ADVANCED CONSTITUTIVE MODELING OF BITUMINOUS MATERIALS BY ENERGETIC APPROACH: FORMULATION, CALIBRATION AND EXPERIMENTAL EVALUATION

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ABSTRACT

This report introduces a generalized constitutive model for the characterization of stress-strain behaviour in asphalts, with particular reference to the problem of the rutting in the bituminous pavements.

With the introduction of the function of density of Helmholtz free energy, for the description of the energy status of visco-elasto-plastic material, and using the concept of "internal variables", it has been possible to express a law of plastic flow in mechanically rigorous terms, which can reliably quantify the permanent deformations.

The three-dimensional formulation of the constitutive relationship has been developed in such a way as to verify the universal dissipation principle, expressed by the Clausius-Duhem dissipative inequality, and bound to the thermodynamic constraints that the system has to satisfy.

The model has been calibrated and validated on the basis of the deformation behaviour of a particular Mastic Asphalt mix, through a specifically developed experimental-numerical procedure, based on the static creep test, both in the unconfined and confined configuration.

Keywords: visco-elasto-plastic constitutive model, rutting, mastic asphalt

1. INTRODUCTION

The definition of a constitutive model, which can grasp the true complexity of the response of asphalts (bituminous mixtures) to mechanical stresses, is an indispensable prerequisite for a rational analysis of the tenso–deformational behaviour of flexible pavements, particularly with reference to the phenomenon of rutting.

The visco-elastic-plastic response of asphalts can be interpreted with the help of rheological models comprising elastic, viscous and plastic components, paired in a more or less complex way, according to the accuracy desired for the simulation and the possibility of surmounting the mathematical complexity resulting from the number of initial components utilised (Bland (1960), Ferry (1970), Giunta (2002), Scarpas et al. (2005)).

This methodology, quite widely used in one-dimensional ambits, is not however easily extended to three-dimensional conditions of stress and deformation, for the study of which it is preferable to follow an energy approach, such as that presented by Simo & Hughes (1997), with which, the mechanical behaviour of a visco-elastic material "point", can be defined calculating a correlation for the Helmholtz free energy (Ψ) in comparison with an appropriate set of internal variables ($\boldsymbol{q}_i$,) which can describe micro-structural state of the point and associate each of these to a viscous process:

$$\psi = \psi\left(\boldsymbol{\varepsilon}, \mathbf{q_i}\right) \qquad i \in \left\{1, 2, ..., n\right\}$$

Introducing the principle of universal dissipation, by means of the dissipative inequality of Clausius-Duhem, is possible to formulate a general, thermodynamically congruent, visco-elastic model, as is full explained in Simo & Hughes (1997).

This paper, taking in to account the mentioned visco-elastic formulation, presents and discusses a law of plastic flow that completes the constitutive modelling of asphalts, in three dimensional terms.

2. FORMULATION OF THE VISCO-ELASTO-PLASTIC CONSTITUTIVE MODEL

In order to take into account any permanent plastic strain $\boldsymbol{\varepsilon}^p$ in the material as a consequence of the loading history, the mentioned function of density of Helmholtz free energy can be formulated as follows:

$$\psi\left(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^p, \mathbf{q}_i\right) = W^0\left(\boldsymbol{\varepsilon}^{ve}\right) - \sum_{i=1}^{n} \mathbf{q}_i : \dot{\boldsymbol{\varepsilon}}^{ve}$$

where $\boldsymbol{\varepsilon}^{ve} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p$ represents the reversible visco-elastic component of the strain. Applying the fulfilment of the dissipative inequality of Clausius-Duhem, which, in pure mechanics, can be formulated as:

$$\dot{\psi} - \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} \leq 0$$

the tension-strain equation and thermodynamic restrictions on the model are obtained:

$$\boldsymbol{\sigma}\left(\varepsilon, \varepsilon^{p}, \mathbf{q}_{i}\right) = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}} = \frac{\partial W^{0}}{\partial \boldsymbol{\varepsilon}} - \sum_{i=1}^{n} \mathbf{q}_{i}$$

$$\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{p}\right) : \dot{\mathbf{q}}_{i} \geq 0$$

taking into account the formulation of the density function of elastic energy $W^{0}(\boldsymbol{\varepsilon}^{ve})=\frac{1}{2}(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{p})\mathbf{:}\mathbf{D}_{0}(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{p})$, where $\mathbf{D}_0$ represents the tensor of the instantaneous elastic moduli, and the law of evolution of the internal viscous parameters, the following is arrived at:

