Blue-box: a device to rebuild the 3D kinematics and the actions on the drivers along a road path

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SYNOPSIS

A new instrument, named blue-box, has been designed and realised to rebuild the complete 3D kinematics of a rigid body, including linear and angular components of velocity and acceleration.
The device is an electronic unit designed to house the sensing elements, i.e four triaxial MEMS accelerometers (by STMicroelectronics) placed inside the box in an optimised configuration.
The errors produced by inaccurate sensor placements and orientations, or by the non-linearity of single sensing components are corrected by preliminary calibration.
The calibration is composed by two main steps: a static procedure firstly corrects the offsets and gains spreads of each accelerometer placed inside the blue-box, and secondly adjusts the errors caused by imprecise orientation of sensor frames. The next step consists in a dynamic procedure, where the box is mounted on a rotating platform, whose angular velocity and acceleration are feedback controlled. Measuring the sensors outputs during rotation, it is possible to reconstruct the blue-box angular velocities and accelerations by the solution of an over determinate linear system according to the pseudo-inverse matrix solution method. The results of the dynamic calibration procedure permit to correct also the errors related to the inexact sensor frames positions. After complete calibration, the blue-box is ready for outdoor trials to acquire the data required to perform kinematics reconstruction of the rigid body to which it has been fixed.
In spite of its versatility, the blue-box has been specifically designed to compute and acquire the kinematics of road vehicles and, by means of a mathematical model of the human body, the actions transmitted to the car occupants. In particular, it will be used for the calibration of a new car driving simulator, which is under development by our research group; in this case the most important parameters to be evaluated are the linear and angular accelerations which are required to compute the inertia actions acting on the subject.
The present paper deals with the calibration procedure hints and describes the transformations to be applied to sensors raw data to obtain the minimisation of the reconstructed kinematics variables errors.

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INTRODUCTION

The need of our research group to develop a new device to rebuild in real time the 3D vehicle kinematics is connected with two researches concerning the road safety. Both researches involve two Departments of the Politecnico di Milano (D.I.I.A.R. and D.Elettrotecnica) and the C.R.I.S.S. (Inter-university Research Center for Road Safety). A brief description of their main objectives and methods is useful to understand the characteristics of the data that must be produced by the device here presented, called “blue-box”.
The first research concerns the use of the virtual reality in order to generate safety indexes for scoring road paths, either for the existing roads and for those under development. At present a few research centers, such as CRISS (Fig.1a), use this approach [Benedetto, 2002] to verify and validate the road design; recent evolutions of this technique also allow to estimate, on statistical bases, the injury severity indexes for the occupants of the cars virtually involved in the accidents [Camposaragna et al., 2003]. Therefore the system can contribute to the estimation of the social cost related to a road design choice. The precision of the data base built on the results of the virtual tests depends on how much the drivers can feel involved into the virtual reality of the driving simulator. To enhance the driving feeling during the simulation, the Politecnico research group designed a new system of actuators for the car cab (Fig.1b) capable to transmit to the driver the portion of inertia actions that are generally perceived during a real driving session. Due to the fact that this simulator will be replicated in various locations - in order to reach special population of drivers such as the truck drivers or the disables - the system must be as simple and small as possible. A completely new design was necessary because the system requirements are very different from those of the very complex and expensive dynamic simulators used by some automotive groups to test the behavior of vehicle components.
The most important kinematic parameters to be reconstructed by the blue-box for the described application are the components of linear and angular acceleration and those of angular velocity; in fact these are the kinematics parameters related to the inertia actions transmitted to the driver.
The data collected during driving tests on a real car on the road will be compared to the data collected by the blue-box in the simulator cab, when the same road is simulated. This test allows to validate the simulator output and to verify its precision and its limits.

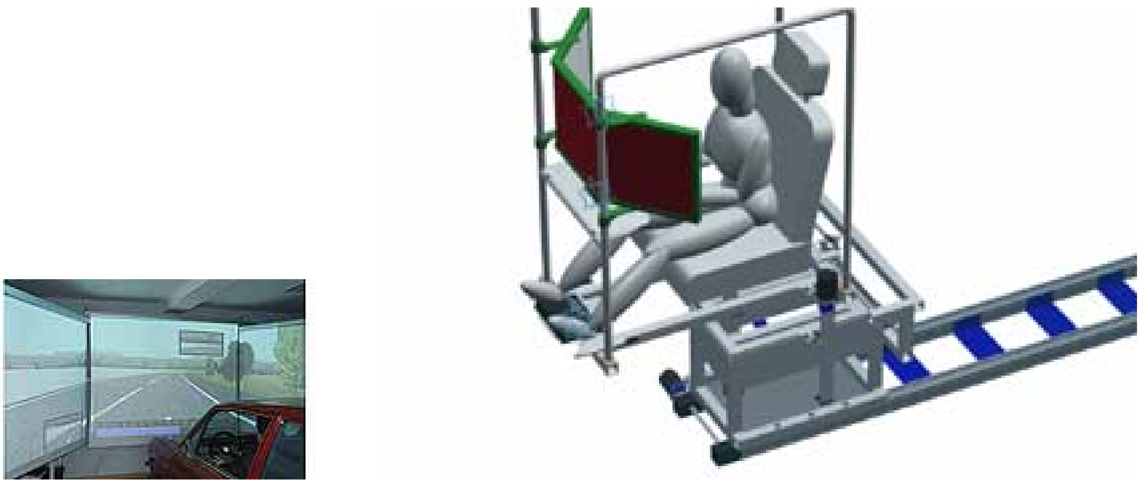

Fig.1: a) CRISS driving simulator at research laboratory of the Roma-tre University
b) virtual prototype of the new actuator for the driving simulator

The second research on road safety under development concerns the active safety: the aim is to collect kinematic parameters continuously and to analyze them in real time detecting dangerous conditions in order to activate alert signals or automatic recovery procedures. The goal is to recognize typical kinematics anomalies of the car trajectory that can be related to driver’s unsafe physical conditions or dangerous road parameters; in other words, the blue-box can be used like an “active” sensor and can inform in real time the driver that the vehicle is near to dangerous conditions. Since in this kind of application the blue-box is constantly active, it may be used also for accident reconstruction purposes. The use of accelerometers produced with the MEMS technology is a key factor for the development of measuring devices with which standard cars may be equipped, because it allows to collect precise data easily and at affordable cost. For the preliminary test on the road the blue-box has been fixed at the frame of an electric city car supplied by the Elettrotecnica Department (Fig. 2).

