *Doctoral Thesis in Railway Engineering*, XXVII Ciclo, Università degli Studi di Roma "La Sapienza".

[10] WU, T.X., THOMPSON, D.J. (2004) – "On the parametric excitation of the wheel/track system" – *Journal of Sound and Vibration*, 278, pp. 725-747.