$$\boldsymbol{\sigma}\left(\varepsilon, \varepsilon^{p}, \mathbf{q}_{i}; t\right) = \mathbf{D_0}\left(\varepsilon\left(t\right) - \varepsilon^{p}\left(t\right)\right) - \sum_{i=1}^{n} \frac{\gamma_i}{\tau_i} \int_{-\infty}^{t} \exp\left[-\frac{(t-s)}{\tau_i}\right] \boldsymbol{\sigma}^{\mathbf{0}}\left(s\right) ds$$

$$\boldsymbol{\sigma}^{\mathbf{0}}\left(t\right) = \mathbf{D_0}\left(\varepsilon\left(t\right) - \varepsilon^{p}\left(t\right)\right)$$

Where γ_i and τ_i are, respectively, the relative stiffness and the relaxation time associated to each viscous process.

The plastic strain $\boldsymbol{\varepsilon}^{p}$ can be re-written in the form $\boldsymbol{\varepsilon}^{p} = \lambda\mathbf{m}$, where λ is the plastic creep, while **m** is a tensor indicative of the directionality of the plastic flow. definition of the visco-elastic-plastic model requires the introduction of a law of evolution of the plastic strains $\boldsymbol{\varepsilon}^{p}$, which is possible through the introduction of a yield criterion:

$$\phi\left(\boldsymbol{\sigma}, \lambda\right) = \pi\left(\boldsymbol{\sigma}\right) - \hat{\pi}\left(\lambda\right) \leq 0$$

where $\phi(\sigma,\lambda)$ is a plastic potential, $\pi(\sigma)$ a function expressing the criticality of the tensional state in relation to the plastic flow processes (this is essentially a Von Mises tension), while $\hat{\pi}\left(\lambda\right)$ is a function describing the limit value that $\pi(\sigma)$ can take without any further plastic creep occurring. In order to obtain a model of the "strain-driven" type, and therefore able to take into account the dependence of the yield conditions on the rate of deformation and the plastic flows caused by creep, it is appropriate that the function π does not depend on the effective stress σ, but rather on the instantaneous elastic stress $\boldsymbol{\sigma}_0(t)=\mathbf{D}_0(\boldsymbol{\varepsilon}(t)-\boldsymbol{\varepsilon}^{p}(t))$. It is normal to introduce the following conditions of complementarity and consistency (Schrefler & Cannarozzi

(1991), Simo & Hughes (1997), Scarpas et al. (2005)) in order to stabilise the conditions so that plastic flow occurs:

$$\phi\dot{\lambda} = 0, \quad \dot{\phi}\dot{\lambda} = 0$$

in this way the law of evolution for plastic creep can be obtained:

$$\dot{\lambda} = \frac{\partial\pi / \partial\boldsymbol{\sigma}^{\mathbf{0}} : \dot{\boldsymbol{\sigma}}^{\mathbf{0}}}{\partial\hat{\pi} / \partial\lambda}$$

With the aim of interpreting the experimental results, it is worth redefining the limit condition $\hat{\pi}(\lambda)$ as a function of the existing yield stress σ_y (stress evaluated in relation to a process of instantaneous loading, therefore exempt from the influence of viscous processes), that is:

$$\hat{\pi}(\lambda) = \breve{\pi}(\sigma_y)$$

$$\sigma_y = \sigma_y^{\,0} + K\lambda$$

where σ^0_y represents the initial yield stress, while K is the hardening modulus.

Considering then the well-established equation $\mathbf{m} = \partial\pi / \partial\boldsymbol{\sigma}^{\mathbf{0}}$, the law of evolution for the plastic creep can be re-written as follows:

$$\dot{\lambda} = \frac{\mathbf{m} : \dot{\boldsymbol{\sigma}}^{\mathbf{0}}}{K\,\partial\breve{\pi} / \partial\sigma_y}$$

Taking into account that $\dot{\boldsymbol{\sigma}}^{\mathbf{0}} = \mathbf{D}_0\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p\right) = \mathbf{D}_0\left(\dot{\boldsymbol{\varepsilon}} - \dot{\lambda}\mathbf{m}\right)$ the following law is obtained:

$$\dot{\lambda} = \frac{\mathbf{m} : \mathbf{D}_0\dot{\boldsymbol{\varepsilon}}}{\mathbf{m} : \mathbf{D}_0\mathbf{m} + K\,\partial\breve{\pi} / \partial\sigma_y} \qquad \text{(Eq. 1)}$$

