INFLUENCE OF THICKNESS AND INTERFACE CONDITION VARIABILITY ON BACKCALCULATED LAYER MODULI FROM SURFACE DEFLECTION

D'Apuzzo; M.
Assistant Professor - University of Cassino , Italy - dapuzz@unicas.it
Nicolosi; V.
Associate Professor - University of Rome "Tor Vergata", Italy - nicolosi@uniroma2.it
Mattarocci; M.
Engineer

ABSTRACT

Evaluation of pavement structural performance by Non Destructive Test (NDT) has been growing since the introduction of the Benkelman Beam at the WASHO Road Test in the early 1950's. The deflection basin data have to be analyzed d to obtain the structural properties of pavement layers and subgrade, that are typically used for pavement evaluation and rehabilitation design. One of the more common methods for analyzing deflection data is the back-calculation, in which theoretical deflections are compared with measured deflections and the assumed moduli are then adjusted in an iterative procedure until theoretical and measured deflection basins match acceptably well. The iterative process assume the pavement model as an input, and the accuracy and the precision of the backcalculation process can be reduced by errors due to the inaccurate input evaluation. Particularly pavement layer thickness and interface condition between layers are affected by a construction and measurement related variability. As matter of fact layer thickness information are measured accurately from core, in a limited number of test points because of their destructive nature, and are rough estimated from Ground Penetrating Radar equipment along the entire profile section. Layer interface properties depends on many factors (such as temperature, coat binder, normal pressure etc.) and are very difficult to be estimated.

In this study a sensitive analysis to evaluate the combined impact of the layer thickness and interface condition variability was performed. A back-calculation program to accurately evaluate the layer moduli, based on a linear elastic multilayer model, was developed and a simulation procedure, based on Monte Carlo method, was applied to represent the thickness variability. The study showed that layer thickness and interface condition variability have a strong impact on the back-calculated results, therefore it was proposed to perform the design of maintenance treatment according to suitable percentile value of their probability distribution. This former could be evaluated by the simulation procedure developed, as a case study reported shows.

Keywords: Deflection Testing, backcalculation, layer thickness variability

1. INTRODUCTION

Structural evaluation of in-service pavements is a key activity for both the project and network level pavement management systems. It provides the necessary information for decision making by highway agencies concerning pavement maintenance programming and rehabilitation planning.

The approaches for structural evaluation of pavements can be classified into destructive and non destructive testing. The most common methods of destructive testing are: coring techniques and test pits. Destructive test methods are rarely used today for normal structural evaluation of in service pavement section because they are costly, tend to cause traffic obstruction and are time-consuming; as matter of fact non destructive tests (NDT) have become the norm for either project or network pavement management systems. Among the NDT adopted for structural evaluation of in service pavement sections two have received the most attention by technical community: deflection-based approach and wave propagation technique. The non-destructive testing of pavement using wave propagation involves the measurement of velocities and wavelengths of surface waves transmitted from vibration source on pavement surface [Hildebrand 2002]. Since the interpretation of test results is not straight forward, and due to the difficulties in relating the computed parameters with those under the different stress and strain states of pavement materials generated by moving traffic loads, this approach has remained largely a research tool [Trefor 1995]. Deflection test methods employing static or slow moving loads were used initially in the 1960s, and since the early 1970s they have been widely adopted as the most practical approach for non-destructive evaluation of pavement properties. In order to meet higher test speed requirements, for minimizing traffic delay , and higher measurement precision needs, a lot of test devices were introduced between the early 1980s and the late 1990s. In conjunction with the development of deflection test devices a large number of empirical and analytical procedures have been developed to estimate the material properties of different pavement layers using the measured deflections. Parameter identification problems involve *forward* as well as *inverse* techniques. In its current form, forwardcalculation only involves the use of certain portions of the FWD deflection basin to derive the modulus (stiffness) of the subgrade and bound surface course, using closed-form solutions. In other words, there is only one directly calculated solution for each of these values, given the deflection data and the layer thicknesses. The forwardcalculation formulae used to deduce the subgrade modulus mainly use deflections measured at larger distances from the load as well as the center deflection, while the surface course modulus is mainly a function of the near-load deflections and/or the radius of curvature of the deflection basin [Stubstad 2005, Jang 2005]. The main drawbacks to this approach are:

- The subgrade and surface course stiffnesses are calculated independently of one another through different forwardcalculation formulae, therefore in combination the values obtained may or may not be reasonable with respect to the total center deflection;
- The forwardcalculated bound surface course modulus has to be a single value, with all bound layers combined into a single, effective surface course layer, and

intermediate layers stiffness are obtained by approximate solution;
- Forwardcalculation produces approximate values (particularly for the base or intermediate layer/s), which should only be used as modulus estimates (for screening backcalculated moduli, for quality assurance/quality control applications, etc.).

The backcalculation procedure involves calculation of theoretical deflections under the applied load using assumed pavement layer moduli. These theoretical deflections are compared with measured deflections and the assumed moduli are then adjusted in an iterative procedure until theoretical and measured deflection basins match acceptably well. Within the past couple of decades, there have been extensive efforts devoted to improving backcalculation of elastic-layer modulus by reducing the absolute error or root mean squared (RMS) error (difference between measured and calculated deflection basins) to values as small as possible, and a lot of computer programs have been developed. Almost all of the programs are based on the following assumptions: surface load is uniformly distributed over a circular area, all layers are homogeneous, isotropic, and linearly elastic, upper layers extend horizontally to infinity, bottom layer is a semi-infinite half-space, layers thickness are constant and there is full friction between all layer interfaces [Ullidtz 1995]. Backcalculation procedures are very popular today, however some critical issues remain because of the fundamental assumptions underlying theoretical models typically used. Particularly it should be noticed that calculated deflections depend on inertia which represents the third power of the pavement thickness. Therefore the back calculated layer moduli are extremely sensitive to layers thickness data, which are one of the most critical elements in the interpretation of deflection testing results. At the same time we have to remark that variability exists in the real world thickness data because of construction defects and measuring errors. As far as layer interfaces are concerned the observed distresses in the pavement structure, as well as field and laboratory tests indicate that the full-bonding between layers does not take place in most situations, and variation of friction in layer interfaces should be also considered in backcalculation analysis.

In this research the possible magnitude of the errors in the backcalculation process generated by uncertainty in layers thickness and friction was evaluated. Moreover new analytical backcalculation procedure was developed in which:

a) frictions are considered as unknowns, together with moduli;
b) a stochastic approach was incorporated into the backcalculation analysis, as proposed by Ullidtz and Coetzee [Ullidtz 1995].

As a matter of fact, if some input variables (e.g. thickness) are accepted as random variables, with given density function, then the output variables (layers moduli) are also random variables defined by a probability density functions [Grogan, 1998]. Once these distributions is known, upper and lower confidence bounds for the predicted moduli can be established [Yusuf Mehta 2003].

2. CONSIDERATION ABOUT THICKNESS AND INTERFACE VARIABILITY

Traditionally, determination of layer thickness relies on destructive tests (coring/bore) or Ground Penetrating Radar technique (GPR); both methods normally introduce an error [Stubstad 2002].

Inaccuracies introduced by coring include:

Data density - Few cores/bores is typically taken from each homogeneous section (conventionally only one measure), due to cost and safety reasons, and thickness is assumed to be constant within each section (variability ignored), therefore the spatial density of thickness data is usually about 7 times less the deflection data.

Data recording - The pavement thickness information is obtained by performing a manual measurement of the core then manually recording this data for subsequent data entry).

