Formulating a safety model and validating it by an experimental investigation on the A3 Salerno-Reggio Calabria

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SYNOPSIS

The purpose of this paper is to present, analyze and validate some models for safety analyses involving roads geometry and design consistency. All the parameters involving the "hardware" of the facility sections (geometry, etc.) are herein called intrinsic and they are divided into two main sets: internal and external to the given i^{th} section. All the parameters non-involving the "hardware" of the facility sections (traffic, driver, etc.) are herein called extrinsic.
As is well known, research reports from different European countries show that more than 30% of accidents can be due to faults in road design.
So, for European Road Engineers too, it becomes more and more important to dispose of models in order to better understand how to operate in their particular countries to reduce accident rates.
By referring to the South of Europe, the A3 Salerno-Reggio Calabria is a very strategic facility; connecting the extreme Southern Italy (and, in particular, the very populous region of Sicily) with the rest of Italy and Europe, it runs through mountainous terrain in a very irksome geographical context and it is going to be upgraded in order to better face the increasing mobility demand and to reduce accident rates.
In the light of the above-mentioned problems, in this paper, safety analysis was initially pursued by analyzing road safety in terms of risk analysis and by formulating some proper models to relate accident rates and some geometric parameters.
A particular analysis was then effected for the A3 highway in order to survey and to analyze accident and traffic data for a period of six years.
Following geometric examination and the statistical analysis of the collected information, traffic and accident data were then analyzed.
After discussing the matter, the formalized models were validated.
The obtained results demonstrate, by quantitative relations, the appreciable influence of some particular geometric characteristics (both internal or external to the single profile element) on risk assessment.

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PROBLEM STATEMENT

Both from a theoretical and from an experimental point of view, Road Safety dependence on intrinsic geometric characteristics is a matter of fact.
Despite this, it is quite evident that the remaining extrinsic factors (human behavior, environment, vehicle) can affect importantly the relation safety-geometry, so introducing "heavy" effects, able to bring into the data a consistent variance, sometimes interpretable as chaotic behavior.
In the light of the above-mentioned, the goal of this paper is to formalize, analyze and validate reliable models for safety analyses involving highways geometry and design consistency.
The target is here pursued through three main phases: 1) problem modeling; 2) experiments; 3) data analysis.
The used symbols are resumed in table 1 in the appendices.

PROBLEM MODELING

This paragraph deals with the logical and analytic modelling of the relation between road safety and geometric parameters in terms of risk analysis.
As is well known, a risk R_a of accident is inherently combined with the motion of a vehicle on an infrastructure (Lamm et al., 1998; Hauer, 2001A; Hauer, 2001B; Polus et al., 2004). R_a derives from the exposure of the user (vehicle, pedestrian, etc) to "the use of the infrastructure". This topic, in principle similar to any risk analysis, in studying road safety, acquires quite unusual characteristics, concisely examined below (more theoretical aspects are discussed in the annex). Both from a theoretical (see the annex) and experimental point of view (international state-of-the art), it seems possible to assess the primary rule of the horizontal radius R for R_a and the various accidents descriptors (such as number of accidents or others, see the annex). Having established this, if one refers to the figure 1, it is possible to introduce the different models here formalized and analyzed.

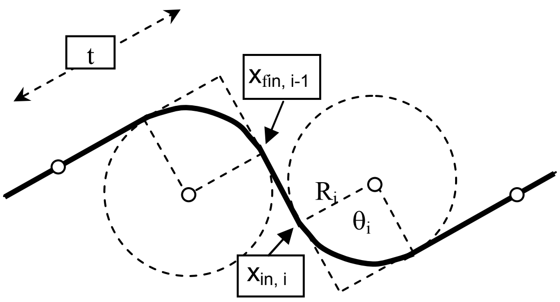
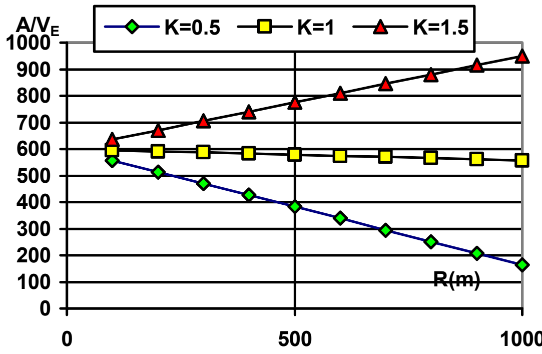
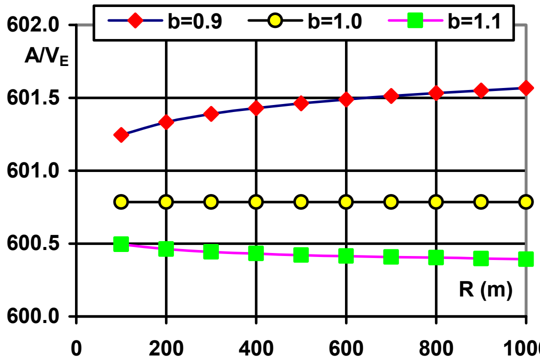


Figure 1 Key symbols

Let A_i be the number of accidents for a given road element (in the unit of time) and $A=A_1+A_2+A_3=\Sigma_i A_i$ a sum of accidents for different elements. Let L_i be the i^{th} length, t_i be the i^{th} tangent length, R the radius and θ the deflection angle. Then $t_i\text{-}R\cdot tg(\theta/2)$ is the Straight Segment length, and $R\cdot\theta$ the Curve length, while the Total length (two straight segment plus one curve) is $t_1+ t_2-2\cdot R\cdot tg(\theta/2)+ R\cdot\theta$. From this, one can obtain that the difference in length when R becomes R' is $(R-R')\cdot(2\cdot tg(\theta/2)-\theta)$. The accidents A_i, referred to the i^{th} element, may be due to intrinsic internal parameters (belonging to the same i^{th} element) or to intrinsic external parameters (non-belonging to the i^{th} element).
Now, if V_E stands for the vehicle number (in millions), r_t for the Accident rate on tangents and r_c is the Accident rate on curves, it is possible to model the problem from an internal or external point of view as in

tables 2 and 3. In this tables "r_c constant" means that $\partial r_c/\partial R=0$, while "$r_c$ variable" means that $\partial r_c/\partial R$ may be different from 0; r_c "quasi-constant" means that, in first approximation, R doesn't appear in r_c formula, but strictly speaking D_T depends on R. Both in table 2 and 3, on the basis of the hypotheses (see the rows A.1.1, A.1.2, A.2.1, A.2.2), the accidents A are calculated (Model Consequences). Moreover, table 2 shows the first derivative of A/V_E as a function of R, both for r_c constant (5th row) and variable (10th row) in function of R.

