A New Simulation Model For The Management Of Unstable Traffic Flow

MATERNINI G.
Professor of Tranportation
Department of Civil Engineering
University of Brescia – Italy
COLOMBO R.M.
Professor of Mathematics
Department of Mathematics
University of Brescia - Italy

ABSTRACT

Unstable traffic flows are caracterized by strong air pollution emissions induced by vehicles and high safety risk before traffic congestion. The most part of existing traffic flow models simulate at a good level stable traffic flows, but there are still big difficulties in simulating unstable traffic flows.
The aim of the paper is to present a new model which is particularly effective in simulating unstable flows situations.
This model, recently appeared in literature, is compared with the classical Lighthill-Whitham and Richards (LWR) model, first in some idealised situations and, secondly, with the help of a set of real traffic data.
The model consists of a systems of partial differential equations. In [D] some models in this class were heavily criticised due to major deficiencies in some of their qualitative properties. At the same time, several good features of the classical LWR model were underlined.
Consequently the implementation of this model in highways could allow to manage in real time the quantity of traffic flows, through variable message signs, influencing vehicle speed (eliminating the “stop and go” effects) and improving road safety.

INTRODUCTION

During periods of stable traffic, with the well-known forms of interaction between flow (number of vehicles per unit of time), vehicle speed and density (number of vehicles per unit of length), the main circulation characteristics are predictable and it is consequently easy to adapt traffic signs and infrastructures in order to improve the efficiency and safety of vehicle circulation.
Apparently, during periods of unstable traffic there is no law that governs the volume of traffic in circulation. This consequently makes it difficult to use the simulation models found in current literature to predict traffic volumes, and it is during this phase that the most critical circulation problems occur, with the subsequent risk of accidents amongst the various road users.
This study presents a mathematical model that simulates the unstable traffic flow when vehicle speed and flow are reduced and the density increases. This model is able to simulate the main traffic volumes, making it possible to implement suitable traffic signs in real-time, reducing the risk of road traffic accidents.
In a suburban environment, the ability to model unstable traffic can make it possible to prevent congestion, or rather the paralysis of the traffic flow, thereby optimizing infrastructure management.
In urban areas, the ability to monitor periods of unstable traffic (which cover several hours a day in the majority of cases) makes it possible to indicate alternative routes to drivers through the implementation of variable message road signs, which are still rarely and insufficiently used, thereby avoiding or noticeably reducing congestion. It is also possible to calculate the optimal driving speed for each route which, if adopted by drivers, would considerably reduce exhaust fume emissions, cutting down on atmospheric pollution and at the same time improving safety conditions.

A 2x2 HYPERBOLIC MODEL FOR VEHICULAR TRAFFIC

Since half a century the Lighthill-Whitham [LW] and Richards [R] model is the starting point for the modeling of vehicular flows. In these years, several other models were introduced, studied and exploited. However, the recent work by Daganzo [D] heavily criticized many of the newer models, underlining their various deficiencies with respect to the LWR model. Now, no new model can be introduced without an accurate comparison with the LWR model.

As a reaction to [D], some pre-existent models were modified and others were developed. In particular, [C1] introduces a new 2x2 model based on a system of 2 hyperbolic conservation laws aiming at the description of flows at high densities.

Here, we present this model first through a comparison with the LWR model in some ideal experiments, secondly testing its attitude to describe real phenomena using experimental data.

As a first step, consider a flat rectilinear one way road, with neither entries nor exits, with only one lane. More realistic situations will be considered below. Let ρ denote the vehicles density (with $\rho \in [0, R]$, *R* being the maximal possible traffic density) and *v* the (mean) traffic speed. The simplicity of the LWR model stems from its being based on these two assumptions:

1. the total amount of vehicles is conserved, and
2. the traffic speed is a function of the traffic density: *v*=*v*(ρ).

As a consequence, the LWR model reads

$$\partial_t \rho + \partial_x [\rho \cdot v(\rho)] = s(\rho)$$

while the presence of entries or exits lead to introduce a suitable right hand side, leading to

$$\partial_t \rho + \partial_x [\rho \cdot v(\rho)] = s(\rho)$$

and possible lack of homogeneity along the road are described through the dependence from *x* and/or *t* of the various terms, that is

$$\partial_t \rho + \partial_x [\rho \cdot v(\rho; t, x)] = s(\rho; t, x) .$$

From the analytical point of view, the latter modifications lead to merely technical difficulties, often considered only at the moment of the final specific implementation.

