# Methodological approach to friction assessment. Use of a well-equipped car

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## SYNOPSIS

The authority, that either owns or runs the road network, is responsible for the tyre-road friction level as an essential parameter to draw up a routine maintenance plan, aimed at providing safe driving conditions, according to the European Union guidelines.
Experimental research has proved a functional relationship between normalized friction force and tyre slip. The roughness (micro, macro, mega) of the road surface has influence on the coefficients regulating that link. The spatial and temporal variability of roughness as well as the complex control of the various factors affecting the friction phenomenon, would involve a statistical account of the necessary data to set out the link between normalized friction force and slip.
Tyre slip can be measured with the different angular speed of wheels (two driving and two non-driving wheels) in a common vehicle. An experimental device will enable us to get these velocities by exploiting the signals coming from the mass-produced sensors of the vehicle.
Once we know the map engine torque, the longitudinal forces acting on the tyre-road contact area can also be drawn from the angular velocities of either driving wheels or drive shaft. Other devices, which directly measure those forces, can be used to check the precision of the forces derived from the above angular speed measures.
It will be shown that, because of phonic wheel characteristics, the variance of slip measure depends on how wide the sampling interval of angular speed is. On the contrary, the variance of force measures can be obtained from the comparison of the information coming from the devices we will describe later on.
The variance of the above measures is an essential input to use and strengthen the statistical model suggested as regards normal driving.

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## INTRODUCTION

It is strongly hoped within the European Union that the rate of road accidents could be reduced. Accurate control, prompt reactivation and the possible increase in friction performance of the road surface may contribute a lot.
The forces acting on the tyre-road contact area actually result in manoeuvrability and stability even in emergency driving. We have to say that, on the same paving and in the same conditions, the friction level may vary in proportion to the structural characteristics of the vehicle, the tyre characteristics and other quality and quantity factors.
Several normalized instruments provide information on the friction level which is made available when the survey takes place. Most of these instruments ground their working principle on the relationship between the longitudinal slip of the test tyre and the acting forces. The same quantities can be detected by means of a common vehicle equipped with specific device.
This device will enable us to measure the angular speed of both wheels and engine every 20 ms. The procedure associates high performance, low costs and easy employment with the minor inconvenience of vehicle flow during survey.
This work will briefly show the working principle of the experimental equipment to be used in order to get slip and, besides, it will illustrate the methodology fixing the engaged forces from the same angular speed measures. A thorough theorical analysis of slip measurement variability lets us a priori evaluate how reliable the output of the statistical model suggested may be.

## FRICTION AND MEASUREMENT TECHNIQUES

The force resultant on the ideal plane of contact between the road surface and a rolling tyre depends on the load orthogonal to that plane (Fig. 1). Differently from what was originally stated in Coloumb definition, the proportional coefficient or friction coefficient may vary in relation to several influential factors (Tab. 1). At present we can not entirely investigate into every single factor as the reciprocal interaction between factors is rather complex.

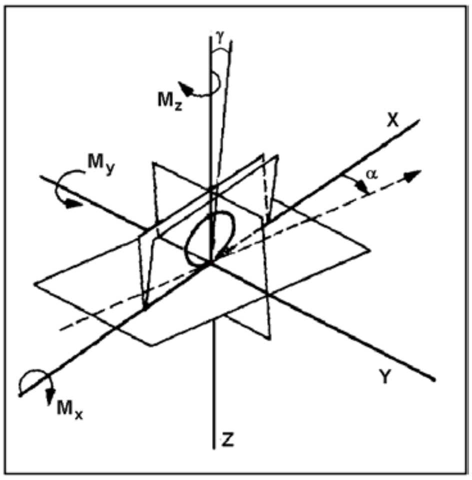

**Figure 1: Tyre axis system**

The friction level in a given moment can be shortly expressed by the following function:

$$\mu = f(S, P, C, V, \mu_{\max}, F_Z)$$

where: $V$ is the forward movement velocity; $\mu_{max}$ shows the maximum level friction coefficient, $F_z$ the vertical force; *S, P, C* sums up the effects respectively related to the road surface, tyres and operating conditions The above equation may be simplified if we point out the longitudinal slip, (hereafter defined) between tyre and road surface.