Table 2 Internal approach

1.	*Major Hypothesis: "the cause of what happens in a section must be found in its internal characteristics"*	
2.	***A.1.1 : @ r_c constant***	$\_r_c=r_t+a$
3.	Model Consequences	
4.	Accidents	$A=[(t_1+t_2-2\cdot R\cdot tg(\theta/2))\cdot r_t+R\cdot\theta\cdot r_c]\cdot V_E=[2\cdot r_t\cdot V_E\cdot(t-R\cdot tg(\theta/2))]+[R\cdot\theta\cdot r_c]\cdot V_E=$ $=[2\cdot r_t\cdot V_E\cdot tg(\theta/2)\cdot(R_{max}-R)]+[R\cdot\theta\cdot r_c]\cdot V_E$
5.	First derivative of A/ V_E, at r_t and r_c constant (A.1.1: @ r_c constant)	$\partial(A/V)/\partial R=-2\cdot tg(\theta/2)\cdot r_t+\theta\cdot r_c=[-2\cdot tg(\theta/2)+\theta\cdot K]\cdot r_t\cong[-1+\cdot K]\cdot\theta\cdot r_t$
6.	R-A/ V_E plot, for different $K=r_c/r_t$, for r_t and r_c constant (A.1.1: @ r_c constant)	 *If r_c is constant and K is greater than about 1, then $\partial(A/V_E)/\partial R>0$*
7.	***A.1.2: @ r_c variable***	$\_r_c=r_t+a\cdot R^{-b}$
8.	Model consequences	
9.	Accidents for $r_c=r_t+a\cdot R^{-b}$	$A=[(t_1+t_2-2\cdot R\cdot tg(\theta/2))\cdot r_t+R\cdot\theta\cdot r_t+\theta\cdot a\cdot R^{-b+1}]\cdot V_E\cong\Sigma_i t_i\cdot r_t+$ $\Sigma_j\theta_j\cdot a\cdot R_j^{-b+1}$
10.	First derivative of A/ V_E, at r_t constant and $r_c=r_t+a\cdot R^{-b}$	$\partial(A/V_E)/\partial R=-2\cdot tg(\theta/2)\cdot r_t+\theta\cdot r_t+\theta\cdot a\cdot R^{-b}(-b+1)\cong(1-b)\theta\cdot a\cdot R^{-b}$
11.	R-A/ V_E plot, for different b, for r_t constant and $r_c=r_t+a\cdot R^{-b}$	 *If $r_c=r_t+a\cdot R^{-b}$ and b<1, then $\partial(A/V_E)/\partial R>0$*

By means of the above formalized simple algorithms it is straightforward to obtain useful information about the range of some of the models coefficients such as K and b (see the plots in table 2, rows 6th and 11th).
Let v be the profile speed (intrinsically "contained" in the geometry of the road, according to the Italian standards too) and let v*=E[v] be the average of the geometrically expected speeds for a given section. So $L^{-1}\cdot\int(v-v^*)ds$ is the average in L of v-v*, while Δs_i is the length in which $v=v_i$ and $L=\Sigma_i\Delta s_i$. If $v_m=0.5\cdot(v_j+v_{j-1})$, $\Delta v=|v_j-v_{j-1}|$, and a is the acceleration/deceleration (a=vdv/ds), $D_{Ti,\ i-1}=v_m\cdot\Delta v\cdot a^{-1}$ is the well known distance used in the construction of the speed profile. So it is possible to formalize two families (A.2.1 and A.2.2) of external intrinsic models as in the following table 3.

Table 3 External approach

Major Hypothesis: "the cause of what does happen in a section must be found both in its internal and external characteristics"	
A.2.1:@ r_c quasi-constant	$r_c=r_t+D_{Tj,\,j-1}\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}$ (quasi-: strictly speaking D_T depends on R too)
Model consequences	$A=[(t_1+t_2-2\cdot R\cdot tg(\theta/2))\cdot r_t+R\cdot\theta\cdot r_c]\cdot V_E=\sum_i l_i\cdot r_t+\sum_j\theta_j\cdot R\cdot[r_t+D_{Tj,\,j-1}\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}]$
A.2.2:@ r_c variable	$r_c=r_t+a\cdot R^{-b}\cdot D_{Ti,\,i-1}\cdot(x_{in,\,i}-x_{in,\,i-1})^{-1}$ ratio: the probabilities are stochastically independent
Model consequences	$A=[(t_1+t_2-2\cdot R\cdot tg(\theta/2))\cdot r_t+R\cdot\theta\cdot r_c]\cdot V_E=\sum_i l_i\cdot r_t+\sum_j\theta_j\cdot R\cdot r_c=$ $=\sum_i l_i\cdot r_t+\sum_j\theta_j\cdot R_j\cdot[r_t+a\cdot R^{-b}\cdot D_{Tj,\,j-1}\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}]=$ $=\sum_i l_i\cdot r_t+\sum_j\theta_j\cdot R_j\cdot r_t+\sum_j\theta_j\cdot a\cdot R^{-b+1}\cdot D_{Tj,\,j-1}\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}]=$ $\cong\sum_i l_i\cdot r_t+\sum_j\theta_j\cdot R_j\cdot r_t+\sum_j\theta_j\cdot a\cdot R_j^{-b+1}\cdot[(g/2a)\cdot(f_{tm}+tg\beta_m)\cdot\lvert R_j-R_{j-1}\rvert]\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}$

The above formalized models constitute the bases on which, in this paper, the target relationships are structured. Figures 2 and 3 resume the exploratory logic here highlighted for Safety models.
Figure 2 shows an exploratory logic to clarify the role of the theoretically-based models in a wide-ranging scenario.