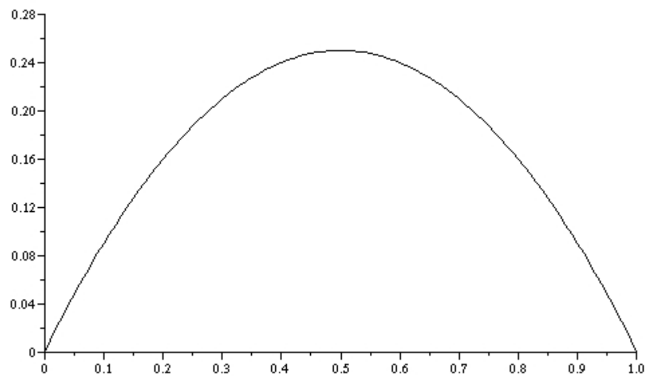

Figure 1

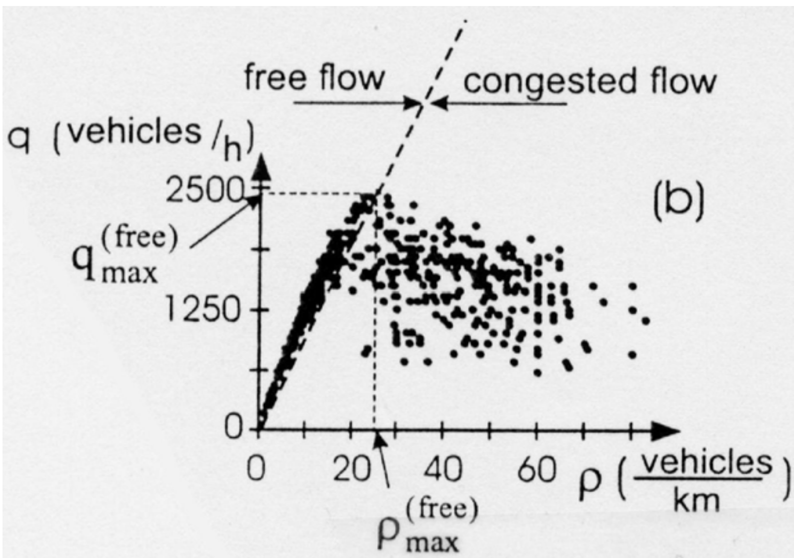

Figure 2

Conceptually, it is more interesting to investigate the functional relation *v*=*v*(ρ). Indeed, a typical *"theoretical"* fundamental diagram is in Figure 1. Minor *variations* on this latter diagram, like the introduction of an inflection point on the left part, do not alter our observations below. In fact, an experimental *"fundamental* *diagram",* taken from [K], is in Figure 2. Behaviour similar to this one are usual and were noticed from the authors also in data coming from Italian highways. It is clear that these data contradict the very existence of the functional dependence $v=v(\rho)$ at high density regimes. In other words, in the region labelled as *"congested"* in Figure 2, density and flow (or speed) need to be considered as independent variables. It follows that 2 equations are necessary. Indeed, the model introduced in [C1] is

$$\partial_t \rho + \partial_x [\rho \cdot v(\rho, q)] = 0$$

$$\partial_t q + \partial_x [(q - Q) v(\rho, q)] = 0$$

where Q is a fixed parameter, characteristic of the *dynamics* of the road, strictly related to the phenomena of *wide jams*, see [K] for a full description of wide jams and [C1]-[C2] for their relations with Q. Above, q is the *weighted flow* and it is related to traffic density and traffic speed through the relation

$$v = \left(1 - \frac{\rho}{R}\right) \frac{q}{\rho}.$$

This equation serves as a sort of *equation of state* in closing the 2x2 model. The fundamental diagram, usually the curve relating density to flow, is replaced by a two dimensional region: of the three variables ρ, v and q, two are independent and the third one is a function of the previous two.

In [AW] and [D], the authors state several requirements that need to be fulfilled by a traffic flow model. Among them, we recall the following properties enjoyed by the present model:

1. No information travels faster than vehicles or, equivalently, information is carried by vehicles;
2. If density and speed are initially non negative and bounded, then they both remain non negative and uniformly bounded for all times;
3. Vehicles stop whenever the maximal density is reached, and only at this density.