**Table 1: Factors influencing friction**

| Tyre (P) | Road surface (S) | Operating conditions (C) | |
|---|---|---|---|
| | | Contaminants | Other factors |
| Dimensions<br>Structure<br>Compounds<br>Shape and depth of tread pattern<br>Pressure inflation<br>Stiffness | Microtexture<br>Macrotexture<br>Megatexture<br>Regularity<br>Draining capacity | Water, Snow, Ice, Dust<br>Film thickness<br>Chemical Composition of contaminants | Surface temperature<br>Tyre temperature<br>Vehicle velocity<br>Acting load |

Then, we have:

$$\mu = f(\sigma_X, V, \mu_{\max}, F_Z)$$

Some tests have underlined the relationship $\mu$-$\sigma_X$. Its qualitative progress, related to different surfaces and forward movement velocities, is recorded on Figure 2. Different tyres or even the same tyre with different inflation pressures would modify the form of the curve $\mu$-$\sigma_X$ in relation to the clear alteration in $\mu_{max}$ values, if road surface characteristics are the same.

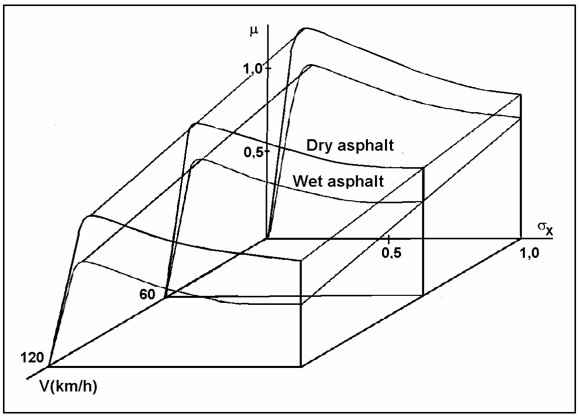


**Figure 2: Curves μ-σX in different road surface conditions and different velocity.**

The upward line in the curve shows a steady tyre provided with a good friction on road surface (the vehicle reacts congruently to driving actions); along the downward or unsteady line, any increase in friction demand may damage the control of both tyres and vehicle.
Technical literature provides several analytical expressions which, starting from different tyre behaviours or experimental observations, let us assess the interaction between road surface and tyres. Table 2 shows the commonly used laws $\mu$-$\sigma_X$.
The World Road Association proposed the use of an universal yardstick for surface friction measurement called the International Friction Index (PIARC 1995), as a further development of the Penn state model:

$$\mu = \mu_0 e^{\frac{S}{S_P}}$$

where $\mu$ is the friction, $\mu_0$ is the intercept of friction at zero speed, $S$ is the slip speed, $S_p$ shows a constant speed.
Based on this relation it is possible to compare the friction level indicators which have been measured with the test devices. This is a further explanation of the fact that any device provides a friction indicator value which may vary according to the working principle used and/or to the specificity of components.
In Tab. 3, as an example, are only reproduced some of the devices used in different countries.
Fixed slip devices can assess the road surface quality on the basis of a single point in $\mu$-$\sigma_X$ curve. According to the method, that point can be located:
- near that line in the curve where the friction coefficient reaches the maximun value (slip 10% - 20%);
- near that line where the test wheel starts locking (slip between 86% and 100%).

We ought to stress that, also for reducing the wear of the test wheel, measures are generally taken after a water film (whose thickness may vary) has been placed between the contact surfaces. In this case, hysteresis, strongly affected by the tyre mixture, plays an active role in fixing the tangential forces working between tyre and road surface. Besides, in the presence of water, those forces are sensibly affected by the tread pattern of the test tyre.

Within the limits related to the deceleration of an ABS equipped vehicle, according to some researchers (NCHRP Web Document 53), test results seem to provide a reliable estimate of the friction level of the road surface even when polluters are acting (water, snow, ice).