Ground penetrating radar was used for the last 30 years as a nondestructive evaluation technique to asses pavement structures. This technique is simply based on sending electromagnetic waves through the pavement structure and then recording the echoes (reflected signals) created at boundaries (dielectric discontinuities) of dissimilar materials. The arrival time and strength of these echoes can be used to calculate pavement layer thickness and other properties, such as moisture content [ASTM D4748 2006, ASTM D6432 2005 and Maser 1994]. Unlike destructive tests GPR results provide nearly continuous layer thickness profiles, which is ideal for backcalculation analysis. However some problems arising from GPR data interpretation for estimating pavement layer thickness: layer interfaces cannot be detected unless a significant differences in the dielectric properties exists between the two layers; there exist a minimum thickness (called depth resolution) that could be reliably detected; thickness estimation depends on dielectric constant of the layers that is greatly affected by moisture content and mixture proportions of different components. A lot of researches analyzed the accuracy of pavement thickness measurement made by GPR, and the range suggested are reported in table 1 [Maser 1994, Lahouar 2002, Selezneva 2002, Maser 2003, Al-Quadi 2005] .

Table 1 Range of accuracy for pavement leyer thickness measurement by GPR

Layer Type	Accuracy (vs Cores)
Hot-mix asphalt	± 3 ÷ 10 %
Granular layers	± 8 ÷ 15 %
Concrete	± 5 ÷ 10 %

As previously illustrated many linear layer elastic backcalculation programs use full friction between all layer interfaces but some researches showed that this assumption in not fulfilled in reality [Lenngren 2003]. Among the few studies conducted to evaluate the possible magnitude of the errors in the backcalculation process generated by the improper assumption about friction (i.e. pavement layers fully bonded), are these by Hakim et al. and Romanoschi [Al Hakim, 1996, Romanoschi 2003]. The former used the structural analysis program BISAR to compute twenty deflection basins for four pavement structures and five values of shear reaction modulus at the interface (Goodman’s model was used in order to represent interlayer friction 10^{-1} - 10^{-5} m^3/MN), and then compare the initial values of moduli to the full friction backcalculated moduli (for the base, subbase and subgrade layers). They find base layer moduli error of 40 percent from the real values and subbase layer moduli of 70 to 140 percent of the real values while the error in modelling the interface condition did not significantly affect the value of the backcalculated subgrade modulus.

The recent study by Romanoschi et al. proposed a new constitutive model in which layer interface restraint is completely described through three parameters: interface reaction modulus K, shear strength τ_{max} and friction coefficient after failure μ. This latter model was used to study the influence of interface condition on the magnitude of surface deflection through a finite element analysis. It should be noticed that Romanoschi and Goodmans models are very similar in field of small shear displacements (i.e. $d_{max}= \tau_{max} / K = 1.6$ mm). As in the previously study Romanoschi et al. compute the deflection and then backcalculated layer moduli, using full friction; the findings were: reaction modulus K of the wearing-binder and binder-base layer interface affect the values of backcalculated layer moduli (asphalt layer moduli are overestimated up to 120% and stabilized base modulus is understimated up to six time), and has little influence on subgrade modulus.
Even though it is of utter importance taking into account interaction between different layers, only few researches investigated this phenomena [Uzan 1978 , Crispino 1997], and they observed very different values of shear reaction moduli (see table 2) and a large variability of experimental results (COV ≈ 14 %).

Table 2 Interface reaction moduli at 15 and 35 °C [Uzan 1978 , Crispino 1997].

Temperature [°C]	Mean Interlayers reaction modulus K [MPa/m]
15	4000 (1) ÷ 16000
35	800 (1) ÷ 14000

3. THE METHODOLOGY

The above mentioned problems due to input data, was addressed by development of a new backcalculation procedure, based on linear elastic layered theory. In this routine the Goodman’s model was used in order to take into account the effect of slippage between layers, and shear reaction moduli was considered as unknowns; that is to say the layer moduli as well as interface reaction moduli was adjusted in an iterative procedure until theoretical and measured deflection basins match acceptably well.