Fig. 2: The blue-box system set up for the test on the road

MATERIALS AND METHODS

The blue-box has been designed and built in order to measure the 3D angular velocities and accelerations of the rigid body to which it has been fixed.
For this purpose, it features a set of four non-coplanar triaxial linear accelerometers, i.e. twelve acceleration acquisitions, as this is the minimum number of sensors to get a complete kinematics measurement during general roto-translations.
The mathematical basis of this assertion is that the acceleration of any point P of a rigid body in space is:

$$\mathbf{a_P} = \mathbf{a_0} + \dot{\omega} \times (\mathbf{P} - \mathbf{O}) + \omega \times (\omega \times (\mathbf{P} - \mathbf{O}))$$

which linearly depends on the position of P with respect to a point O used as a reference.
Hence, due to the fact that the position of any point in space can be calculated as a linear combination of the known positions of four non-coplanar points:

$$\mathbf{P} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\boldsymbol{C} + \delta\boldsymbol{D}$$

its acceleration is calculated as follows:

$$\ddot{\mathbf{P}} = \alpha\ddot{\mathbf{A}} + \beta\ddot{\mathbf{B}} + \gamma\ddot{\mathbf{C}} + \delta\ddot{\mathbf{D}}$$

which brings to the twelve scalar inputs.
As above said, we decided to combine the twelve mono-axial inputs into four triaxial sensors, based on MEMS technology and capable of measuring the accelerations along three orthogonal axes in a single chip, as indicated in fig. 3. The three axis of the sensor frame form a left frame, while all the following calculations are performed in the right-handed frame convention.

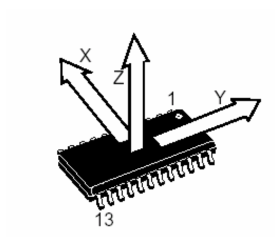

Fig. 3: measuring axes of the MEMS accelerometer (left frame)

The sensors used are the STMicroelectronics LIS3L02AS4, featuring a maximum bandwidth of 4.0 kHz on the two axes x and y and 2.5 kHz on the axis z; moreover they allow a user-selectable full-scale equal to ±2g or ±6g, where g is the gravity acceleration g≈9.81[m/s^2].
Naming S_0, S_1, S_2, S_3 the four no-coplanar triaxial accelerations acquired by each accelerometer:

$$S_i = \lfloor a_{xi}\ a_{yi}\ a_{zi} \rfloor \quad i = 0,...,3$$

and naming P_0, P_1, P_2, P_3 the sensors position vectors:

$$P_0 = [0\ \ 0\ \ 0]^T$$
$$P_1 = [x_1\ \ y_1\ z_1]^T$$
$$P_2 = [x_2\ y_2\ z_2]^T$$
$$P_3 = [x_3\ y_3\ z_3]^T$$

the acceleration $\boldsymbol{a_0}$ of the origin is measured by S_0. Inverting the system which relates the accelerations measured in four points to the unknowns ω and $\dot{\omega}$, it is possible to determine completely the kinematics of the rigid body [Legnani, 2001].

$$\begin{bmatrix} 0 & z_1 & -y_1 & y_1 & z_1 & 0 & 0 & -x_1 & -x_1 \\ -z_1 & 0 & x_1 & x_1 & 0 & z_1 & -y_1 & 0 & -y_1 \\ y_1 & -x_1 & 0 & 0 & x_1 & y_1 & -z_1 & -z_1 & 0 \\ 0 & z_2 & -y_2 & y_2 & z_2 & 0 & 0 & -x_2 & -x_2 \\ -z_2 & 0 & x_2 & x_2 & 0 & z_2 & -y_2 & 0 & -y_2 \\ y_2 & -x_2 & 0 & 0 & x_2 & y_2 & -z_2 & -z_2 & 0 \\ 0 & z_3 & -y_3 & y_3 & z_3 & 0 & 0 & -x_3 & -x_3 \\ -z_3 & 0 & x_3 & x_3 & 0 & z_3 & -y_3 & 0 & -y_3 \\ y_3 & -x_3 & 0 & 0 & x_3 & y_3 & -z_3 & -z_3 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \omega_x\omega_y \\ \omega_x\omega_z \\ \omega_y\omega_z \\ \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \end{bmatrix} = \begin{bmatrix} a_{x1/0} \\ a_{y1/0} \\ a_{z1/0} \\ a_{x2/0} \\ a_{y2/0} \\ a_{z2/0} \\ a_{x3/0} \\ a_{y3/0} \\ a_{z3/0} \end{bmatrix}$$

where the data $a_{xi/0}$, $a_{yi/0}$, $a_{zi/0}$ (i=1,2,3) are the acceleration differences along the *x, y, z* axes of the *i*-th sensor with respect to the origin O.
The coefficient matrix is non-singular if the four accelerometers are non-coplanar, for the demonstration given above.
The optimae location of the four sensors inside the blue-box has been determined on the base of numeric calculus considerations. In fact, it can be proved that the quality of the computed values of an unknown vector X by a linear system A·X=B is dependent on two indicators: the modulus of the determinant of the coefficient matrix *det(A)*, which has to be maximized, or the condition number *cond(A)*, which has to be minimized [Ljung, 1987]. The two indicators have then been calculated for the possible sensor arrangements inside the blue-box, extracting two best solutions: the first maximizing the modulus of the determinant *det(A)*, the second minimizing the condition number *cond(A)*. The two solutions are shown in Fig. 4, and have been both adopted in the present configuration in order to compare them on an experimental basis.