Given that $\boldsymbol{\sigma}(t) = \mathbf{D}_0\,(\boldsymbol{\varepsilon}(t) - \boldsymbol{\varepsilon}^{\mathrm{p}}(t)) - \sum\mathbf{q}_{\mathrm{i}}(t)$, $\dot{\boldsymbol{\sigma}} = \mathbf{D}_0\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p\right) - \sum\dot{\mathbf{q}}_{\mathbf{i}}$ is obtained, from which it follows that $\dot{\boldsymbol{\sigma}} = \mathbf{D}_{\mathrm{ep}}\,\dot{\boldsymbol{\varepsilon}} - \sum\dot{\mathbf{q}}_{\mathbf{i}}$, where $\mathbf{D}_{\mathrm{ep}}$ represents the constitutive elastic-plastic tensor:

$$\mathbf{D_{ep}} = \left[1 - \frac{\mathbf{m} : \mathbf{D}_0 \mathbf{m}}{\mathbf{m} : \mathbf{D}_0 \mathbf{m} + K \, {}^{\partial \bar{\pi}} \! / \! {}_{\partial \sigma_y}} \right] \mathbf{D}_0$$

the following tension-stress equation is then obtained:

$$\boldsymbol{\sigma}(t) = \int_{-\infty}^{t} \mathbf{D_{ep}} \dot{\boldsymbol{\varepsilon}} \, ds - \sum_{i=1}^{n} \mathbf{q}_i(t)$$

$$\mathbf{q}_i(t) = \frac{\gamma_i}{\tau_i} \int_{-\infty}^{t} \exp\left[-\frac{(t-s)}{\tau_i} \right] \boldsymbol{\sigma}^{\mathbf{0}}(s) \, ds$$

With the hypothesis of isotropic and homogeneous material, the model is thus characterized by a parameter for the elastic response (Elastic Modulus E^0), a pair of parameters for each viscous process it is wished to introduce (relative stiffness γ_i and relaxation times, τ_i) and a further pair of parameters for the plastic component (yield strain ε^0_y and hardening modulus K).

A series of preliminary calibrations, in which the number of viscous processes was varied, demonstrated that it is worth introducing four of these processes to reliably capture the various phases of viscous relaxation, both the briefer ones and those in the medium- and long-term. Consequently, eleven constituent parameters are associated to the model, whose determination is described in the next section.

3. CALIBRATION OF THE CONSTITUTIVE MODEL

For the determination of the constitutive parameters the data relating to the deformational response of a particular bituminous mix (Mastic Asphalt) were elaborated (Baldo (2006)), obtained from static creep tests at free lateral expansion on Marshall specimens, with a loading time of 500 seconds and unloading time for visco-elastic recovery set at 1500 seconds. for each of the four temperatures considered (40 °C, 20 °C, 10 °C and 5 °C) a set of six tension levels was selected (25 kPa, 50 kPa, 75 kPa, 100 kPa, 125 kPa, 150 kPa), so that the deformational response of the asphalt could be studied in a representative way. The Poisson's coefficient has been fixed at a constant value of 0.35 for every temperature.

The mastic asphalt used, specific for the pavements of cement decks, has a grading curve within the MA 8 envelope of the Swiss standard SN 640 440, with a filler–bitumen ratio of 1.3.

The calibration procedure was developed following a one-dimensional approach, in both the schematizing of the creep test, and in the elucidating of the constitutive visco-elastic-plastic model. The procedure is split into two phases: in the first one, the yield stress (ε_y^0) and a mechanical parameter depending on the initial Elastic Modulus (E^0)

and hardening Modulus (*K*) are obtained, and the values of total stress are calculated in the second, starting from the effective stresses and the values of the remaining constituent parameters