**Table 2: $\mu$-$\sigma_X$ laws**

| | |
|---|---|
| Pacejka<br>(1991) | $Y(x) = D \sin [C\, ar \tan (Bx\text{-}E(Bx\text{-}ar \tan (Bx)))]$<br>where: Y(x) may reproduce either the longitudinal force $F_x$ or the transversal force $F_y$ or the couple of self-alignment $M_z$, $D$ is the maximum value of the curve, $C$ the constant fixing the curve type, $B$ checks the slope of a road, $E$ checks the slip abscissa we get the maximum friction. (Pacejka. et al.1991) |
| Ben Amar<br>(1994) | $F_x = \frac{1}{2} k_x g_l \omega_R (\Delta l + l_R{}^2) + \mu F_z \cos(\chi)(1 - \frac{3}{4}\frac{(\Delta l + l_r)^2}{l_R{}^2} + \frac{1}{4}\frac{(\Delta l + l_R)^3}{l_R{}^3}$<br>$F_y = \frac{1}{2} k_y g_t \omega_R (\Delta l + l_R{}^2) + \mu F_z \sin(\chi)(1 - \frac{3}{4}\frac{(\Delta l + l_r)^2}{l_R{}^2} + \frac{1}{4}\frac{(\Delta l + l_R)^3}{l_R{}^3}$<br>where: $\omega_r$ is the width of contact area, $2l_r$ the length of contact area, $2l_r+\Delta l$ abscissa of detachment point; $g_l$, $g_t$ longitudinal and transversal slip, $k_x$, $k_y$ longitudinal and transversal stiffness. (Stephant et al. 2002) |
| LuGre<br>(1995) | $\dot{z} = v_r \quad \frac{\sigma_0 \lvert v_r \rvert}{g(v_r)} z$; $\qquad g(v_r) = \mu_c + (\mu_s - \mu_c) e^{-\left\lvert \frac{v_r}{v_s} \right\rvert^{1/2}}$<br>$F = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) F_n$<br>where: $\sigma_0$ is the normalized rubber longitudinal lumped stiffness, $\sigma_1$ the normalized rubber longitudinal lumped damping, $\sigma_2$ the normalized viscous relative damping, $\mu_c$ the normalized Coulomb friction, $\mu_s$ the normalized static friction, $v_s$ the Stribeck relative velocity, $F_n$ the normal force, $v_r$ the relative velocity, z the internal friction state, $q$ a parameter introduced to capture changes in road characteristics; it can be interpreted as being the coefficient of road adhesion. (Canudas-De-Wit C.et al. 1999) |
| Kiencke and Nielsen<br>(2000) | $\mu^{ij}(g^{ij}) = (c_1(1 - \exp(-c_2 g^{ij})) - c_3 g^{ij}) \exp(-c_4 g^{ij} V_G^{Sa})(1 - c_5 (F_z^{ij})^2)$<br>where: $c_1$, $c_2$, $c_3$ are parameters resulting from paving type and condition, $c_4$ a parameter function of driving velocity, $c_5$ a parameter resulting from wheel load , $g^{ij}$ the global slip (the vector addition of longitudinal and transversal slip),$V_G{}^{Sa}$ the vehicle velocity in the reference system, $F_z$ the normal load. (Kiencke U. and Nielsen L. 2000) |

**Table 3: Devices for the assessment of friction indicators**

| Working principle | Device |
|---|---|
| 100% Slip<br>(Locked wheel) | LCPC SKID TRAILER ADHERA<br>ASTM E274 TRAILER<br>SKID RESISTENCE TESTER |
| Fixed slip | SKIDDOMETER BV8 (20%)<br>SKIDDOMETER BV11 (20%)<br>DWW TRAILER (86%)<br>GRIPTESTER (14.5%) |
| Variable slip | NORSEMETER ROAR<br>NORSEMETER SALTAR<br>DAGMAR/PETRA TRAILER<br>KOMATSU SKID TESTER |
| Fixed slip angle<br>(Side force) | MUMETER (7.5°)<br>SCRIM (20°)<br>SUMMS (20°)<br>WALLON ODOLIOGRAPH(15°)<br>CRR ODOLIOGRAPH (20°)<br>STRADOGRAF (12°) |

| | |
|---|---|
| Deceleration | C-MU<br>CORALBA<br>TOPLEY-METER<br>BOWNMANK-METER |

More detailed information on road surface conditions can be surely obtained through variable slip measures. For every single cycle of measurement, this technique provides the whole friction-slip curve. During "linear" braking of the test wheel, data are recorded since free rolling until locking.

## HOW TO MEASURE FRICTION WITH A WELL-EQUIPPED CAR

### Statistical Model For Friction Measure

The meaningful formal differences in the expressions in Tab. 2 do not involve substantial alterations in the progress of $\mu$ and $\sigma_X$ relationship. It has been proved that, if we introduce congruent values of the coefficients in the various expressions, the highest deviation in values would not exceed 10% (Stephant et al. 2002).
It has also been noticed that, unless the longitudinal friction $\mu$ gets closer to its highest value ($\mu_{max}$), a direct proportionality does exist between $\mu$ and $\sigma_X$, then, we can write:

$$\mu = B\sigma_X$$

This is not to be seen as a mathematical link between the two variables. We must take into account the possible errors in the measurement of variable due to either the methodology or the instruments used to get them. The tyre road contact characteristics may vary in space and in time for the same road.
As we will say further on, we would get both the slip measure and the longitudinal nomalized force by equipping an ABS fitted out modern car.
We can get these measures, at fixed intervals, if the vehicle is driven at constant velocity or moderate speeding or slowing down. These conditions are surely included in the linear section of $\mu$-$\sigma_X$ relationship.
Figure 3 diagram shows the necessary steps to draw the so-called slip slope (β) by making use of only the angular speed measures of vehicle wheels.