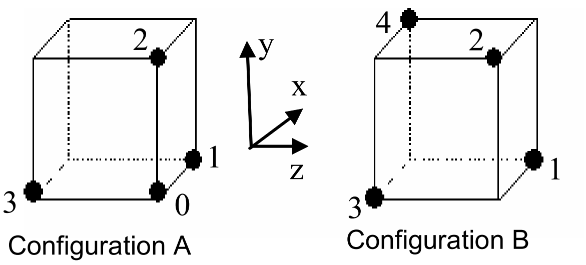

Fig. 4: The two chosen sensors locations

In Fig. 4, the layout called "Configuration A" minimizes the condition number *cond(A)*, which is equal to $\mathrm{cond}(A) = \left(1 + \frac{5}{2}|L|\right) \cdot \max\left(1, \frac{3}{|L|}\right)$, where *L* is the box side, while the second indicator *det(A)* is $16L^9$.

The configuration B maximizes *det(A)*, which results equal to $128L^9$, while the condition number in this arrangement is equal to $\mathrm{cond(A)} = (1+6|\mathrm{L}|)\cdot \max\left(2, \frac{3}{|\mathrm{L}|}\right)$.

The following step in the blue-box design was the definition of its side dimension *L*. This has been done on the base of experimental data on maximum angular accelerations and velocities on a road path.
Acceleration measurements were carried out with a Renault Laguna car on Renault's test tracks, in a road section used for vehicle handling driving experiments [Reymond, 1998]. The maximum linear and angular accelerations components measured on this test vehicle (Fig. 5) can be summarized as follows, where *g* is the gravity acceleration.

- surge: - 0.6 g ÷ 0.4 g
- sway: - 0.7 g ÷ 0.7 g
- heave: - 0.8 g ÷ 1.1 g
- roll: ± 320 °/s² (5,56 rad/s²)
- pitch: ± 360 °/s² (6,28 rad/s²)
- yaw: ± 45 °/s² (0,79 rad/s²)

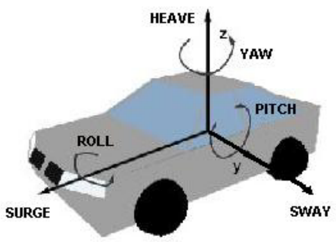

Fig. 5: Car degrees of freedom nomenclature

These average values can be considered a relevant envelope for most of driving situations. In highway driving however, the lateral accelerations limits are generally much lower, typically below *0.1 g*.
The minimum resolution required for the angular acceleration data should be at least an order of magnitude lower than the smallest component to be collected, that here is the yaw angular acceleration; consequently the minimum resolution of the data for this kind of applications is 0.08 [rad/s²].
It is well known that, processing real-world analog data, because of noise superimposed to the desired signal, there is trade-off between the bandwidth of the signal and the maximum resolution that can be achieved. The noise of the implemented accelerometers has the characteristic of being a white Gaussian noise and provides the same contribution to all frequencies with a spectral noise density of 20 [μg/√Hz]. In order to provide the maximum achievable resolution it is important to limit the bandwidth to the maximum frequency required by the application. In this case, the allowed maximum frequency has been set to 750 [Hz]. The resolution and the bandwidth are correlated by the following formula:

$$\varepsilon = 20\left[\frac{\mu g}{\sqrt{Hz}}\right]\cdot\sqrt{750\cdot\frac{\pi}{2}}\left[\sqrt{Hz}\right] = 686\mu g = 0.686mg = 6{,}73\mathrm{e}^{-3}\left[{}^{m}\!/\!{}_{s^2}\right]$$

The sensor resolution is 2ε=1,35e^{-2} [m/s^2]. The required resolution for the angular acceleration is 0.08[rad/s^2].
The angular acceleration, knowing two accelerometers' outputs (a_1; a_2) and their distance (*L*), is:
$\Delta a = a_2 - a_1 = \dot{\omega}\cdot L$
Therefore the minimum relative distance *L* is:

$$L = \frac{\Delta a}{\dot{\omega}} = \frac{0.0135\left[{}^{m}\!/\!{}_{s^2}\right]}{0.0790\left[{}^{rad}\!/\!{}_{s^2}\right]} = 0.17[m]$$

Setting then *L*=0.20 [m], which is the maximum distance between the sensors suitable to a standard car set up, the resolution becomes:

$$\dot{\omega} = \frac{\Delta a}{L} = \frac{0.0135\left[{}^{m}\!/\!{}_{s^2}\right]}{0.200[m]} = 0{,}0675\left[{}^{rad}\!/\!{}_{s^2}\right]$$

which satisfies the minimum requested value for this application.
On the base of the fixed length L, the blue-box sensors position and orientation is reported in Figure 6. The position vectors of the five sensors are:

P_0=[-0.1 0.1 -0.1]
P_1=[0.1 0.1 -0.1]
P_2=[-0.1 -0.1 -0.1]
P_3=[-0.1 0.1 0.1]
P_4=[0.1 -0.1 0.1]

where the configuration A includes the accelerometers placed in the positions P_0, P_1, P_2, P_3, while the configuration B includes P_4, P_1, P_2, P_3 .However it must be pointed out that these are the theoretic position values, because during the system assembling some small misplacements can occur. The effect of these errors have to be corrected by a static and a dynamic calibration proedures as explained below.

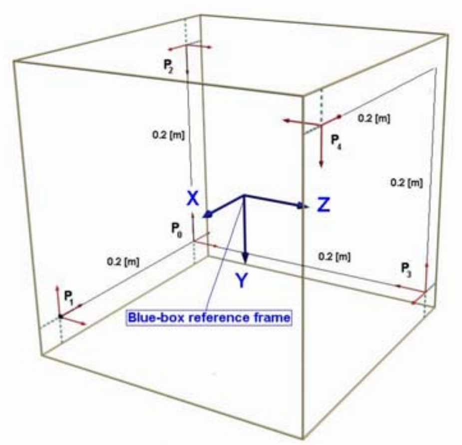

Figure 6 The blue-box sensors' positions

THE BLUE-BOX STATIC CALIBRATION

The blue-box needs to be calibrated in order to minimize the errors in the accelerometers pose and in the electronics.
The most frequent causes of errors are:

- the drift: a constant offset added to measures;
- the gain error;
- the pose error: manufacturing errors in positioning and orienting the accelerometers;

There is another possible cause of error, related to the use of a MEMS triaxial sensor, i.e. the three integrated axes could be not perfectly orthogonal. However, in the present work it has been assumed that this error is at least an order of magnitude lower than the other ones and then it has been ignored.