In the first part of the calibration, for each experimental curve, the pair of values is considered comprising the maximum unitary axial deformation (strain) and the permanent. Reporting these values onto a Cartesian plane, a cluster of points is obtained, one for each of the curves analyzed. As illustrated in Figure 1, the least squares interpolation of these points produces a straight line, the slope of which corresponds to the quantity $E^0/(E^0/K)$, while the intercept on the y axis allows ε_y^0 to be obtained. It is possible to demonstrate what is affirmed, observing that, taking *m*=-1 for the compressive stresses, equation (Eq. 1) can be rewritten as follows:

$$\dot{\lambda} = -\frac{E^0 \dot{\varepsilon}}{E^0 + K}$$

Given that $\varepsilon^P = \lambda m$, the *rate* of permanent creep can be determined as:

$$\dot{\varepsilon}^P = \frac{E^0}{E^0 + K}\dot{\varepsilon}$$

Calling *H* the quantity $\frac{E^0}{E^0 + K}$, and expressing the derived functions gives:

$$\frac{d\varepsilon^P}{dt} = H \cdot \frac{d\varepsilon}{dt}$$

From which, integrating with respect to time, it is concluded that:

$$\varepsilon^P = H\varepsilon^{\max} + \alpha \qquad \text{(Eq. 2)}$$

In which α is an integration constant. It can be observed that in correspondence to the yield conditions, the plastic deformation still has to begin to develop, that is $\varepsilon^{max} = \varepsilon_y{}^0 \Rightarrow \varepsilon^P = 0$. Introducing these conditions in (Eq. 2), the value of the integration constant $\alpha = -H\varepsilon_y{}^0$ can be obtained, which allows us to particularise the expression of ε^P as:

$$\varepsilon^P = H\varepsilon^{\max} - H\varepsilon_y{}^0$$

from which it follows that:

$$H = \frac{\varepsilon^P}{(\varepsilon^{\max} - \varepsilon_y^{\,0})}$$

With reference to Figure 1, the interpolation curve of the experimental data can now be written in the form:

$$\varepsilon^P = a\varepsilon^{max} + b \qquad \text{(Eq. 3)}$$

a and *b* being the interpolation coefficients. On the basis of simple trigonometric considerations, the slope *a* of the curve results as being equal to the quantity $\frac{\varepsilon^P}{(\varepsilon^{\max} - \varepsilon_y^{\,0})}$; so it can be concluded that $a = H$.

Therefore, as anticipated, the interpolation coefficient *a*, the slope of the curve, is equal to the quantity $\frac{E^0}{E^0 + K}$. Introducing the condition $\varepsilon^{max} = \varepsilon_y^{\,0} \Rightarrow \varepsilon^P = 0$ also in (Eq. 3), expression of the yield strain can be obtained as a function of the interpolation coefficients:

$$\varepsilon_y^{\,0} = -\frac{b}{a}$$

In summary, the first phase of calibration, with the interpolation to the least squares of the pairs of values $(\varepsilon^P, \varepsilon^{max})$, yields the constitutive parameter $\varepsilon_y^{\,0}$ and the parameter *H*.

The second part of the calibration is of the iterative type, and is also divided in two sub-procedures. The first is constituted by the visco-elasto-plastic model, expressed in one-dimensional terms which, having the values of $\varepsilon_y^{\,0}$ and *H* as inputs, those of the first attempt at the remaining constituent parameters (Elastic Modulus, relative stiffness and relaxation times) and effective stresses, calculates the total model strains.

The second sub-procedure instead, having the above-determined model values and those experimental of the total strain as inputs, yields the updated values of the constituent parameters through a process of optimization, to be used for a subsequent iteration of the second calibration phase. The procedure stops when the preset tolerance is reached, providing the final values of the constituent parameters.

Figure 2 presents the results of the second phase of the calibration procedure at a temperature of 40 °C (full details are given in Baldo (2006)): as can be seen, the interpolation and experimental curves are essentially overlapping, with a slightly more pronounced difference in correspondence to 150 kPa. The values of the constituent parameters of the Mastic Asphalt are summarized in Table 1, for all the temperatures investigated.