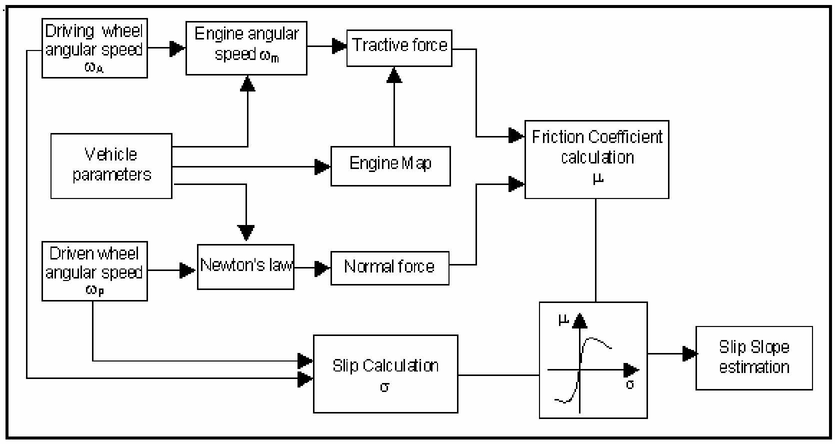


**Figure3: Scheme of slip slope estimation**

While a vehicle is driving along a given road section, we can get n pair of measures ($m_i$, $s_{X,i}$) in place of n real unknown values ($\mu_i$, $\sigma_{X,i}$). We get the following linear regression model:

$$m_i = \mu_i + e_\mu$$
$$s_i = \sigma_{X,i} + e_\sigma$$
$$\mu_i = B\sigma_{X,i} + \varepsilon_i$$

$e_\mu$, $e_\sigma$ express the errors in measured values and $\varepsilon_i$ shows the stochastic variability of real values. The errors $\varepsilon_i$ express that, during driving, any alteration in one of the influencing factors causes a jump from a friction curve to another (Canudas-De-Wit et al. 1999).
On account of the link between measurement errors and the explicative variable, the classical least square method provides estimations of parameters $\alpha$ and $\beta$ that are not consistent, whatever the sample dimension may be. We can still get consistent values out of these parameters once we have additional information on how large the variance of measure errors may be. If we have information about the variance, the second order moments are stochastically convergent to their average values (Cicchitelli 1984), (Kendall 1989).
Here is, in fact, the target of this work: a close research of the procedures used to get the $m_i$, $s_{X,i}$ measures so as to draw, even through simulation, reliable information on the variance of measure errors.

Once we take the variance as our starting point, the comparison of the angular coefficients ($\beta$) of the regression in different road sections would let us write down a list of the sections as regards friction performance. Yet our results would depend on the variance of stochastic terms we can only assess after careful experimental research.
Besides, it can be stated that, $\beta$ being equal, the bigger variance $\beta$ proves to be, the worse friction performances of road surface are. Both $\beta$ and Var($\beta$ ) are to be taken into consideration for an effective comparison of the surface characteristics of two separate road sections.

## Longitudinal Slip

In order to work out the longitudinal slip $\sigma_X$ we can suggest several expressions, which only differ in the form not in the substance. If $V_X$ shows the absolute velocity of the forward wheel movement, $\omega_X$ the angular velocity, $R_d$ the rolling radius, the slip will be:

$$\sigma_X = \frac{\omega_X R_d - V_X}{V_X}$$

Following the above definition, the slip $\sigma_X$ would get a value ranging from -1 to+ $\propto$ and in particular (Genta 2000b):

- $\sigma_X >= -1$: would be recorded during braking ($V_X > \omega_X R_d$); $\sigma_X = -1$ would coincide with wheel locking ($V_X > 0$; $\omega_X R_d = 0$); if this is the case, the wheel moves forward but doesn't roll and the ratio between the longitudinal force and the one orthogonal to the plane of contact is equal to the coefficient of kinematic friction and certainly lower than the rolling friction coefficient;
- $\sigma_X = 0$: would prove the pure rolling motion in the absence of applied moments ($V_X = \omega_X R_d$); in real life, the $V_X$ and $\omega_X R_d$ values are never exactly the same; but for driven wheels we can assume $V_X .= \omega_X R_d$;
- $\sigma_X > 0$: the slip gets positive values as soon as a driving moment is applied on the wheel; for $V_X < \omega_X R_d$, the slip becomes $\propto$ and a total slip takes place (for example: a start on icy road).