We defer to [C1] and [C2] for the analytical proofs of these statements.

As in the LWR case, the effects of entries and exits can be added through suitable terms on the right hand sides. Moreover, the presence of the second equation for the weighted flow allows to describe further phenomena where the total number of vehicles is conserved, while other factors affect speed. Possible examples are: ascents, descents or stretches with low visibility. In these cases, a suitable term is added on the right hand side of the second equation and the descriptions obtained are out of the scope of the LWR.

We end this short description of the model with the remark that [C2] is devoted to show that it allows also a good description of *phase transitions*, as introduced by Kerner, see [K] and the reference therein.

COMPARISON WITH THE LWR MODEL

The comparison is organised as follows. First, we equip the LWR model with the usual speed law

$$v = V \cdot \left(1 - \frac{\rho}{R}\right).$$

As it is usual, V is the maximal possible speed. For the sake of simplicity, we also let

$$R = 1\frac{vehicles}{length}, \quad Q = 1\frac{vehicles}{time} \quad \text{and} \quad V = 1\frac{length}{time}.$$

These choices does not diminish the generality of our observations below.

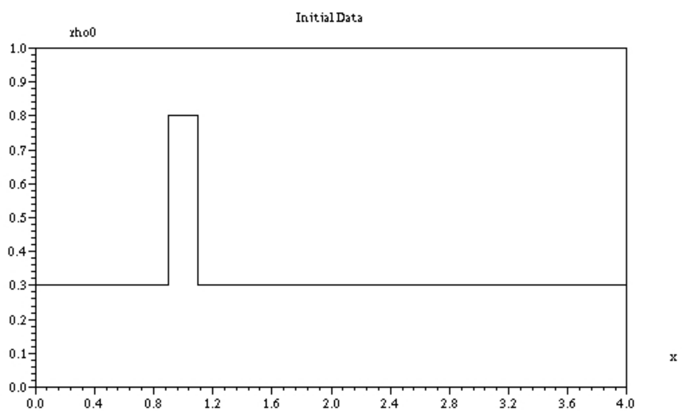

Figure 3

We choose as the initial density distribution the *square wave* in Figure 3. Corresponding to this initial data, the LWR model gives at time t=0.4 the density shown in Figure 4. Note that the evolution predicted by the LWR is typical of scalar conservation laws with a concave flux. Indeed, the downward jump on the right in the initial data becomes smooth (Lipschitz) as soon as t>0 (it is a centered rarefaction wave). At the same time, the upward jump in the initial density is not smoothed: it is a shock and translates along the road with vehicles crossing it from left to right.

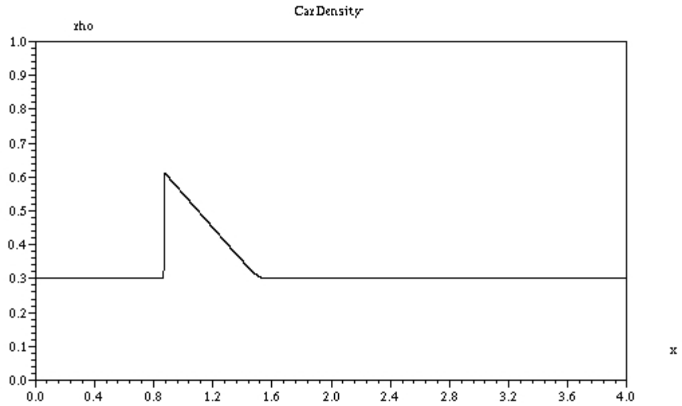

Figure 4

Below, this result is compared with three different numerical integrations of the 2x2 model In all of them, the initial density is the same square wave shown in Figure 3, left., but the other initial data is chosen according to three different criteria.