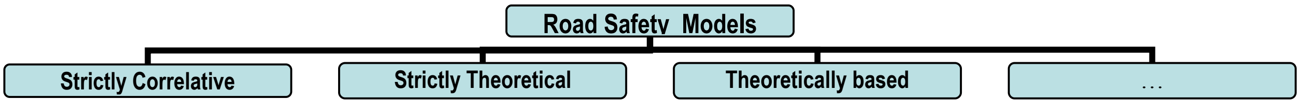


Figure 2 An exploratory logic for safety models

Following the inferences in table 2 and 3, in figure 3, for each of the four classes of models above introduced (A.1.1, A.1.2, A.2.1, A.2.2, see tables 2 and 3) some relationship are hypothesized.

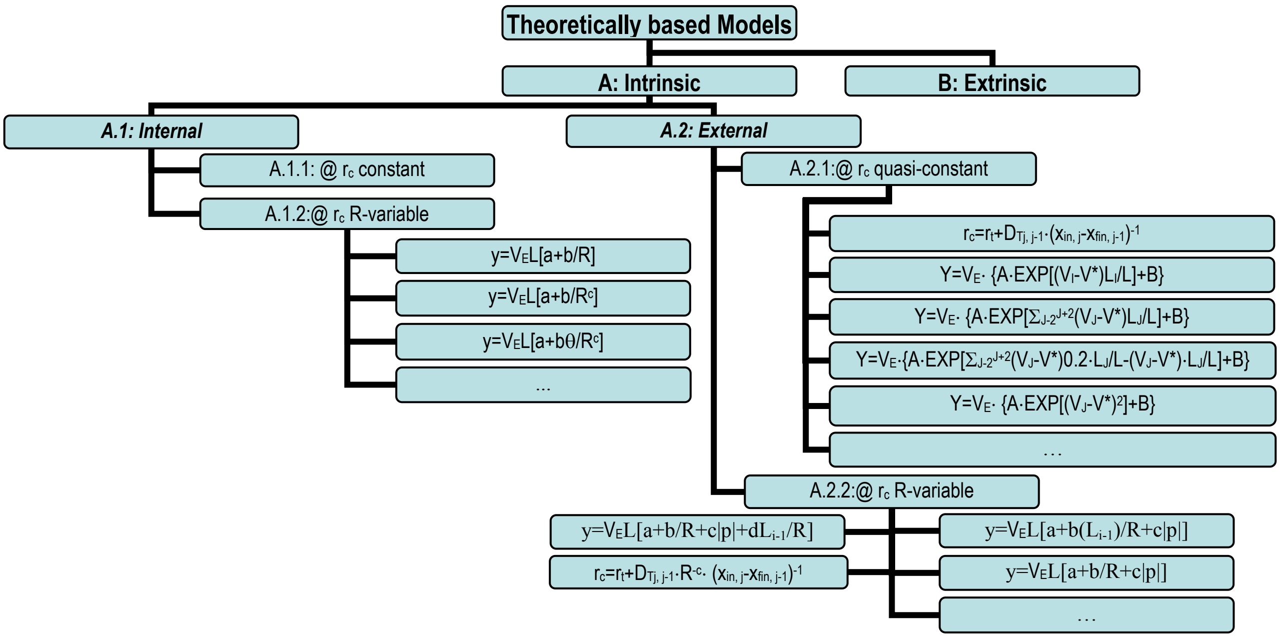


Figure 3 An exploratory logic for safety intrinsic/extrinsic models (see tables 1 to 3)

EXPERIMENTS AND ANALYSES

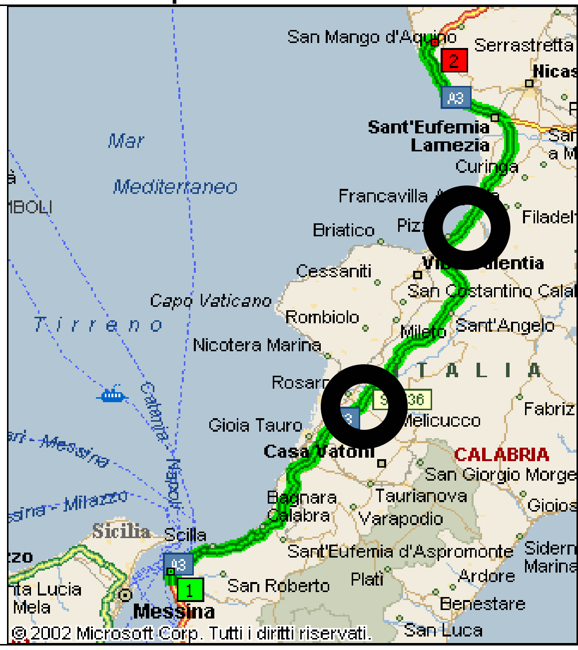
This paragraph deals with experiments, outputs analyses and validations following the above reported theoretical approach.
In order to validate the actual significance of the above discussed, the formalized models were applied to the A3 Salerno-Reggio Calabria, a strategic Italian motorway that runs from the Centre to the South of the West Coast of Italy. About 100 Km were examined, according to the Design Of Experiments (DOE) summarized in Table 4.

I and II phase
In the first phase (cfr. Table 4) the examined road was numerically interpreted and transformed in a data file, both for the north and the south carriageway. About 50·2 Km were examined. For each year and for each section, traffic was then estimated by utilising some specific data base. Spot traffic surveys and checks were

effected in order to validate the obtained data and to compare them by applying current demand evolution models.