Moreover in this backcalculation procedure a stochastic approach was used to account for the variability in layer thickness (within homogeneous pavement sections or due to measurement error), as illustrated below. It have to be remarked that the suggested stochastic approach can be incorporated in any other backcalculation procedure.

3.1 Framework used in the analysis

The basic flowchart that represents the fundamental elements in the stochastic backcalculation program is shown in Figure 1. Briefly, these elements include:

Data input Includes, for a specific test location, the measured pavement surface deflections and associated distances from the load, pavement type, load levels and frequency, mean and variance of layer thicknesses.

Generation of data Layer thickness are generated by Monte Carlo simulation.

Seed range moduli The seed ranges of moduli are estimated by predictive models in order to prevent program from converging to unreasonable moduli values (either high or low). Furthermore the middle values of the ranges are used as initial estimate of moduli in the computer program to calculate surface deflections.

Deflection calculation Layered elastic computer programs are used to calculate a deflection basin for generated thickness.

Error check The measure of how well the calculated deflection basin matches (or converges to) the measured deflection basin.

Search for new moduli The linear methods is employed to converge on a set of layer moduli and interface shear reaction moduli which produces an acceptable error between the measured and calculated deflection basins.

Results analysis The simulation results are analyzed in order to represent the stochastic distribution of layer moduli (i.e. mean and variance are evaluated and the normality of distribution are tested).

In the following paragraph some of these elements are illustrated in more detail.