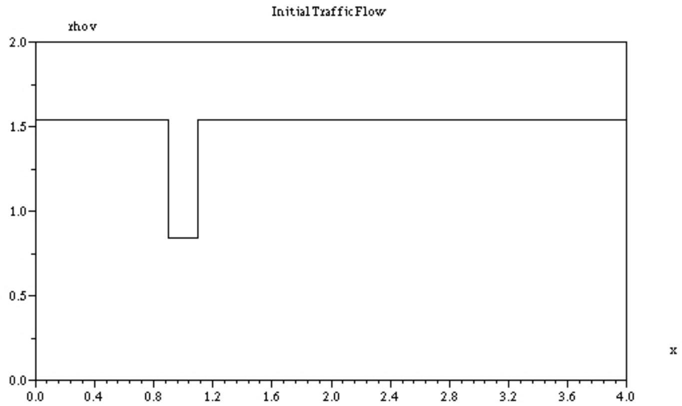

Figure 5

The first comparison aims to show that the above behaviour of the LWR model falls within the scope of the present model. Indeed, assign initial data along a 1 Lax curve or, in other words, assign the initial traffic flow distribution as in Figure 5, always keeping as initial traffic the one in Figure 3. Note that, as in the LWR case, the steepness of the downward jump decreases while the upward jump is a stable shock. As a consequence, the density at time t=0.4 is distributed in a way qualitatively similar to that obtained through the LWR model. This example shows that the 2x2 model *extends* the LWR model.

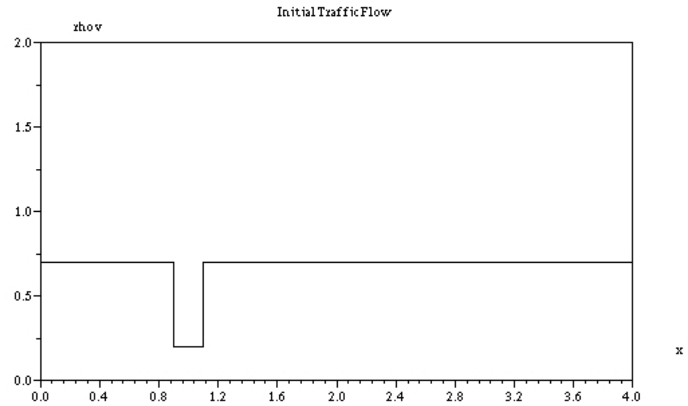

Figure 6

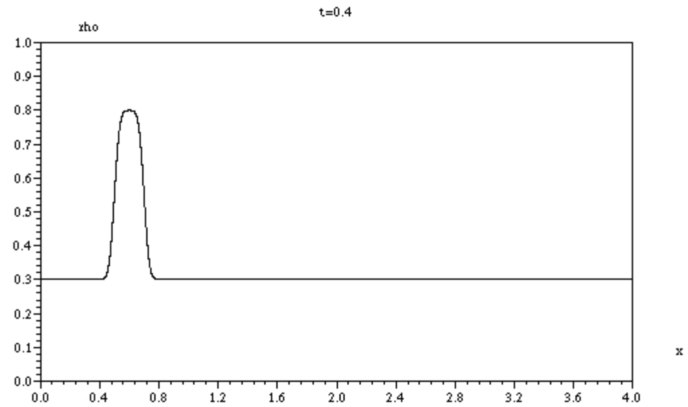

Figure 7

Secondly, always maintaining the same initial density distribution in Figure 3, we assign now an initial weighted momentum constantly equal to *Q*. The corresponding initial traffic flow distribution is in Figure 6. This choice leads to a wide jam, a persistent wave in a sort of dynamical equilibrium with respect to the surrounding vehicles, see Figure 7 (the partial smoothing of the square wave is due to the unavoidable numerical viscosity). Indeed, the square wave in the density moves backward, while vehicles enter it from the left and exit to the right. Remark that the present model, due to its analytical structure, allows both upward and downward persistent waves in the density moving both forward or backward. We underline that this example shows that persistent behaviours can be described by the present model. On the contrary, scalar conservation laws with a strictly concave flux may not describe such phenomena, due to the well known decay of negative waves.