Table 4: Main parameters and phases of the experiments

Majors	
Motorway	A3 Salerno Reggio Calabria.
Sections	From Km 350+100 to Km 400+200
Time period analysed	From 1997 to 2002
Level Of Service	C, D
Carriageway	Both North (i.e. North oriented) and South. Transverse dimensions: lanes: 4X3.75m; shoulders: 2X1.50m; central: 1.10m
Main phases	
I Phase	Surveying, assessing and organizing geometric and traffic data
II Phase	Single-varied analysis for accidents, geometry, etc. – outliers treatment (*)
III Phase	Choice of the set – Poly-varied analysis and optimization by different optimization methods. Outliers treatment (*)
IV Phase	Results analysis and interpretation
(*) synergic	



During the second phase the geometric data (radius of the horizontal curves, length of the horizontal section, longitudinal grade) were statistically and phenomenologically examined.
By referring to the statistics (see table 5), it is possible to highlight that Radii result quite low (if compared with the typology of the motorway), while north and south carriageway grades seem on average somewhat symmetric.
The coefficient σ/E (i.e the ratio between variance and average) appears quite high both for radii and lengths. Moreover, the grade is importantly affected by the average.

Table 5 Statistics

	Length [m]	Radius [m]	Curvature [1748/R]	Grade (North)	Grade (South)
Average E	315,05	1184,43	2,23	0,12	-0,12
Variance σ^2	35427,08	716722,92	1,92	5,01	5,01
Max	950,91	5000,00	5,83	4,10	4,28
Min	12,38	300,00	0,35	-4,28	-4,10
σ/E	0,60	0,71	0,62	18,65	-18,65

By referring to the traffic, each carriageway of the entire motorway was divided into 13 main segments (Pontecagnano, Eboli, Sicignano, Sala Consilina, Mormanno, Frascineto, Cosenza Nord, Falerna, S.Eufemia, Rosarno, Palmi, Villa S.Giovanni are the correspondent focal points), with given heavy vehicle percentages and total traffic (Testaguzza, 2001).
The total number of vehicle/day is below reported in figure 4, for different sections and for different years, while in figure 5 the accidents per year are reported.
This information was obtained by collecting all the accident reports in the different police stations along the above-mentioned motorway.
Since the studied sections were between the Km 350 and 400 (see table 4), attention was paid to take into account the traffic in the sections 350Km~382Km and that in the sections 382Km~400Km (see figure 4) and to consider eventual yard effects.

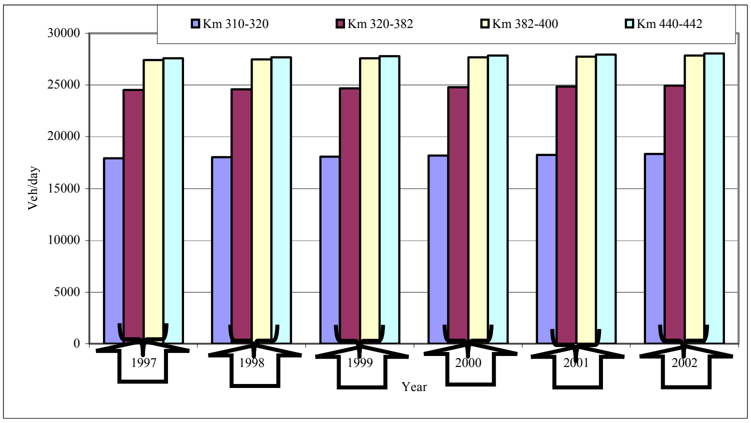


Figure 4 Vehicles for day over the years 1997 to 2002 (in different sections)

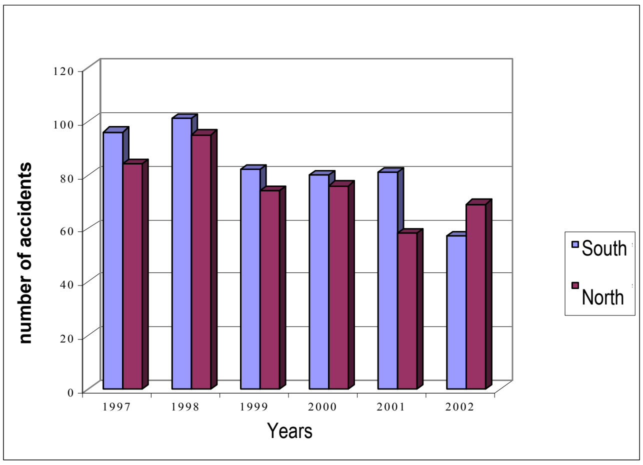


Figure 5 Main statistics on accidents

III and IV Phases

The optimization was carried on by two different methods: Least Square Method (LSM) and Negative Multinomial Likehood Function (NMLF). Each model was applied by following the logical scheme in figure 6. Two main steps may be showed: selecting the set to which the model is applied and choosing the maximization method (LSM or NMLF).

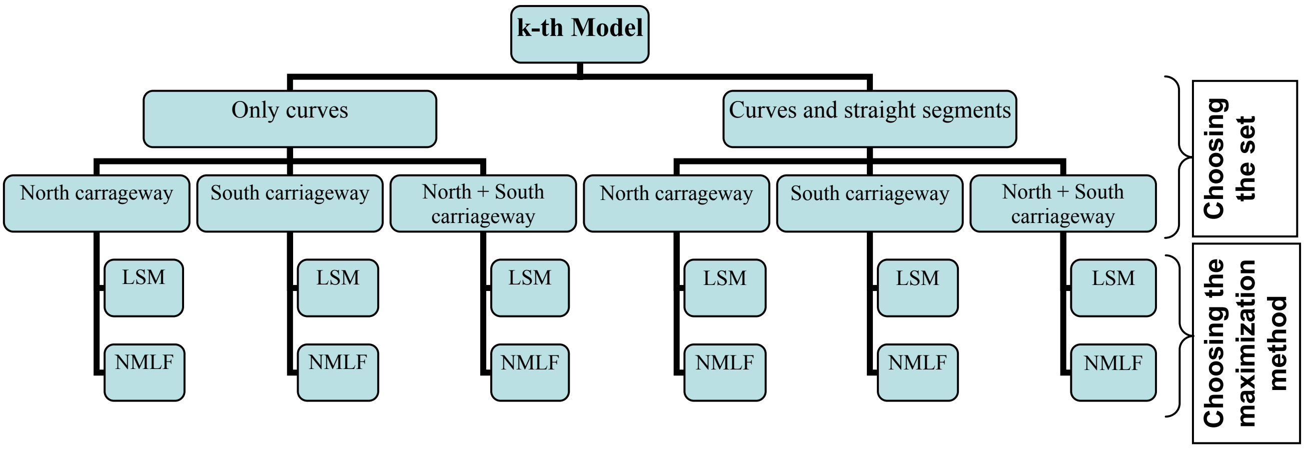


Figure 6 Validating the k-th model by considering a given set of sections and a particular maximization method (LSM or NMLF)

The Negative Multinomial Likehood Function (NMLF in figure 6) was formalized by supposing that accident counts come from a Poisson or Negative Binomial distribution but with some slight modifications in order to take into account that traffic changes over the years (Hauer, 2003).