Offset And Gain Errors

A linear accelerometer supplies an output voltage V which is related to the acceleration a to be measured by the linear equation:
$a = \lambda V + \delta$
where λ is called gain, δ is called offset. Remembering that an accelerometer measures its actual acceleration plus the gravity acceleration, the offset δ and the gain λ can be calculated on the base of the procedure and observations here described. The accelerometer is pointed upward in such a way that it measures the gravity acceleration g, and its output voltage is called V^+. Then the sensor is rotated by 180° in such a way that it measures the reversed gravity acceleration -g, and the measured voltage is called V^-. It is so possible to write two equations:

$$\left.\begin{aligned} \lambda V^+ + \delta = g \\ \lambda V^- + \delta = -g \end{aligned}\right\} \qquad (\alpha)$$

Thus the value of λ and δ can be simply obtained by:

$$\lambda = \frac{2g}{V^+ - V^-} \qquad ; \qquad \delta = 2g\frac{V^+ + V^-}{V^+ - V^-}$$

A triaxial accelerometer is composed by three linear devices, so the calibration procedure described above requires presently an independent calibration of the three channels.
There is another possible procedure, which is the one adopted in the present research. Each triaxial sensor has to be placed at least in six different poses, and the corresponding output voltages have to be collected. Indicating with a_{xi}, a_{yi}, a_{zi} the accelerations measured by the three orthogonal sensor channels in the *i-th* pose, they are constrained by the equation:

$$a_x^{\,2} + a_y^{\,2} + a_z^{\,2} = g^2$$

and so:

$$(\lambda_x V_x + \delta_x)^2 + (\lambda_y V_y + \delta_y)^2 + (\lambda_z V_z + \delta_z)^2 = g^2$$

where the λ_x, λ_y, λ_z are the above said gains for each axis, and δ_x, δ_y, δ_z are the offsets for each axis.
The value of the offsets and of the gains are determined by the minimization of the index:

$$\xi_k = \sum_{i=1}^{n} \left((\lambda_{x,k} \cdot V_{ix,k} + \delta_{x,k})^2 + (\lambda_{y,k} \cdot V_{iy,k} + \delta_{y,k})^2 + (\lambda_{z,k} \cdot V_{iz,k} + \delta_{z,k})^2 - g^2 \right)^2$$

where *n is* the number of poses and *k* is the index that identifies each sensor. To have good results, it has been proven that the minimum number of poses is *n*=6, chosen in a way that the vertical direction is parallel in succession to the *x*, *-x*, *y*, *-y*, *z*, *-z* direction axis. However, it has to be noticed that this procedure is not sensitive to orientation errors, thanks to the fact that it is based on the matching of the gravity vector modulus.
The accelerometers in use must be supplied by a voltage between 0 [V] and 3.333 [V] (V_{DC}), and can measure a range of acceleration between *-2g* and *+2g*.
For zero acceleration it should be measured an output voltage of $V_{DC}/2$=1.665 [V], with an error of ±10% (offset error), as declared by the device builder.
As the theoretical gain declared in the device datasheet is $\lambda=5g/V_{DC}$=14.71 [m/Vs²] with an error of ±10% (gain error) and the theoretical offset is δ=- *2.5 g*= -24,525 [m/s²], the acceleration of *+1g* should generate an output equal to ($V_{DC}/2 + V_{DC}/5$)=2.331 [V], while for *-1g* the output should be ($V_{DC}/2 - V_{DC}/5$)=0.999 [V].
The data have been collected using a Data Acquisition (DAQ) electronic board plugged to a PC, featuring 16 analog 16 bits inputs at a multiplexed frequency of 333 [kSamples/s]. The six measuring steps of a calibration session are reported in Tab. 1, where *k* is the index relative to the accelerometer, and *n* indicates the pose.

Tab. 1 Accelerometers outputs [V] relative to the six test poses used for the static calibration

k=1	n=1	n=2	n=3	n=4	n=5	n=6
Vx	0.9141	2.1979	1.5625	1.5422	1.5558	1.5474
Vy	1.6521	1.6296	2.2915	0.9956	1.6406	1.6486
Vz	1.7069	1.6951	1.7056	1.6955	2.3534	1.0410

k=2	n=1	n=2	n=3	n=4	n=5	n=6
Vx	0.8892	2.1712	1.5324	1.5208	1.5275	1.5234
Vy	1.5805	1.5653	2.2267	0.9316	1.5722	1.5801
Vz	1.6396	1.6292	1.6368	1.6308	2.2866	0.9856

k=3	n=1	n=2	n=3	n=4	n=5	n=6
Vx	2.2087	0.9244	1.5669	1.5637	1.5552	1.5590
Vy	1.6107	1.6107	0.9755	2.2524	1.6146	1.5997
Vz	1.5983	1.5967	1.6000	1.5935	2.2385	0.9676

k=4	n=1	n=2	n=3	n=4	n=5	n=6
Vx	2.1444	0.8491	1.4942	1.4743	1.4870	1.4894
Vy	1.6406	1.6548	2.2995	0.9971	1.6464	1.6567
Vz	1.6165	1.6176	1.6129	1.6161	0.9662	2.2738

k=5	n=1	n=2	n=3	n=4	n=5	n=6
Vx	0.8810	2.1764	1.5209	1.5293	1.5202	1.5292
Vy	1.6021	1.5925	0.9524	2.2468	1.5989	1.5890
Vz	1.6199	1.6147	1.6083	1.6230	0.9753	2.2629

The gains and the offsets of the five triaxial accelerometers have been evaluated by the minimization of the index ξ_k *u*sing the Nelder-Mead Simplex Algorithm implemented in the MATLAB®. The results are reported in Table 2.