If the subscript 1 and 2 shows the two front driving wheels and 3 and 4 the rear driven wheels, the slip related to each wheel is:

$$\sigma_1 = \frac{\omega_1 R_1 - V_X}{V_X} \qquad \sigma_2 = \frac{\omega_2 R_2 - V_X}{V_X}$$

$$\sigma_3 = \frac{\omega_3 R_3 - V_X}{V_X} \cong 0 \qquad \sigma_4 = \frac{\omega_4 R_4 - V_X}{V_X} \cong 0$$

As long as we know the four wheel angular velocity of an ABS equipped car we can get the absolute velocity $V_X$, without any special devices. While driving on a straight line, when driven wheel slip doesn't take place, we have:

$$\omega_3 R_3 = \omega_4 R_4 \cong V_X$$

Therefore, given $\omega_3 R_3 = \omega_4 R_4 = \omega_P R_P$, the longitudinal slip of one of the front wheels, generically marked by the subscript "A", is shown as:
In case a braking moment is applied on driven wheels, the translatory velocity would be affected by a

$$\sigma_A = \frac{\omega_A R_A}{\omega_P R_P} - 1$$

mistake due to the slip speed of those wheels. The previous conditions still hold unless the secondary circuit of back brakes oil is closed or supplementary wheel is used to assess $V_x$.
While driving on a curve, all four wheels have different angular velocities. In this case, we need to go over the previous relations as well as the following ones based on dynamic equilibrium.

## Reliability Of Slip Measurement

The angular velocity of each wheel can be measured with the signals coming from the four variable reluctance (or any type) detectors which can be found on any ABS equipped vehicle. The detectors, located near the phonic wheels integral with hubs, produce a variation in sinusoidal wave tension, amplified and squared by a simple device expressly manufactured at DIIV.
Then the state of the signal is tested at 10khz frequency so as to step up a special pulse counter, when the square wave goes up (Fig. 4). Every 20 ms the difference in counting is sent to a computer recording these differences as well as the value of signals coming from other detectors the vehicle is equipped with. In order

to minimize any counting errors, it is assumed that, during data transmission, the signal state checking will not be interrupted.

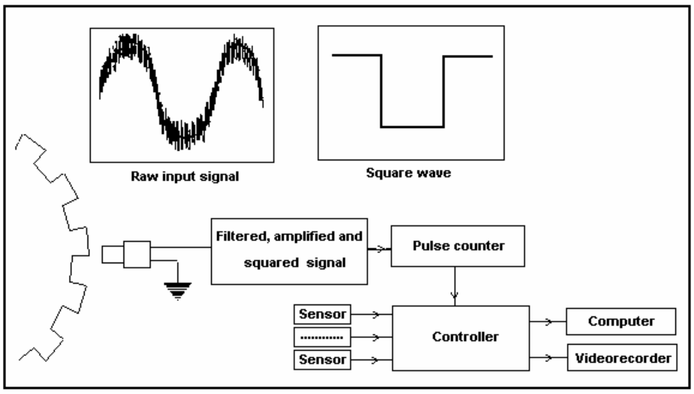

**Figure 4: Scheme of signal elaboration**

If $D_x$ shows the quantity of teeth in the phonic wheel and $N_x$ the quantity of teeth which can be read in a given time $T$, the real value $\omega_X$ would be included in the interval:

$$\frac{2\pi N_X}{TD_X} \leq \omega_X \leq \frac{2\pi(N_X+1)}{TD_X}$$

The estimate of the slip is given by:

$$\hat{\sigma}_A = \frac{N_A D_P}{N_P D_A} K - 1$$

$K$ refers to the rolling radii ratio, which, at that moment, is expected to be steady and exact. If we share this assumption, we will draw the intervals including the real slip value:

$$\hat{\sigma}_A - \frac{N_A D_P K}{(N_P+1) N_P D_A} \leq \sigma_A \leq \hat{\sigma}_A + \frac{D_P K}{N_P D_A} \quad ; \quad \hat{\sigma}_A - \varepsilon_i \leq \sigma_A \leq \hat{\sigma}_A + \varepsilon_s$$

where:

$$\varepsilon_i = \frac{N_A D_P K}{(N_P+1) N_P D_A} \quad ; \quad \varepsilon_s = \frac{D_P K}{N_P D_A} \quad ; \quad \varepsilon_i = \varepsilon_s \frac{N_A}{(N_P+1)}$$