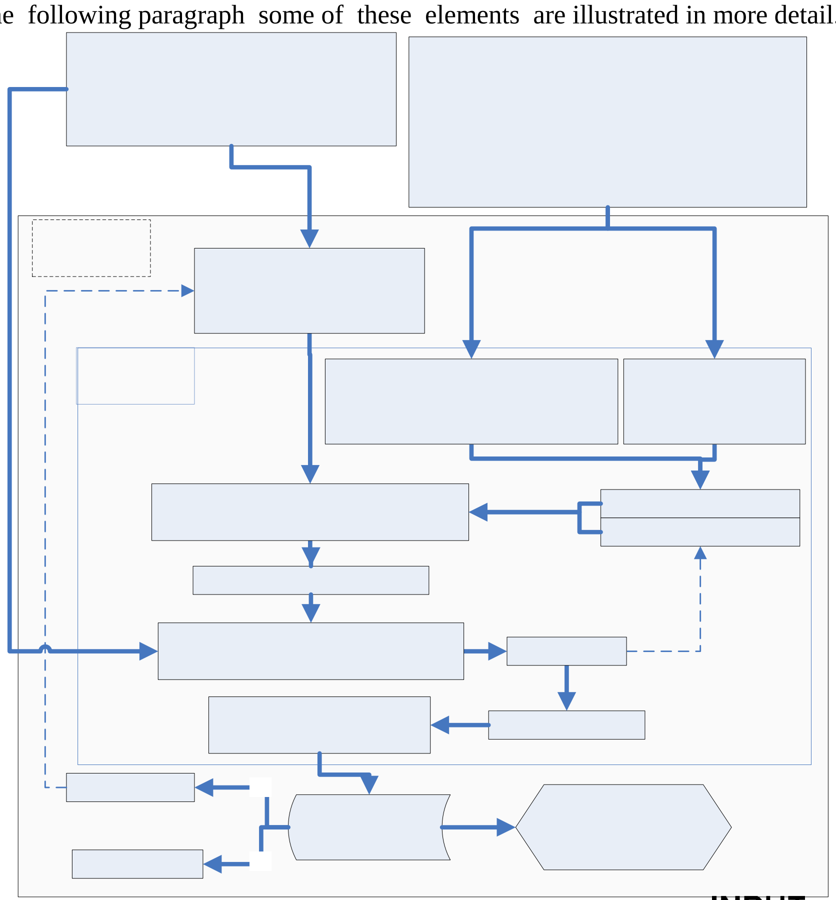

Figure 1 Flowchart of the stochastic backcalculation procedure

3.1.1 Input data generation

As previously stated the stochastic approach proposed aim at investigating how does backcalculated layer moduli vary when the data about layer thickness vary, according to some assumed probability distribution (error distribution for GPR measures and construction variability for cores/bores). A Monte Carlo simulation, based on a constrained sampling scheme well-known as "Latin Hypercube sampling", was applied [Wei-Liem 1996], in which the range of each variable is divided into m non-overlapping intervals on the basis of equal probability, and one values from each interval is selected at random with respect to probability density in the interval. The m values thus obtained for each "h_i" variable (i=1 to k layer thickness) are paired in a random manner (equally likely combinations), the pairing was done by associating a random permutation, of the first m integers, with each input variable (see figure 2). To help clarify the sampling process, consider m = 5 intervals (each interval correspond to a 20% probability) , let be h_{ij} the values of i-th layer picked in the j-th interval, use a permutation to order the values and form a vector (e.g. let be the permutation 3, 1, 5, 2, 4 the vector for the i-th variable is h_{i3}, h_{i1}, h_{i5} , h_{i2}, h_{i4}). Next form an (m × k) matrix of input where the column are the vector formed as previously illustrated; the i-th row of the matrix contains specific values of each of the k thickness layers to be used on the i-th backculation run. The process was repeated n times in order to generate n × m input vectors. This approach yields reasonable estimates for the distribution of the dependent variable (i.e. layer moduli) even with reduced dimension sample, unlike non constrained sampling. In the framework proposed we assume that random variables "h_k" have a normal distribution with a mean equal to measured thickness and standard deviation equal to measurement error.

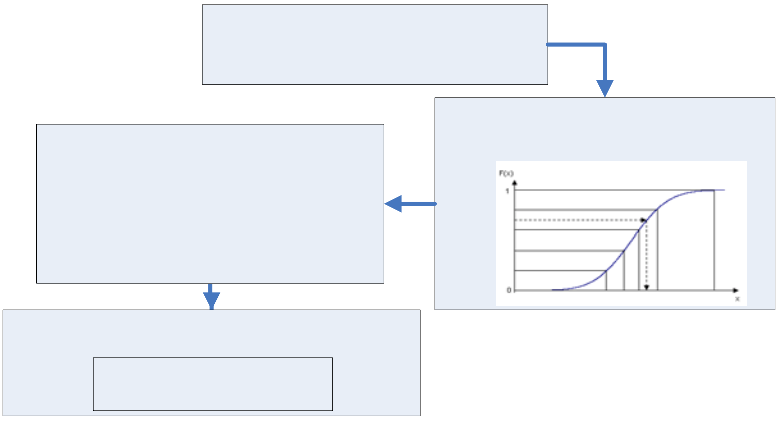

Figure 2 Sampling scheme for generating input thickness values

3.1.2 Seed range moduli

The seed range of dynamic moduli of asphalt concrete layers were evaluated by two predictive models: Asphalt Institute "AI" (compressive test) and Shell (bending test) Method [Dragos 1999, Bonnaure 1977]. By using a bi-model evaluation, including modulus found both in tension and compression we were able to better represent asphalt concrete range behaviour under field condition.

The seed range of interface reaction moduli were evaluated according to Uzan et al. (1978) and Crispino et al. (1997) as function of temperature and interface normal stress.

3.1.3 Pavement response model.

A layered linear elastic system has been assumed as pavement model, taking into account the interaction between the layers. The Poisson coefficient was calculated as function of modulus as suggested in [Dragos 1999].

3.1.4 Error check and search for moduli

The sum of the squared relative errors of deflection was used as measure of convergence (of how well the calculated deflection basin matches the measured deflection basin):

$$RMSE = \sqrt{\sum_{i=1}^{n_d}\left[\frac{1}{n_d}\left(\frac{d_{ci} - d_{mi}}{d_{mi}}\right)^2\right]} \qquad \text{(Eq. 1)}$$

where n_d is the number of deflection measured, d_{ci} and d_{mi} are the calculated and measured deflection in the i-th point.