Tab.2 Gain (λ) and Offset (δ) for the five triaxial accelerometers. λ are expressed in [m/Vs²], σ in [m/s²]

k	λ_x	λ_y	λ_z	δ_x	δ_y	δ_z
1	15.2639	-15.1218	14.9327	-23.7514	24.8527	-25.3434
2	15.2857	-15.1334	15.0649	-23.3914	23.8970	-24.6477
3	15.2608	-15.3494	15.4184	-23.9068	24.7736	-24.2786
4	15.1307	-15.0436	14.9868	-22.6467	24.7937	-24.2786
5	15.1296	-15.1408	15.2198	-23.1288	24.2182	-24.6428

The parameters λ and δ also reverse the direction of the sensor reference axis *Y* in order to obtain a right-handed frame as shown in Fig. 7.

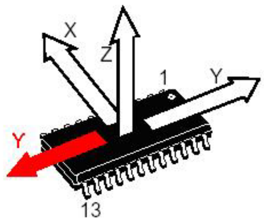

Figure 7 Sensor's new Y reference axis

To check the quality of the calibration, the equations *(α)* are utilized to reconstruct the accelerations a_x, a_y, a_z sensed by the five accelerometers in the six calibration poses, as reported in Table 3. The acceleration modulus is also evaluated as:

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

It has then been verified that after calibration the acceleration modulus is equal to the gravity acceleration g≈9.81[m/s^2].
The maximum acceleration error is 0.3395 [m/s²] (accelerometer=4, pose=4) that means the 3% of *g*. The mean acceleration error is 0.0271 [m/s²] that means the 0.3% of *g*.

Tab. 3 Accelerations in [m/s²]

k=1	n=1	n=2	n=3	n=4	n=5	n=6
ax	-9.7981	9.7977	0.0982	-0.2119	-0.0039	-0.1315
ay	-0.1301	0.2108	-9.7987	9.7977	0.0434	-0.0776
az	0.1459	-0.0307	0.1258	-0.0254	9.7999	-9.7988

k=2	n=1	n=2	n=3	n=4	n=5	n=6
ax	-9.7996	9.7972	0.0321	-0.1457	-0.0433	-0.1055
ay	-0.0212	0.2083	-9.7999	9.7986	0.1040	-0.0147
az	0.0524	-0.1034	0.0102	-0.0794	9.7994	-9.7994

k=3	n=1	n=2	n=3	n=4	n=5	n=6
ax	9.7992	-9.7994	-0.0052	-0.0441	-0.1736	-0.1153
ay	0.0503	0.0497	9.7999	-9.7988	-0.0093	0.2195
az	-0.0736	-0.0980	-0.0467	-0.1477	9.7984	-9.7985

k=4	n=1	n=2	n=3	n=4	n=5	n=6
ax	9.7992	-9.7994	-0.0389	-0.3395	-0.1474	-0.1111
ay	0.1129	-0.1004	-9.7993	9.7939	0.0253	-0.1296
az	-0.0520	-0.0357	-0.1067	-0.0580	-9.7989	9.79887

k=5	n=1	n=2	n=3	n=4	n=5	n=6
ax	-9.7999	9.7992	-0.1179	0.0096	-0.1283	0.0070
ay	-0.0384	0.1071	9.7979	-9.7998	0.0097	0.1591
az	0.0118	-0.0677	-0.1651	0.0593	-9.7992	9.7987

Orientation Errors

Analyzing the results reported in Table 3, it is clear that there is a residual error in sensors orientation, otherwise in each pose we should have obtained an acceleration equal to 9.81[m/s^2] on one acceleration channel, and 0[m/s^2] on the other two channels.

The following procedure allows to minimize the errors related to the imprecise angular positioning of the accelerometers axes relative to the reference axes of the blue-box, unavoidable during the blue-box assembling procedure. The method requires only two static acquisitions for each accelerometer in order to compute the rotational matrix *R (3x3)*, that gives the angular position of each accelerometer axes relative to the blue-box reference frame. The two acquisitions must be carried out by rotating the blue-box around an axis of the reference frame. The advantage of this approach is that it is not required to know the amount of the rotation.

The steps of this procedure are two:

1. to place the blue-box on a plane with a reference axis directed as the gravity vector (i.e. *Z* axis). In this position, the gravity can be detected by each accelerometer and the measured components are stored in the vector $a^{(z)}$. The versor $u^{(z)}=a^{(z)}\ x\ ||a^{(z)}||$ identifies the direction of the gravity acceleration, and of the reference frame *Z* axis, with respect to the accelerometer reference frame;
2. to rotate the blue-box around a reference frame axis normal to *Z* axis (i.e. *X* axis) and carrying out a new acquisition of the gravity acceleration. The gravity vector $a^{(yz)}$ lies on the plane *yz* of the reference frame and the versor $u^{(yz)}=a^{(yz)}/||a^{(yz)}||$ identifies a versor normal to the *X* axis of the reference frame seen in the accelerometer frame. It's important to rotate the blue-box only around the *X* axis, minimizing the rotation around other axes.

By these data, it is possible to calculate the corrected sensor reference frame versors relative to the blue-box reference frame:

- the versor $u^{(z)}$ is the one evaluated in the above point 1.
- the versor $u^{(x)}$, identifying the *X* axis of the reference frame seen in the accelerometer frame, is normal to the versors $u^{(z)}$ and $u^{(yz)}$: $u^{(x)}= (u^{(yz)}\ x\ u^{(yz)})/||\ u^{(yz)}\ x\ u^{(yz)}\ ||$.
- the versor $u^{(y)}$, identifying the *Y* axis of the reference frame in the accelerometer frame, can be computed as follows: $u^{(y)}= (u^{(z)}\ x\ u^{(x)})/||\ u^{(z)}\ x\ u^{(x)}\ ||$.

The rotation matrix *R* of the accelerometer frame relative to the blue-box reference frame can then be constructed by the three vectors as: $R = \left[u^{(x)} \quad u^{(y)} \quad u^{(z)}\right]^T$

The results of this procedure are five rotational matrices *Ri*, one for each sensor, which project the accelerations measured with respect to the sensor frames into the blue-box reference frame, taking into account the potential mounting errors.