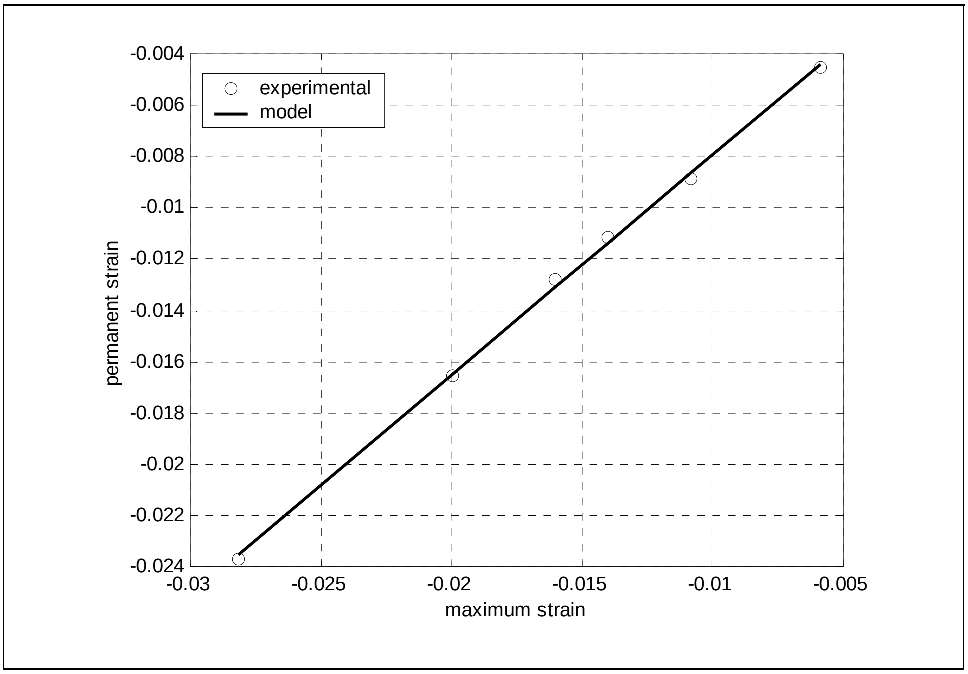

Figure 1 First calibration phase @ 40°C

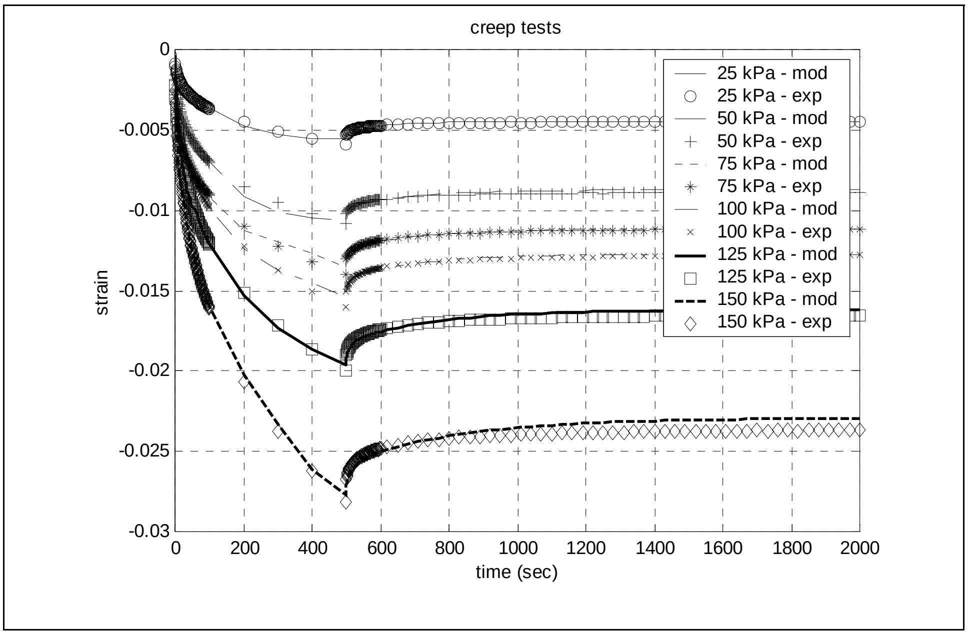

Figure 2 Second Calibration phase @ 40°C

Asphalts are well known to be highly temperature-sensitive materials, which become stiffer with the lowering of the temperature. Indeed, the values obtained, for both the Modulus of Elasticity and that of work-hardening, increase with the lower temperature, thus denoting the expected stiffening of the mix and demonstrating the physical congruence of the values themselves.