If we consider both the quantity of teeth in the phonic wheel of the test car (*$D_P$=40 e $D_A$=47*) and $N_A > N_P + 1$, it follows that $|\varepsilon_i| > |\varepsilon_s|$. The utmost value of the percentage error in $\sigma_A$ is:

$$\Delta\,\sigma_A = \frac{\max|\sigma_A - \hat{\sigma}_A|}{\sigma_A} = \frac{\varepsilon_i}{\sigma_A} \leq \frac{\varepsilon_i}{\hat{\sigma}_A - \varepsilon_i}$$

getting to:

$$\Delta\,\sigma_A \leq \frac{\hat{\sigma}_A + 1}{N_P \hat{\sigma}_A - 1}$$

The last expression lets us state that, estimate values being equal, the fluctuation in the real value comes down as soon as the quantity of teeth $N_P$ goes up. If $\Delta\sigma_A$= 10%, the lowest number of cogs to be read is given in Tab 4.

**Table 4: Theorical lowest number of cogs in $\Delta\sigma_A$= 10%**

| | | | | | | |
|---|---|---|---|---|---|---|
| $\sigma_A$ | 0.015 | 0.020 | 0.025 | 0.030 | 0.035 | 0.040 |
| $N_P$ | 743 | 560 | 450 | 377 | 324 | 285 |

Consequently we have to expand the time interval when the sampling takes place so as to get the measurement of the angular velocity ratio in case we want it to be affected by a fixed percentage error. Then, we add up the differences of teeth recorded every 20 ms in a fixed number of intervals. Alternatively, by making use of Wald and Bartlett's findings and following a specific procedure, the whole sample can be isolated into several pair sub-groups and the estimate of regression parameters can be drawn by fixing the average of equal number sub-groups.

We still need to assess how deeply the $\sigma_A$ calculation will be affected by the variable ratio of radii $R_A/R_P$. On this point we ought to remember that for a common wheel equipped with radial tyres (external nominal radius $R_e$), the dynamic radius $R_d$ does not differ much from $0.98R_e$. (Genta 2000a). Following the procedure suggested by some researchers (Frank 2000), we would get a reliable estimate of $R_A/R_P$ ratio during driving time when no moment is working on wheels. In any case, the difference between the estimated value of radii ratio and the real one is thought to be rather poor except for occasional dynamic transient.
In order to analyse how slip measurement variance may be owing to the variability of radii ratio, we have:

$$\hat{\sigma}_A = \frac{\hat{\omega}_A}{\hat{\omega}_P}\hat{K} - 1 \quad ; \quad \frac{\omega_A}{\omega_P} = \frac{\hat{\omega}_A}{\hat{\omega}_P}(1+\gamma) = \frac{N_A D_P}{N_P D_A}(1+\gamma) \quad ; \quad K = \hat{K}(1+\alpha)$$

then:

$$\sigma_A = \frac{\omega_A}{\omega_P}K - 1 = \frac{\hat{\omega}_A \hat{K}(1+\gamma)(1+\alpha)}{\hat{\omega}_P} - 1 = (\hat{\sigma}_A + 1)(1+\gamma)(1+\alpha) - 1$$

If we disregard higher order terms, the last expression results in:

$$\sigma_A \cong \hat{\sigma}_A + \gamma - \alpha$$

## Simulated Slip

A simulation research can lead us to evaluate the variability of slip measurement. Owing to the vehicle structure and the tyre pressure (both front and rear tyres), the variability of rolling radii ratio is due to the vertical load which may vary in case of an even road surface and viscoelastic suspension.
The functional relation between the tyre deflection and the vertical load will enable us to assess, thanks to casual but coherent alterations in the wheel load, the variability of the measurement error in the slip.
This is what we know about the wheels:
a) the radial stiffness $J_A$ e $J_P$; is thought to be constant in the two wheels and not far from each other;
b) the vertical loads $Z_A$ e $Z_P$ (coinciding with the static load);
c) the forward movement velocity $V_X$ of vehicle;
d) the real value of slip $\sigma_A$;
e) the ratio between nominal and dynamic radii $K_A$ e $K_P$.
We can draw the average value of rolling dynamics radii and consequently the four wheel angular velocity if we provide the casual value of load moving for a 0,05 ms period.
Considering the specific phonic wheels, we would go back, then, to the simulated value of the slip with regard to a fixed sampling interval. The lower value T* of that interval would cause, at the given velocity, the number of recorded teeth to exceed 400. This is because the simulated slip measurement, in case teeth are fewer than 400, is so variable as the results of the simulation are not meaningful.
The histogram in Figure 5 sums up a simulation. The reason why we have no observations in some areas is due to the fact that only integers are used to estimate angular speed. The simulated slip was expected to be lower than the real one, if we stick to the findings related to the variable errors connected to the angular speed ratio. Besides, simulation findings show that the estimate is not reliable as the sampling interval gets sensibly higher. If we considerably reduce the error related to the measurement of angular speed ratio, we still have a variance of $\sigma_A$ tightly linked to the ineliminable variability of radii ratio.

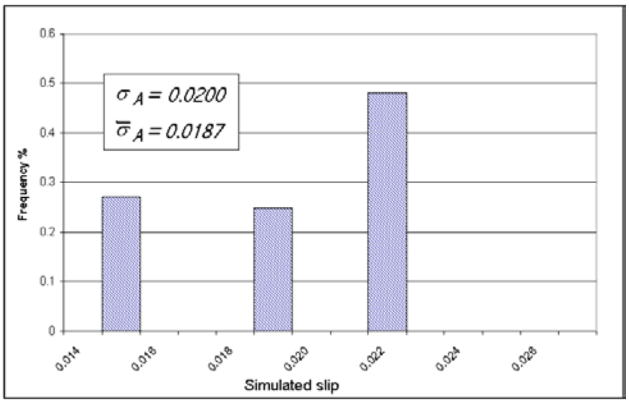


**Figure 5: Histogram of simulated slip.**

If we have a maximum load transfer equal to 20% of the static load as well as a number of read teeth equal to 750 (Tab. 4), slip values and the percentage differences are recorded in the diagram of Figure 6. From this, we can infer that:

a) percentage differences are considerably affected by the variability of radii ratio; this influence comes down as soon as slip value goes up;
b) the simulated average value is always lower than the input value;
c) for $\sigma_A > 0.02$ the simulated average value is about 5% different from the input value;
d) for $\sigma_A > 0.03$ the error in the estimate does not exceed 20%;
e) for $\sigma_A \leq 0.03$ the percentage deviation is so high as to increase the observation time.

In the last case a test is definitely necessary to make sure that real radii ratio is equal to the simulated one. Once we have checked the aim of the assessment, the pseudo-steadiness of the percentage deviation between input and output values, during simulation, should not involve any relevant concequences for the comparisons to be made.

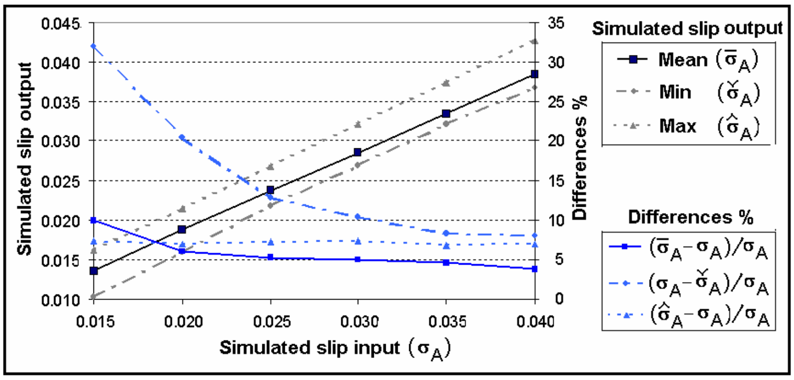


**Figure 6: Slip output vs. slip input**

## Engaged Forces Assessment

For a vehicle simplified model, the longitudinal and vertical forces are outlined in Figure 7.