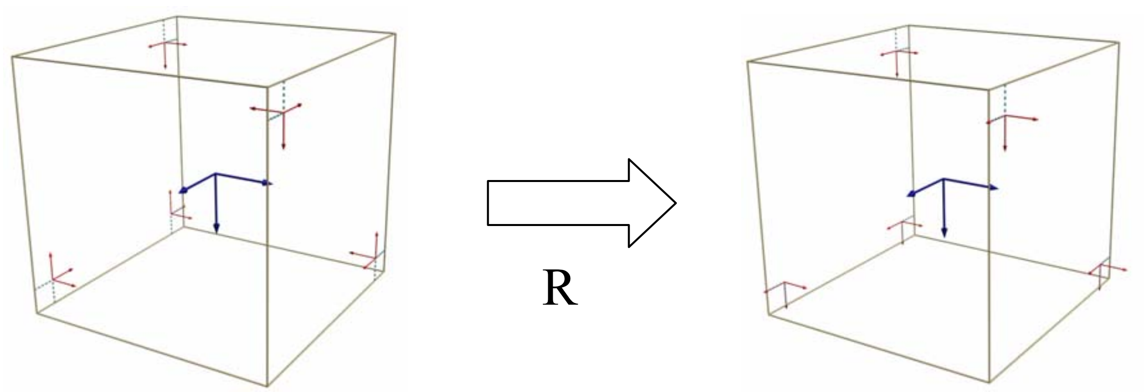

Fig. 8 Final sensor reference frames after static calibration

Knowing the accelerations a_s calculated in the sensors frame, the same accelerations in the blue-box reference frame a_R can be simply computed as follows:

$$a_R = R \cdot a_s$$

The five *R* matrices computed for the tested blue-box are:

$$R_{(0)} = \begin{bmatrix} -0.9993 & 0.0385 & -0.0006 \\ -0.0305 & -0.9992 & 0.0044 \\ -0.0004 & 0.0044 & -1.0000 \end{bmatrix} \qquad R_{(1)} = \begin{bmatrix} -0.9990 & 0.0444 & -0.0049 \\ -0.0444 & -0.9990 & 0.0104 \\ -0.0044 & 0.0106 & 0.9999 \end{bmatrix}$$

$$R_{(2)} = \begin{bmatrix} 0.9994 & 0.0304 & 0.0177 \\ -0.0304 & 0.9995 & 0.0004 \\ -0.0177 & -0.0009 & 0.9998 \end{bmatrix} \qquad R_{(3)} = \begin{bmatrix} 0.9999 & -0.0083 & -0.0151 \\ -0.0084 & -1.0000 & -0.0025 \\ -0.0150 & 0.0026 & -0.9999 \end{bmatrix}$$

$$R_{(4)} = \begin{bmatrix} -0.9987 & -0.0491 & 0.0130 \\ -0.0490 & 0.9988 & 0.0016 \\ -0.0131 & 0.0010 & -0.9999 \end{bmatrix}$$

This result concludes the static calibration of the blue-box. When used to acquire the road data, the raw accelerometers outputs have to be corrected in two steps: first they are adjusted using the parameters λ and δ above calculated; next the resulting data have to be transformed through the matrices *Ri* in order to correct also the orientation errors (Fig. 8).
Another kind of errors, related to inexact positioning of the sensor frame origins, has to be compensated by a dynamic calibration procedure, as explained below.

THE BLUE-BOX DYNAMIC CALIBRATION

A dynamic calibration procedure has been set up with the aim to adjust the errors due to the mislocation of the accelerometers' frame origins relative to the blue-box reference frame, arising during the blue-box assembling. This method requires a precise system to move the blue-box following a preset law of motion. A rotating platform (Fig. 10) actuated by a DC Motor feedback controlled, has been designed and realised for the dynamic tests of the blue-box. During the tests, the motor encoder signals and the five accelerometer outputs are simultaneously acquired by the previously mentioned DAQ board: the encoder signals feed the counter inputs giving motor position, velocity and acceleration, the accelerometers' outputs are collected by the analog inputs.
The relation between the relative position of two accelerometers ($x_{oP}\ y_{oP}\ z_{oP}$), the platform angular velocity ω and acceleration $\dot{\omega}$, and the sensors acceleration a_P and a_0 is described by the following equation:

$$\begin{bmatrix} a_{xOP} \\ a_{yOP} \\ a_{zOP} \end{bmatrix} = \begin{bmatrix} a_{xP} \\ a_{yP} \\ a_{zP} \end{bmatrix} - \begin{bmatrix} a_{xO} \\ a_{yO} \\ a_{zO} \end{bmatrix} = \begin{bmatrix} -\omega_y^2 - \omega_x^2 & \omega_x\omega_y - \dot{\omega}_z & \omega_x\omega_z + \dot{\omega}_y \\ \omega_x\omega_y + \dot{\omega}_z & -\omega_z^2 - \omega_x^2 & \omega_y\omega_z - \dot{\omega}_x \\ \omega_x\omega_z - \dot{\omega}_y & \omega_y\omega_z + \dot{\omega}_x & -\omega_x^2 - \omega_y^2 \end{bmatrix} \begin{bmatrix} x_{OP} \\ y_{OP} \\ z_{OP} \end{bmatrix}$$

which in matrix form can be written as
$A = \omega \cdot X$

The same equation can be written for all the sensors (the index *n* represents the number of sensors) and at different time steps (the index *m* indicates the number of time samples):