Backcalculation problem can be formulated as a minimization problem to find the "best" solutions E[E_1,…., E_k, $K_{1\text{-}2}$,……., $K_{(k\text{-}1)\text{-}k}$]: Minimize RMSE (E) (Eq. 1)

satisfying the constraints $E_i^L \le E_i \le E_i^U$

where [E_i^L ; E_i^U] is the seed range for the i-th modulus.

In the routine developed the Hooke and Jeeves method was used to solve the problem. These methods, which is one of the most commonly used direct search methods, assume that there is only one minimum in the region of search. In this method, an initial step size is chosen and the search direction is initiated from a given starting point. A combination of exploratory and pattern moves is made iteratively to find the most profitable search directions. An exploratory move is employed first to find the best point around the initial point. If the exploratory move leads to a decrease in the value of function, it is regarded as a success; otherwise, it is considered a failure. Then a pattern move is made to find the next point.

3.1.5 *Results analysis*

Starting with FWD test data the backcalculation analysis was performed using each of the set of thickness generated by MC simulation, and a sample of layer moduli were obtained. Then the hypothesis that specifies the probability law for the random modulus being sampled was tested. Once the probably laws of backcalculated moduli were known a certain percentile could be used for rehabilitation design.

4. CASE STUDY

A case study was carried out in order to test the process developed. The pavement structure selected is illustrated in table 3, and the deflection basin was calculated assuming: asphalt layer moduli by predictive models (see paragraph 3.1.2), resilient moduli of granular materials equal to Mr=200 MPa (subbase) and Mr=100 MPa subgrade, interdface reaction moduli as suggested by Uzan et al..

As far as interface reaction moduli is concerned a parametric study was performed and results showed a dramatic variation in backcalculated asphalt layer moduli (ΔE) for reaction moduli "K" less than the values suggested by Uzan et al. (ΔK see figure 3).

Table 3 Pavement structure and thickness [mm]

Surface (HMA)	Binder (HMA)	Base (HMA)	Subbase (Granular)
50	60	170	150

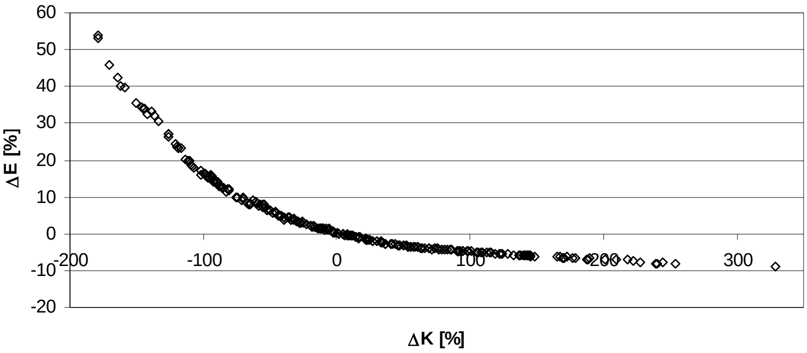

Figure 3 Variation of backcalculated moduli (ΔE) vs vartion of shear reaction moduli (ΔK) (with reference to values suggested by Uzan et al.)

The stochastic backcalculation was performed by MC simulation assuming four values for coefficient of variation of layer thickness: 2.5, 5 , 10 and 15%. The simulation showed that:

- A square relationship exist between coefficient of variation of backcalculated moduli "COV_E " and that of thickness "COV_T " (see figure 4)

  $COV_E = a\ COV_T^2 + b\ COV_T$

  where a and b are coefficients (a=0.08÷0.065 b= 3.7÷2.8)
  It should be noticed that COV_E were about 2.5 times COV_T;
- Backcalculated asphalt moduli distribution may be best described by a Log-normal random variable, as the Chi-square test confirmed. This result is not unexpected since moduli are related to thickness through a multiplicative relationship (see figure 5)
- The sum of the squared relative errors of deflection could be reduced to less than 1% if interface reaction moduli are backcalculated (i.e. considered as unknown).