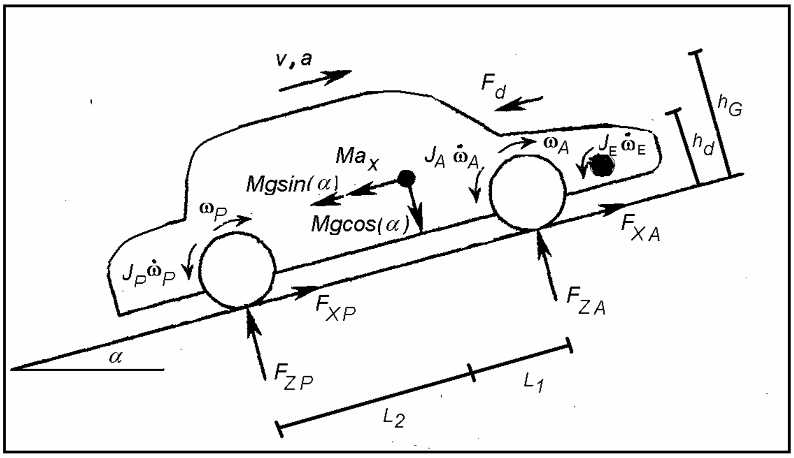


**Figure 7: Acting forces**

Where $M$ is the mass of the vehicle; $F_d$ the aerodynamic resistance, $F_{ZP}$, $F_{ZA}$ the vertical force applied on front and back wheels, $F_{XP}$, $F_{XA}$ the acting longitudinal forces, $a_x$ the vehicle longitudinal acceleration.
If we disregard the force of inertia of rolling masses, as a result of moment balance around point W, the normal force $F_{ZA}$ is given by:

$$F_{ZA} = \frac{MgL_2\cos(\alpha) - Mgh_G sen(\alpha) - F_d h_d - Ma_X h_G}{L_1 + L_2}$$

The only unknown is acceleration $a_x$ whose approximate measure is drawn by:

$$a_X = R_d \dot{\omega}_P \quad ; \qquad \dot{\omega}_P = \frac{\omega_{P,t+1} - \omega_{P,t}}{\Delta t}$$

where $\omega_{P,t}$ and $\omega_{P,t+1}$ are the angular velocities of rear wheels in the instant *t* and *t+1*.
However, the acceleration value $a_x$ can be directly measured as follows:
a) with a simple accelerometer;
b) on the basis of the data drawn by a multi-axis inertial platform (IMU, AHRS);
c) processing the data provided by a GPS system which is precise enough.
However we can leave the term $Ma_xh_G$ out; actually the employment of a static vehicle model produces an error which is not bigger than 3-4% if compared to the dynamic case. This error might be compensated by the pitching of the vehicle body owing to the ineliminable defects of road surface.
The traction force $F_{XA}$ on the driving wheels can be measured with engine torque map. It is known in fact, that a link does exist between the engine power and the rev number. Engine rev number can be easily drawn from the following relation:

$$\omega_E = \frac{\omega_A}{\tau_C \tau_T}$$

where $\tau_C$, $\tau_T$ are respectively the ratio to both gear and differential . Once we know the number of engine revolutions $\omega_E$ and the gear and transmission efficiency, we get the engaged torque. The angular speed $\omega_E$ of drive shaft can be directly detected:
d) by processing the data coming from the Hall effect sensors currently placed on vehicles.
As the kind of signal produced by these sensors does not differ from the above description, it will not be so hard to get them. This procedure will actually involve the processing of data coming from these sensors to check, at the same time, how accurate the angular speed measure in driving wheels is.
$F_{XA}$ forces can be measured:
e) with sensors providing the torque value on wheels.

## CONCLUSIONS

A procedure for estimating tyre-road friction during normal driving of commercial passenger car has been delevoped. This procedure involves the employment of mass-produced sensors as well as a simple device to get and record the measure of wheel angular speed, of drive shaft and transmission shaft.
A reliable and consistent estimate of the so-called slip slope can be drawn by properly choosing the sampling interval with regard to the characteristics of phonic wheels and to the speed the test is carried out. In the later processing phase we decide how long that interval can be. Then, the data we acquire every 20 ms can be assembled so as to get as fewer slip measure errors as possible.
The assessment of slip measure variability has been fully dealt with in this work. On the contrary, the variability of force measures can be fixed by tests only.
Therefore, the employment of more sophisticated devices (GPS, IMU, AHRS...) will be used not only to confirm the described procedure but also to introduce any possible changes to the statistical model suggested. These changes could actually consider the influence of specific parameters (surface temperature, type of tyre and so on) on slip slope.
In this preliminary phase of the work, the methodological approach to assess slip slope will not be seen as an alternative to the existing well-tested procedures. On the contrary it has to be regarded as an instrument for deepening the knowledge of the complex balance between friction demand and road surface capability to meet that demand.

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