So it was possible to formalize the following equation (to be maximized): $LnI=\Sigma_i lnI_i$, with $lnI_i=\varphi_i\cdot ln(\varphi_i)+[\Sigma_j a_{ij}\cdot ln(y_{ij})]+ln[\Gamma(a_i+\varphi_i)]-ln[\Gamma(\varphi_i)]-(a_i+\varphi_i)\ \cdot ln(y_i+\varphi_i)$. In it, Γ is the well known Gamma function, $\varphi_i=\varphi\cdot L_i$ (where L_i stands for the length of the i-th element), the a_{ij} are the observed accidents for the i- th element in the j-th year, the y_{ij} stand for the predicted accidents for the i-th element in the j-th year.

So the variables to be determined are the model parameters and φ (variance).

Slight differences in R-square coefficients (less than 10%) were obtained by applying both the least square and the negative likehood methods.

So, in the following table 6, the obtained R-square coefficients are organized in the following four main ranges: <0,1; 0,1~0,3; 0,3~0,5; 0,5~0,7.

Table 6 R-square coefficients

Model		South		North		South + North	
		CU	**CU+ST**	**CU**	**CU+ST**	**CU**	**CU+ST**
1.	$Y=V_E L[a+b/R]$	0.3~0.5	0.3~0.5	0.3~0.5	0.3~0.5	0.3~0.5	0.3~0.5
2.	$y=V_E L[a+b\theta/R^c]$	0.3~0.5		0.3~0.5		0.3~0.5	
3.	$y=V_E L[a+b/R^c]$	0.3~0.5		0.3~0.5		0.3~0.5	
4.	$Y=V_E\cdot \{A\cdot EXP[(v_j-v^*)^2]+B\}$	<0.1		<0.1		<0.1	
5.	$Y=V_E\cdot \{A\cdot EXP[(v_i-v^*)L_l/L]+B\}$	<0.1		<0.1		<0.1	
6.	$y=V_E L[a+b/R+c\|p\|]$	0.3~0.5	0.3~0.5	0.3~0.5	0.3~0.5	0.3~0.5	0.3~0.5
7.	$Y=V_E L[a+b(L_{i-1})/R+c\|p\|]$	0.1~0.3		0.1~0.3		0.1~0.3	
8.	$y=V_E L[a+b/R+c\|p\|+dL_{i-1}/R]$		0.5~0.7		0.5~0.7		0.5~0.7
9.	$Y=V_E\cdot \{A\cdot EXP[\Sigma_{J-2}^{j+2}(v_j-v^*)L_j/L]+B\}$	<0.1		<0.1		<0.1	
10.	$Y=V_E\cdot\{A\cdot EXP[\Sigma_{J-2}^{J+2}(v_j-v^*)0.2\cdot L_J/L-(v_j-v^*)\cdot L_J/L]+B\}$	<0.1		<0.1		<0.1	
11.	$r_c=r_t+D_{Tj,\,j-1}\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}\cdot R^{-1}$	0.1~0.3		0.1~0.3		0.1~0.3	
12.	$r_c=r_t+D_{Tj,\,j-1}\cdot(x_{in,\,j}-x_{fin,\,j-1})^{-1}$	<0.1		<0.1		<0.1	
CU: only curves; ST: only straight sections; CU+ST: both the ones and the others; a, b, c, d, A, B: coefficients							

By considering the above reported R-square coefficients (see table 6), it is possible to appreciate that the models at R constant or quasi-constant (classes A.1.1 and A.2.1 in figure 3 and 4th, 5th, 9th,10th, 11th, 12th model in table 6) have a non-negligible variance in the dominions of both the high and low accident numbers. So, a large part of them reveals, in practice, an inadequate level of explained variance.

Moreover, it appears quite evident that by introducing some external parameters (e.g. L_{i-1}, alike a condensed case-history for the i-th element) some of the models can achieve a considerable improvement.

Importantly, the models of the classes A.1.2 and A.2.2 (i.e. @ r_c R-variable) may constitute a quite reliable base for accident prediction (see the 1st, 2nd, 3rd, 6th, 8th model in table 6).

Figure 7 shows the 8th model in table 6, for a given grade |p|, for a L_{i-1}/R ratio varying from 0.1 m/m to 10m/m (V_E and L being constant; a=0.068; b=61.31; c=0.003; d=0.037; ρ^2=0.64; surveyed set: CU+ST, South and North carriageway).

Figure 7 shows that, for ratios L/R≤1, the influence of L/R on the number of accidents is quite negligible. This fact shows an agreement with the well known constraint R>L, which is a recurrent characteristic of many European standards.

In particular, for a condition with R=150m and L=1500 m, the same benefit (about “-1,4 accidents”) can descend from changing the previous straight section from 1500 to 150m, by changing the radius from 150 m to 600m, with a L=6000m, or simply by doubling the radius (R=300, L=1500).

At L/R constant, the influence of the radius ($|\partial Y/\partial R|$) decreases as the radius itself increases. So, for L/R=1, by changing the radius from 200m to 400m, the benefit is quite “-0,6 accidents”, while by changing it from 600 to 800m the benefit is about “-0,2 accidents”.

Finally, by finding the first derivative, the different importance of the parameters in the equation reported in figure 7 is quite evident. By changing the grade p in $y=V_E L[a+b/R+c|p|+dL_{i-1}/R]$, for example from 0 to 1, it will be a change in y equal to a given quantity ΔA.