$$\begin{bmatrix} [W_{1,0}]_{3x3} & [0]_{3x3} & [0]_{3x3} & [0]_{3x3} \\ [0]_{3x3} & [W_{2,0}]_{3x3} & [0]_{3x3} & [0]_{3x3} \\ [0]_{3x3} & [0]_{3x3} & [W_{3,0}]_{3x3} & [0]_{3x3} \\ [0]_{3x3} & [0]_{3x3} & [0]_{3x3} & [W_{4,0}]_{3x3} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & [W_{i,j}]_{3x3} & \vdots & \vdots \\ \vdots & \vdots & [W_{i+1,j}]_{3x3} & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ [W_{1,m}]_{3x3} & [0]_{3x3} & [0]_{3x3} & [0]_{3x3} \\ [0]_{3x3} & [W_{2,m}]_{3x3} & [0]_{3x3} & [0]_{3x3} \\ [0]_{3x3} & [0]_{3x3} & [W_{3,m}]_{3x3} & [0]_{3x3} \\ [0]_{3x3} & [0]_{3x3} & [0]_{3x3} & [W_{4,m}]_{3x3} \end{bmatrix}_{(3\times n\times m)\times(3\times n)} \cdot \begin{bmatrix} \begin{bmatrix} x_{01} \\ y_{01} \\ z_{01} \end{bmatrix} \\ \begin{bmatrix} x_{02} \\ y_{02} \\ z_{02} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} x_{0n} \\ y_{0n} \\ z_{0n} \end{bmatrix} \end{bmatrix}_{(3\times n)\times 1} = \begin{bmatrix} [a_{01}]^0{}_{3\times n} \\ [a_{02}]^1{}_{3\times n} \\ \vdots \\ [a_{0n}]^m{}_{3\times n} \end{bmatrix}_{(3\times n\times m)\times 1} \quad where: \quad [a_{0i}]^j{}_{3\times n} = \begin{bmatrix} a_{x0i} \\ a_{x0i} \\ a_{x0i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where one of the sensors, named 'O', is assumed as reference, and the distances from the others are estimated with respect to it. So the relative acceleration between the *k-th* accelerometer and the reference one is given by $[a_{Ok}]=a_k-a_O$.
In the equation above, $W_{i,j}$ indicates the blue-box acceleration matrix relative to each sensor frame, *i* identifies the sensor, and *j* identifies the time sample. The *X* vector contains the coordinate of the relative position of each accelerometer frame with respect to the first sensor frame. The vector A contains the relative accelerations of each sensor.
This equation system is linear but over determinate, so it can be solved with a least squares criteria by means of the pseudo inverse matrix method [Legnani, 2003] following the procedure here described.

If $\Lambda \cdot X = b$ is a over determinate linear system, it is impossible to invert the Λ matrix, but it can be found a solution using the pseudo inverse matrix Λ^+ calculated, for example, by the recursive Greville algorithm. The estimated solution is given then by:

$$X^{\wedge} = \Lambda^+ \cdot b$$

and an error vector associated to this solution can be computed:

$$e = \Lambda \cdot X^{\wedge} - b$$

On the base of this method, three cases may occur:

- the system has only one solution, then the error vector is null and *X^* coincide with the solution *X*;
- the system has infinite solutions and *X^* is the one with the minimal Euclidean norm;
- the system has not solution and *X^* is the best approximation using the method of the maximum likelihood principle, i.e. the solution is the one with the minimum Euclidean norm of the vector *e*;

In the present case, the *b* vector contains the measured accelerometer outputs (acquired through the analog inputs of the DAQ board) and Λ contains the blue-box angular velocity and accelerations, obtained through the synchronized acquisition of the encoder outputs equipping the DC motor moving the rotating platform. The linear system solution gives the vector *X^* corresponding to the corrected sensors position vectors which compensate the errors related to the misplacements in the sensor frames.
On the base of the theoretic procedure just explained, let us see the steps necessary to completely calibrate the blue-box.
First of all the sensors' acquired accelerations are corrected by means of λ and δ parameters calculated above (Tab.2). The resulting accelerations are then multiplied by the *Ri* matrices to correct also the orientation errors. The static calibration results are used as starting point of the dynamic calibration procedure where a sinusoidal law of motion has been applied to the blue-box through the DC motor, feed-back controlled. It's important to observe that to have a full-rank well conditioned matrix, it s important to include in the dynamic calibration procedure at least two rotations around two orthogonal axes. This is obtained repeating the motion after repositioning the blue-box on the rotating platform with a different orientation. In the present research, the blue-box has been rotated around the *Z* axis, obtaining the best solution for all sensors in the *z=0* plane. Then the blue-box has been rotated around the *Y* axis to have the best solution for all sensors in the *y=0* plane. The two vectors obtained by the two calibrating rotations are then combined as follows:

$$\hat{\boldsymbol{P}}_{Z\_rotation} = [\boldsymbol{P}_{0x} \quad \boldsymbol{P}_{0y} \quad \boldsymbol{0} \quad \boldsymbol{P}_{1x} \quad \boldsymbol{P}_{1y} \quad \boldsymbol{0} \quad \boldsymbol{P}_{2x} \quad \boldsymbol{P}_{2y} \quad \boldsymbol{0} \quad \boldsymbol{P}_{3x} \quad \boldsymbol{P}_{3y} \quad \boldsymbol{0}]$$

$$\hat{\boldsymbol{P}}_{Y\_rotation} = [\hat{\boldsymbol{P}}_{0x} \quad \boldsymbol{0} \quad \hat{\boldsymbol{P}}_{0z} \quad \hat{\boldsymbol{P}}_{1x} \quad \boldsymbol{0} \quad \hat{\boldsymbol{P}}_{1z} \quad \hat{\boldsymbol{P}}_{2x} \quad \boldsymbol{0} \quad \hat{\boldsymbol{P}}_{2z} \quad \hat{\boldsymbol{P}}_{3x} \quad \boldsymbol{0} \quad \hat{\boldsymbol{P}}_{3z}]$$

$$\boldsymbol{P} = [\boldsymbol{P}_{0x} \quad \boldsymbol{P}_{0y} \quad \hat{\boldsymbol{P}}_{0z} \quad \boldsymbol{P}_{1x} \quad \boldsymbol{P}_{1y} \quad \hat{\boldsymbol{P}}_{1z} \quad \boldsymbol{P}_{2x} \quad \boldsymbol{P}_{2y} \quad \hat{\boldsymbol{P}}_{2z} \quad \boldsymbol{P}_{3x} \quad \boldsymbol{P}_{3y} \quad \hat{\boldsymbol{P}}_{2z}]$$