Table 1 Constitutive parameters value @ various temperatures

Constitutive parameters	40°C	20°C	10°C	5°C
E^0 (MPa)	8.81277E+01	9.34428E+01	1.11316E+02	1.13343E+02
ε_v^0	7.52600E-04	6.30624E-04	3.06016E-04	1.96794E-04
K (Mpa)	1.44587E+01	2.38192E+01	3.85274E+01	3.96426E+01
γ_1	2.08793E-01	2.86080E-01	2.54617E-01	2.19884E-01
τ_1 (s)	1.18967E+00	9.52086E-01	4.64431E-01	7.88007E-01
γ_2	2.21940E-01	1.53490E-01	1.01905E-01	1.97438E-02
τ_2 (s)	1.42936E+01	8.30171E+00	1.20735E+01	2.74039E+01
γ_3	1.65343E-01	1.43086E-01	1.46407E-01	2.07013E-02
τ_3 (s)	3.00000E+01	5.04838E+01	6.70736E+01	9.56912E+01
γ_4	2.87439E-01	2.00262E-01	5.65114E+02	3.09428E-01
τ_4 (s)	5.65941E+02	5.73955E+02	1.65295E-01	6.13565E+02

4. EVALUATION OF THE CONSTITUTIVE MODEL

In order to verify the capacity of the model to represent the salient points of the deformational behavior of the Mastic Asphalt (qualitative trend, maximum and permanent strains), numerically static creep tests were simulated at a stress of 100 kPa, with load application and recovery times of 3600 s, in both confined conditions and with free lateral expansion. To reproduce the effect of lateral confinement, cylindrical specimens 150 mm in diameter and 60 mm tall had a static axial load applied by a 100 mm diameter plate above. the specimen was thus divided into a “virtual” cylinder 100 mm in diameter, directly loaded by the plate above, and a “virtual” ring of surrounding material, not axially loaded, with a radius of 25 mm, which develops an effective containing action that impedes lateral expansion of the specimen. For the free lateral expansion, traditional Marshall specimens were used.

The simulation was done utilizing a general purpose FEM code, in which the model was implemented thanks to the possibility offered by the software to utilize constitutive laws defined by the user, through a sub-routine written in the specific program language. Exploiting the characteristic double symmetry of a cylindrical specimen, it was possible to develop a three-dimensional FEM model that reproduces only one quarter. three-dimensional elements were utilized for discretising with six nodes of the wedge type, i.e. triangular-based prisms, with three degrees of freedom at translation per node and three at rotation. two models were thus produced, one for the Marshall specimen, composed of 2000 elements and 1331 nodes, the other for the 150 mm diameter specimen, with 4000 elements and 2486 nodes. As regards the constraints, the specimen resting on the lower plate was schematized, for both FEM models, by canceling the vertical and horizontal displacements of all the nodes belonging to the base section. a constraint was also set for both FEM models: the horizontal displacements were cancelled at the nodes belonging to the section of the upper surface in contact with the 100 mm load-bearing plate. The loading conditions, transmitted by the plate above, are applied directly to the upper section of the FEM model by uniform pressure. Figure 3 presents the result of the validation at 40 °C, while Tables 2 and 3 summarize the results for all the analyzed temperatures. only the differences between the visco-elasto-plastic model (VEP) and experimental data were considered; the values of strain of the visco-elastic model (VEL) are also indicated.

The qualitative trend of the curves of the VEP model is entirely similar in general to that of the experimental ones, in both the non-confined (NC) and confined (C) case, with a tendency to overestimate the evaluation of the maximum strain and underestimate the permanent one. The maximum difference is just over 15%, and this at the temperature of 10 °C. The results are even more satisfactory at all the other temperatures, in particular at 40 °C, i.e. the most significant temperature, among those considered, for the phenomenon of rutting.

It should be noted that the VEL model wildly underestimates the deformability of the material compared to experimental reality: from almost 5 times at 40 °C, to more than twofold at 5 °C, in terms of maximum strains. as was to be expected, the difference as regards permanent strains is even greater, given that these ones cannot be evaluated with a purely visco-elastic model.