On the contrary, by changing the radius R of one unit there will be a change in y equal to about $\Delta A/10$. So, for example, if one compares, from a construction point of view, a change of the radius from 350m to 450m and a change in longitudinal grade of one point percent, it results that the decrease in accidents given to the change in radius is ten times greater than that caused by the change in longitudinal grade. This fact may confirm the priority of R on p in controlling intrinsically road safety.

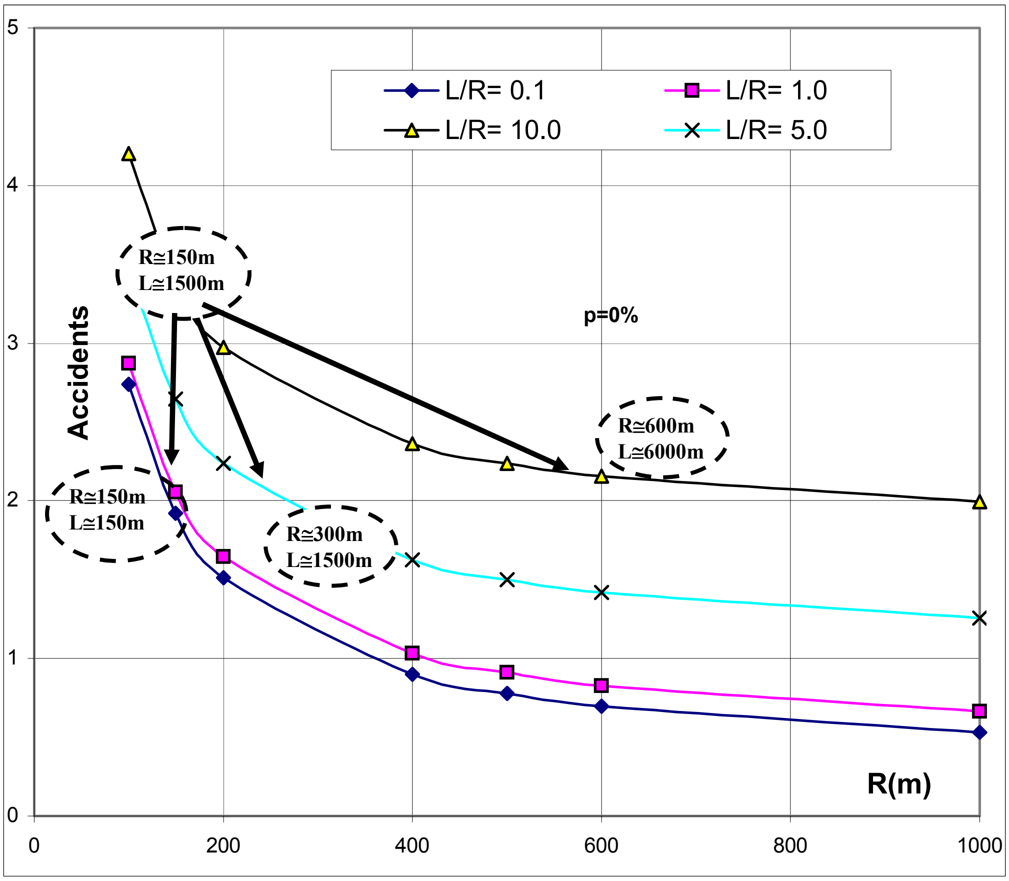


Figure 7 Influence of L/R ratio on the number of accidents

In the following figures from 8 to 9, some biplots are reported. They concern the relationship between the number of accidents predicted by a given model ("Predicted") and that effectively surveyed ("Actual"). These plots, which may be considered as representative for the classes A.1.2 and A.2.2 (i.e. @ r_c R-variable), show a good accuracy but a quite high variance.

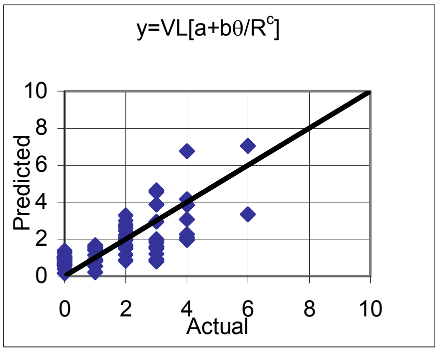


Figure 8 Biplot for the 2-nd model in table 6

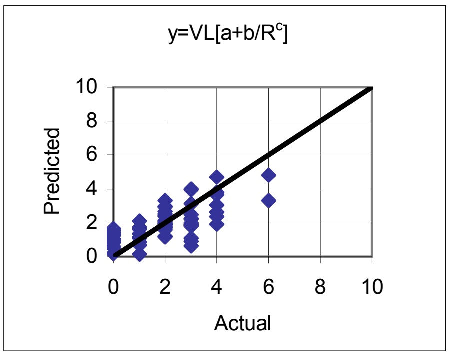


Figure 9 Biplot for the 3-rd model in table 6

It is significant to remark that by operating a statistic analysis in order to discriminate the outliers, it was possible, in relation to the different statistical techniques and relations to upgrade importantly (+0.1~+0.3) the R-square coefficients. It is noteworthy to highlight that all the results and plots here reported consider all the points, without any outlier suppression.

On the basis of the above it is possible to outline some considerations:

1. The experiments and the statistical processing of the data confirm the leading rule of R. This circumstance results from the analysis of the R-square coefficients (table 6) and from the analysis of the first derivatives of the model represented in figure 7. Despite this, both the longitudinal grade and the ratio L/R may sometimes have a consistent importance;
2. The relation between 1/R and r seems to be non linear. Anyhow, by a linear correlation, the R-square coefficient stands in the same dominion of table 6;

3. In these conditions, the overall influence of the longitudinal grade can be considered a minor topic if compared with other parameters;
4. The influence of the deflection angle of the curve on the accident rate seems to be quite negligible;
5. The coexistence of the two different independent variables L_{i-1}/R and $1/R$ can optimize importantly the correlations (see table 6); in particular, the presence of the external parameter L_{i-1}/R, which concerns the design consistency, can cause an appreciable growth of the R-square coefficient;
6. The indicators depending on the delta (v_i-v*) seem to have, in this case, a negligible correlation with the accident rate (cfr. Table 6);
7. The indicator $[\Sigma_{j-2}^{j+2}(v_j-v^*)0.2\cdot L_j/L-(v_j-v^*)\cdot L_j/L]$, though sensible to some peaks, seems not well correlated with the accident rate.