5. CONCLUSION

In this paper a new methodology for the analysis of non destructive deflection testing through backcalculation was proposed, in which uncertainty about thickness and interface bound are accounted for. Particularly in the procedure proposed the interface reaction moduli are considered as unknowns and they are backcalculated together layer moduli.

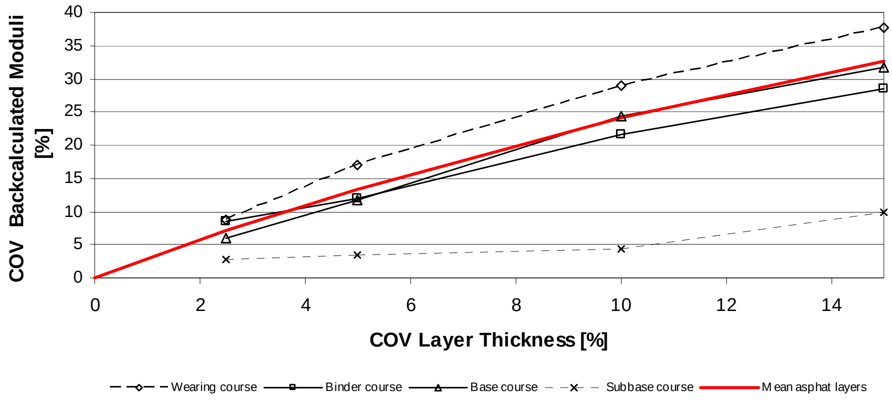

Figure 4 Coefficient of variation of layer thickness vs coefficient of variation of layer moduli

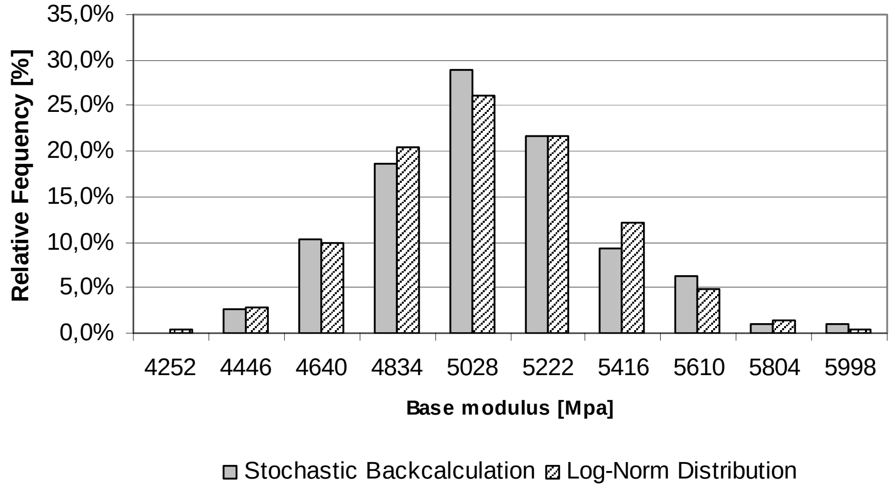

Figure 5 Percentage frequency distribution of backcalculated modulus of base layer for a COV of layer thickness of 2.5 %.

Moreover a stochastic approach was incorporated into the backcalculation analysis, in order to manage the variability of input thickness data due measurement error (GPR measures) or construction defect (measurement by core/bores).

An implementation example was presented in this paper, in which the probability distribution of the layer moduli was determined along with the probability law (Log-normal density function). Therefore it was suggested to use a certain percentile from distribution of backcalculated layer moduli in the overlay thickness design in order to consider the reliability due to uncertainty in thickness data.

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