The position vector *P* gives the final corrected sensor frames positions.
The dynamic calibration procedure has been carried on in both the chosen configurations A and B shown in Figure 4. The corrected sensors' positions for the A configuration are:

P_0=[-0.0924 0.1049 -0.0938]
P_1=[0.1074 0.1040 -0.0935]
P_2=[-0.0996 -0.0981 -0.0950]
P_3=[-0.0950 0.1063 0.1056]

While for the B configuration it has been found:

P_4=[0.1076 -0.0952 0.1062]
P_1=[0.1074 0.1040 -0.0935]
P_2=[-0.0996 -0.0979 -0.0950]
P_3=[-0.0949 0.1063 0.1056]

In Figure 9, it has been reported as an example the behavior of the blue-box angular acceleration for each calibration step. In red the angular acceleration imposed to the blue-box is plotted, in green the blue-box angular acceleration collected without any calibration, in blue the angular acceleration after the static calibration procedure only and finally, in cyan, the blue-box angular acceleration corrected at the end of both static and dynamic procedure.
The graph shows that the dynamic calibration procedure significantly improves the acceleration reconstruction: this result had to be expected considering that the angular acceleration linearly depends on the inexact sensor frame position.

The final results obtained after complete calibration also permit to evaluate the better sensor configuration between A or B solutions of figure 4. The statistical parameters extracted from the difference between the acceleration reconstruction and the acceleration imposed to the blue-box are reported in Tables 4 and 5.

Tab. 4 Statistical parameters for configuration A

	Mean error (rad/s^2)	Standard deviation (rad/s^2)	Peak value (rad/s^2)
Blue-box angular accelerations with no calibration	1.7112	0.5262	2.9969
Blue-box angular accelerations after static calibration	0.3608	0.3423	1.3828
Blue-box angular accelerations after static and dynamic calibration	0.1102	0.2660	0.8568

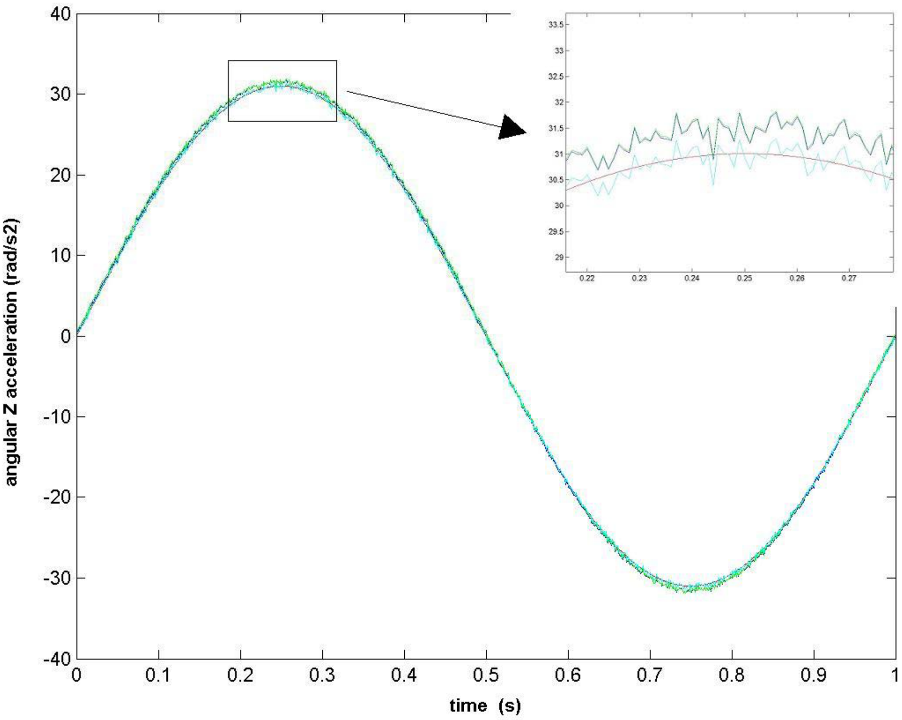

Figure 9 The blue-box angular acceleration pre and post calibration

Tab. 5 Statistical parameters for configuration B

	Mean error (rad/s^2)	Standard deviation (rad/s^2)	Peak value (rad/s^2)
Blue-box angular accelerations with no calibration	0.2022	0.4341	1.0798
Blue-box angular accelerations after static calibration	0.0576	0.4137	1.0089
Blue-box angular accelerations after static and dynamic calibration	0.0027	0.1906	0.5990

The comparison between the results reported in Tabs. 4 and 5 shows the superiority of the B configuration with respect to A. In fact the configuration B has lower mean error, standard deviation and peak error, thus it has been chosen as the final configuration for the acceleration acquisitions in road paths.

Fig. 10: Dynamic calibration set up: blue-box mounted on a rotating platform close loop controlled

CONCLUSIONS

A new low-cost device called “blue-box” to acquire the complete 3d kinematics of a rigid body has been designed produced and tested. The measure of the motion is obtained by four triaxial MEMS accelerometers. A calibration procedure has been designed and implemented to correct the raw data obtained by the sensor outputs from the errors in the position and in the orientation in the sensors placement, and also in the offsets and gains values proper of each electronic sensor, connected to production spread.
Different sensor configurations have been analyzed with the aim to select the one which minimizes the measuring errors.
The final mean error is equal to 0.0027 (rad/s^2) as reported in Tab. 5: considering that the blue-box has been dimensioned in order to obtain an angular acceleration resolution equal to 0.0675 (rad/s^2), the residual mean error on this resolution after calibration is equal to 4%.
The present measuring device allows to acquire the kinematic variables and process them during the car motion. In this kind of tests the blue-box is fixed to the car chassis (as shown in Fig. 2) and it is connected to the DAQ board for sensor outputs acquisition. The DAQ board is connected to a notebook for data storage, and all the system is supplied by the car battery. The first version of the blue-box prototype is mainly designed to collect the acceleration parameters required to compute the inertia actions exerted on the driver of a driving simulator and those exerted on a car driver. For future applications mainly focused on displacement analyses other components, like magnetometers and “gyroscopes”, will be added integrated in the device to minimize also the position errors.

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