CONCLUSIONS

Despite the considerable time of observation (six years), the length of the trunk (about 100 Km), the probable appropriateness of both traffic and accidents surveys and statistics (many direct and indirect validations), the obtained results must be considered just a small sub-set, neither sufficient to assure model transferability nor to suggest too general conclusions.
Conversely, in the light of the formalized models and effected experiments, some inferences may be outlined:

a) Some intrinsic parameters (e.g. R, L_{i-1}, p) may explain a considerable part of the entire safety process; so, they can be useful in risk assessment;
b) the horizontal radius affects importantly the stochastic process of road accidents and may control a considerable part of the phenomena;
c) some external characteristics (such as L_{i-1}), which add some of the information not contained right in the i-th road element where the accident does happen, may appreciably optimize the correlations.

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APPENDICES

Annex A - Symbols

Table 1: Symbols	
$\Gamma(\varphi_i)$	Gamma function
$\Gamma(\varphi_i)$	Gamma function
σ^2	Variance
θ_i, θ	Deflection angle
φ_I, φ	$\varphi_i=\varphi\cdot L_i$ (where L_i stands for the length of the i-th element); φ is the variance
β_m	Transverse slope
Δs_i	Space
a, a_T, a_L, **a**	Accelerations
a, b, C, A. B, d	Real numbers
A_i, A	Accidents
a_{ij}	Observed accidents for the i- th element in the j-th year

$D_{Ti,\ i-1}$	Transition distance
$D_{Ti,\ i-1}$	Transition distance
E	Average
EXi	Facility exposure for the i^{th} section
F	Force
f	Number of injured
f_{tm}, f_t, f_a	Friction coefficients
G	Risk magnitude
g	Gravitational acceleration
h_G	Height of the centre of gravity of the masses
i, **n**	Vectors with modulus equal to 1
IS	Accident indicator
K	Ratio between r_c and r_t
K_g	Coefficient in risk formula
L, l_i, L_{i-1}	Section length
L, l_i, L_{i-1}	Section length
L_a	Distance from the right to the left wheel
m	Mass
m_d	Number of deaths
o_t	Other damages
p	Longitudinal slope
P	Weight of the car
p_i	Probability
R_a	Risk
R_{es}	Resistances
R_i, R, R’, $R_{2.5}$, R_m, R_{min}	Radii
r_t, r_c	Accident rate in tangent or in curve
t_i, t	Tangent length
v*, v, v_m, v_j, Δv, v	Speeds
V_E	Number of Vehicles
$x_{fin,\ i-1,}$ $x_{in,\ i}$	Abscissa
$x_{fin,\ i-1,}$ $x_{in,\ i}$	Abscissa
y_{ij}, y_i	Predicted accidents

Annex B - Horizontal Radii in Road Risk Theory

This annex concerns the rule that the horizontal radii can play in terms of Risk theory applied to road safety.
The risk R_a is attributable to the infrastructural exposure EX_i, whose concept “is” to that of “cause” as the concept of accident “is” to that of “effect”.
So, the Attributable Risk is the rate of the disease in exposed individuals that can be attributed to the exposure to the road. This measure is derived by subtracting the rate (usually incidence or mortality) of the disease among nonexposed persons from the corresponding rate among exposed individuals.
In particular, with reference to the exposure EX_i, it is possible to say that it concerns the movements in the facility, so the circulation, the distances covered, the travel times; the respective parameters, subjected to *exposure assessment*, derive from these concepts.
It is possible to make some subdivisions in classes of risk exposure (professional or non-professional driver, type of vehicle, etc.), or in Levels (greater or smaller) of risk exposure; anyhow, despite this, for road safety, it is quite complex to define an exposure threshold (log-probit), below which there is no “response” (that is to say a negative effect, for example, an accident).
Because of the extreme variability of the infrastructural risk exposure, it is very difficult to hypothesize a non-variance condition in the exposure time, that is a *steady state exposure*.
Moreover, in particular, the association of a constant “dose-response”, following a fixed exposure level, is very problematical; in the specific case, the constraints and modes of risk exposure (that is, infrastructural variables, traffic variables, etc.) control the effect of the accident number following a given “dose” of exposure (for example, a 1-km travel, a 1-hour travel, etc).
Therefore, the correlation between a given exposure and its respective effect (that is *response*, e.g. accident), i.e. the so-called *dose-response*, resulting from an analysis of the number of accidents data and from a specific *dose-response-assessment* isn’t always very significant, unless it refers to intermodal comparisons (for example roads-railways, etc.).