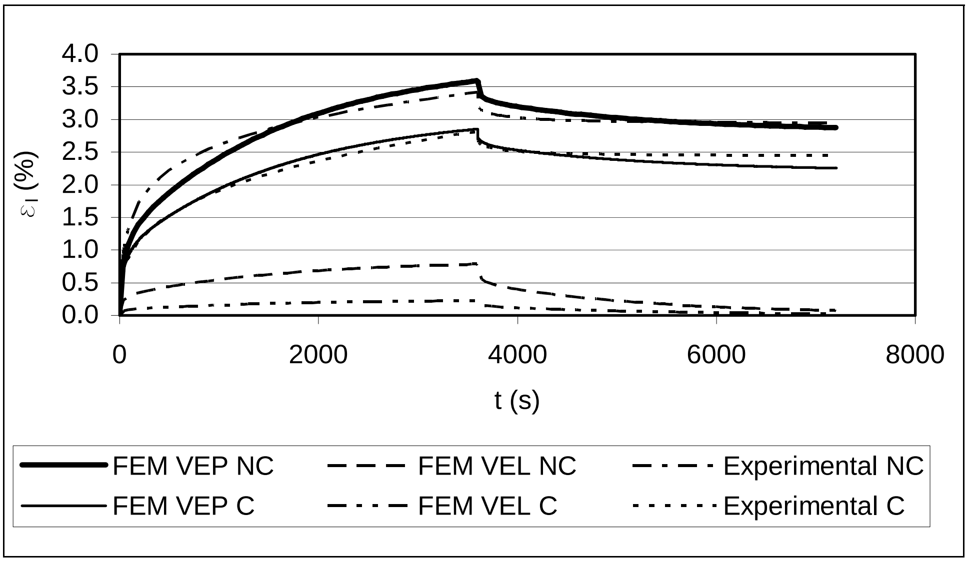

Figure 3 Static Creep @ 40°C. Longitudinal strain (ε_l) *vs* Time

Table 2 Model validation: creep without confinement

T(°C)	Type of data	ε^{max} (%)	ε^{p} (%)	$\Delta\varepsilon^{max}_{VEP-EXP}$ (%)	$\Delta\varepsilon^{p}_{VEP-EXP}$ (%)
40	VEP	3.57	2.87	+4.69	-2.32
	VEL	0.76	0.07		
	Experimental	3.41	2.94		
20	VEP	1.81	1.33	+7.74	-9.02
	VEL	0.50	0.02		
	Experimental	1.68	1.45		
10	VEP	0.98	0.65	+6.52	-12.31
	VEL	0.34	0.01		
	Experimental	0.92	0.73		
5	VEP	0.52	0.33	+6.12	-9.09
	VEL	0.20	0.01		
	Experimental	0.49	0.36		

Table 3 Model validation: creep with confinement

T(°C)	Type of data	ε^{max} (%)	ε^{p} (%)	$\Delta\varepsilon^{max}_{VEP-EXP}$ (%)	$\Delta\varepsilon^{p}_{VEP-EXP}$ (%)
40	VEP	2.84	2.25	+1.07	-8.44
	VEL	0.63	0.06		
	Experimental	2.81	2.44		
20	VEP	1.32	0.92	+5.60	-10.87
	VEL	0.41	0.02		
	Experimental	1.25	1.02		
10	VEP	0.74	0.46	+8.82	-15.22
	VEL	0.28	0.01		
	Experimental	0.68	0.53		
5	VEP	0.38	0.22	+5.56	-13.63
	VEL	0.16	0.01		
	Experimental	0.36	0.25		

5. CONCLUSIONS

A general visco-elasto-plastic constitutive model has been developed in the research described above, which can simulate the principal aspects of the deformational response of asphalts to mechanical stresses. The model is based on the latest constitutive theories typical of irreversible mechanical processes, described by means of appropriate internal variables. Following an energy approach, with the introduction of the Helmholtz free energy to describe the energy status of the material, the law of plastic flow has been expressed in mechanically rigorous terms.

The formulation has been developed in such a way that the basic requisites defined by the laws of thermodynamics can be verified a priori, in particular the practical conditions of the instantaneously-dissipated energy.

The model has been calibrated on the basis of the deformational response of a particular bituminous mix (mastic asphalt), by means of an experimental-analytical procedure developed ad hoc.

A rigorous experimental-numerical validation has demonstrated that the proposed model can interpret the fundamental aspects of the response of the Mastic Asphalt, in terms of both maximum and permanent strains at different temperatures.

The comparison between the visco-elasto-plastic and visco-elastic models has clearly demonstrated the importance of reliably quantifying the permanent strains of asphalts, to avoid a totally unrealistic estimate of the deformability of the material, particularly at higher temperatures.

Original experimental protocols have been developed to support the calibration and validation phases. The solution used to reproduce lateral confinement in the creep tests is particularly worthy of mention, because it is as simple and economical as it is efficacious.

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