As for the estimation of the Risk parameter R_a, it is traditionally linked to the product between a quantity p, which estimates the "breakdown" probability, and another G, related to the magnitude of the event. In strictly road terms, the subject can be interpreted by the trilogy probability-consequence-magnitude too.
Now, if one hypothesizes the independence between each involved probabilities, in $R_a=G \cdot p$, it is possible to state $p=\Pi_i p_i$, where $\Pi_i p_i=p_1 \cdot p_2 \cdot \ldots\ldots\ldots \cdot p_n$, i=1, .., n, $0 \leq p_i \leq 1$.
Relating to each p_i and the numerousness of the respective set, we can hypothesize a logical organization in independent components with reference to the infrastructure ($p_{\text{"road breakdown"}}$), the vehicles interference ($p_{\text{vehicle interf.}}$), the behaviours of the users ($p_{\text{behaviour}}$), the vehicle ($p_{\text{"vehicle breakdown"}}$), the environmental effect, etc..
In particular, by referring to the explication of the single p_i, it is possible to observe that, according to some Authors, for low changes in the traffic volumes, there is a linear correlation between accident frequency and traffic volumes.
On the contrary, with reference to the magnitude (G) of the accident, this can be expressed as a number of dead, injured or in terms of material damage. Therefore, for this subject, the important elements are:
-The accident rate relating to the "class of injury"; the analysis of this spectrum of parameters is the base for a "direct regulation" for road safety;
-The number of dead and injured and material damage for a 1-km road; this quantity leads to a classification "per exposure level" of the road;
-The number and the distribution of the different types of injuries according to the group of users, or per million of users; in this case, the parameter concerns a classification of the exposures on the basis of circulation characteristics;
-The number of deaths compared to that of the injured (dead or injured).
On the other hand, G is a function of the relative kinetic energy $0.5 \cdot m \cdot \Delta v^2$ (considering the vectors projections); among the expressions for the respective estimation, it is possible to quote $G=K_g \cdot 0.5 \cdot m \cdot \Delta v^2$, where K_g can depend on the measures of the passive safety for the infrastructure and the vehicle.
Another technique of estimation can be the monetization of the damages caused. In this case, G can be expressed in euro (or another currency).
The indicators of the number of accidents IS, formalized for the valuation and the study of the number of accidents phenomena, derive from the opportunity of a comparative and synthetic estimation of the total magnitude of the damages and of the traffic and/or journey and /or time competence "load". Among them one can put in evidence the following ones: IS_1=(number of victims)/(km·vehicles); IS_2=Cost in euro/(Km·vehicles), with Cost in euro= $a \cdot m_d+b \cdot f+c \cdot o_t$, where m_d represents the number of deaths, f the injured and o_t the remaining damages; in this expression a, b, c are positive numbers which take into account the different "value" of m_d, f, o_t; IS_3=N.of victims/(n. people·circulation hours); IS_4=a·(number of accidents in a year)/(length of the considered section·Annual Daily Average Traffic); IS_5=b·(number of accidents in a year)/(length of the considered section).
From a general point of view, IS can be expressed as a function of the Risk R_a, as $IS=R_a \cdot C$, where C is a real number. Despite this (and some attempts to analyse some problems in terms of stochastic geometry), the passage from $IS=R_a \cdot C$ to an algorithm, useful for applications, seems quite difficult.
From a road Engineer point of view, the accident frequencies (e.g. IS_4) may be studied by correlations with substantially intrinsic factors (that is to say road geometry, pavement, etc.) or by referring them to extrinsic ones (driver behavior, vehicle, etc.), although this difference isn't always very rigorous. Intrinsic factors may be internal or external to the considered road section (see figures 2 and 3).
Both internal and external factors may be dependent or not dependent on R.
This organization in terms of R-dependence may be considered both theoretically and experimental based and it is due to the particular relevance of the plane curvature in relation to design, vehicle dynamic, and accident frequencies (and probability). It may be seen as a direct consequence of Newton's Theorem, as follows. In fact, as is well known, being $\mathbf{F}=m \cdot \mathbf{a}$, $\mathbf{a}=d(\mathbf{i} \cdot v)/dt=\mathbf{i} \cdot dv/dt+v \cdot d\mathbf{i}/dt= \mathbf{i} \cdot a_L+\mathbf{n} \cdot a_T$ (where a_T stands for v^2/R and, for a motorway $R \approx R_{horizontal}$, being $R_V >> R_{horizontal}$), vehicle stability in curve is essentially governed by radius and crossfall. The approach of Italian standards may be resumed in three segments on a two-logarithmic plot (R-tgβ) as in figure 12.

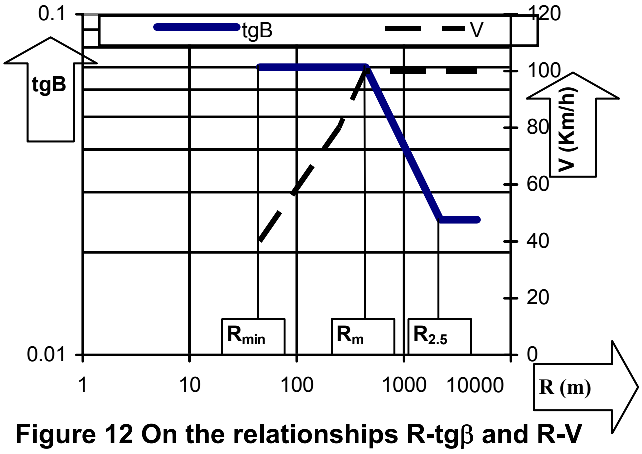

Figure 12 On the relationships R-tgβ and R-V

This plot descends from being $-(P/g)\cdot(v^2/R)\cdot\cos\beta+P\cdot\mathrm{sen}\beta+P\cdot f_t\cdot\cos\beta$; so it comes that $v^2/(g\cdot R)\cong tg\beta+f_t$. The value of R_{min} corresponds to $R_{min}=R[\max(tg\beta), v_{pmin}, ft(v_{pmin})]$ and it is $Tg\beta=cost\ \forall R\in(R_{min}, R_m)$. The value of R_m is determined by the following relation: $R_m=R[\max(tg\beta), v_{pmax}, ft(v_{pmax})]$. From R_m to $R_{2.5}$ the behavior is based on the following hypothesis: $Log_{10}R\cong -a\cdot Log_{10}(tg\beta)+b$, where $a>0$, $b>0$, $\forall R\in(R_m, R_{2.5})$. Finally it is $R_{2.5}=5R_m$. It must be highlight that a particular value R' exists (not represented in the plot) for which it is: R': $tg\beta=\pm 2.5\%$, with $R'>R_{2.5}>R_m>R_{min}$ and $f_t\cong[f^2_a-(R_{es}/P)^2]^{0.5}=F(v, \text{road type})$. Italian Standards pose $\max(tg\ \beta)=0.035\sim0.05\sim0.7$ and $\min(tg\ \beta)=0.025$, while approximately it results $v\cong(2.5\cdot R)^{0.5}$. Finally one must precise that the other relation $v^2/(g\cdot R)\cong(h_G\cdot tg\beta+0.5\cdot L_a)\cdot(h_G\text{-}tg\beta\cdot 0.5\cdot L_a)^{-1}$ usually doesn't control R-assessment. In the light of the above, intrinsic safety dependence on R may be considered as a theoretically based statement too.