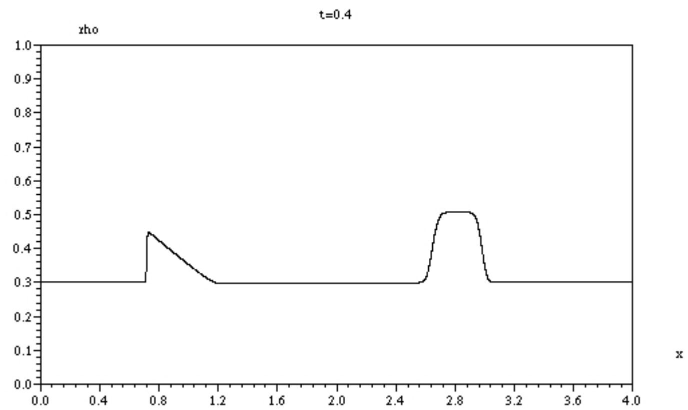

Figure 8

Third, always with the same initial density distribution in Figure 3, the initial traffic flow is chosen constant, see Figure 7. A sort of superposition of the previous behaviours is obtained, see Figure 8. Here, the initial square wave leads to waves that interact, leading first to a rather complex dynamics, then to the formation of two easily identifiable waves. Indeed, the asymptotic configuration of the solution consists of a first wave similar to the LWR one moving backward, and a second one, similar to the wide jam case, propagating at the speed of the main traffic. As above, the partial smoothing of the wave on the right is due to the unavoidable numerical viscosity. This example is meant as a glance to the variety of behaviours that the present model can describe.

In general, any initial data for the present model can be approximated through the juxtaposition of initial data of the types considered above. Therefore, a generic solution consists of a sort of non linear superposition of the behaviours considered above.

EXPERIMENTAL VERIFICATION THROUGH AN APPLICATION ON A MOTORWAY IN THE VENICE AREA

We examine now the behaviour of the present model with respect to the real situation of the Venice Freeway. This road was chosen first thanks to the availability of good traffic data and, secondly, due to the typical high traffic load usually present. The freeway segment considered is sketched in Figure 9. The data received is very detailed and rather accurate. Vehicles are classified in three different classes according to their length. Then, for each lane, the number and speed of vehicles in each class is measured. On the basis of these measures, various other quantities are available. It is remarkable that these data are available also along entries and exits, so that the 2x2 model above could be tested in the case of non zero right hand sides.

Due to the geometry of the considered segment of the Venice freeway we neglected the lack of homogeneity. The present simulations were carried out with all the parameters and functions independent from the space variable. The introduction of such dependence may clearly and easily improve the agreement between data and model, but at the expenses of several rather arbitrary choices.

The numerical algorithm adopted is the classical Lax Friedrichs method, see [L, §12.1]. We stress here that this numerical method is conservative. Indeed, it provides approximate solutions that satisfy the balance of vehicles with an extremely high accuracy, usually higher than that provided by experimental data. Numerical analysis provides several more recent and efficient methods to numerically integrate systems of conservation laws as the one here considered. The Lax Friedrichs algorithm was chosen for its simplicity, however it turned out to be sufficient for the scopes of the present analysis.

The model was provided with the initial and boundary data. The former is the density measured at the initial time of the integration. The latter consists of the flows measured at the beginning and at the end of the considered segment. At each integration step, the measured inflows at entries and the outflows at exits were uniformly distributed along the entry or exit.

Aiming at the description of the traffic evolution, we first introduced a right hand side only in the first equation. To compute it, we simply distributed the measured inflow (for entries) or outflow (for exits) along the due width. The choice of this source term is classical and, in the case of the LWR model, also found in textbooks, see [H, §83]. However, the resulting comparison between measured data and numerical simulation turned out rather unsatisfactory.

A dramatic improvement was obtained through the introduction of a source term also in the second equation. With reference to the above comparison between the 2x2 and the LWR model, recall that such a second source term can not be inserted in the latter model. More precisely, the role of this second right hand is to describe the influence of the entry/exit on the global traffic flow of the freeway near the entry/exit. Even a qualitative evaluation of this influence is not straightforward, for it is subject to contradictory effects. As an example, consider the case of an entry. On one side, the increase in the vehicles density leads to an increase of the flow, if the speed were constant. On the other hand, the vehicles entering the freeway are often slower than those already in it and their arrival may well cause also the others to slow down. Similar observations hold also in the case of an exit.

An accurate data analysis, *a priori*, and the numerical experiments, *a posteriori*, suggest the following reasonable description. Near to entries and exits, with a remarkable similarity between the two cases, drivers seem to adjust their speed to the speed limit (w=80 km/h) more carefully than in other points along the freeway. Therefore, we added the following term that models a kind of "*relaxation*" towards the speed w.

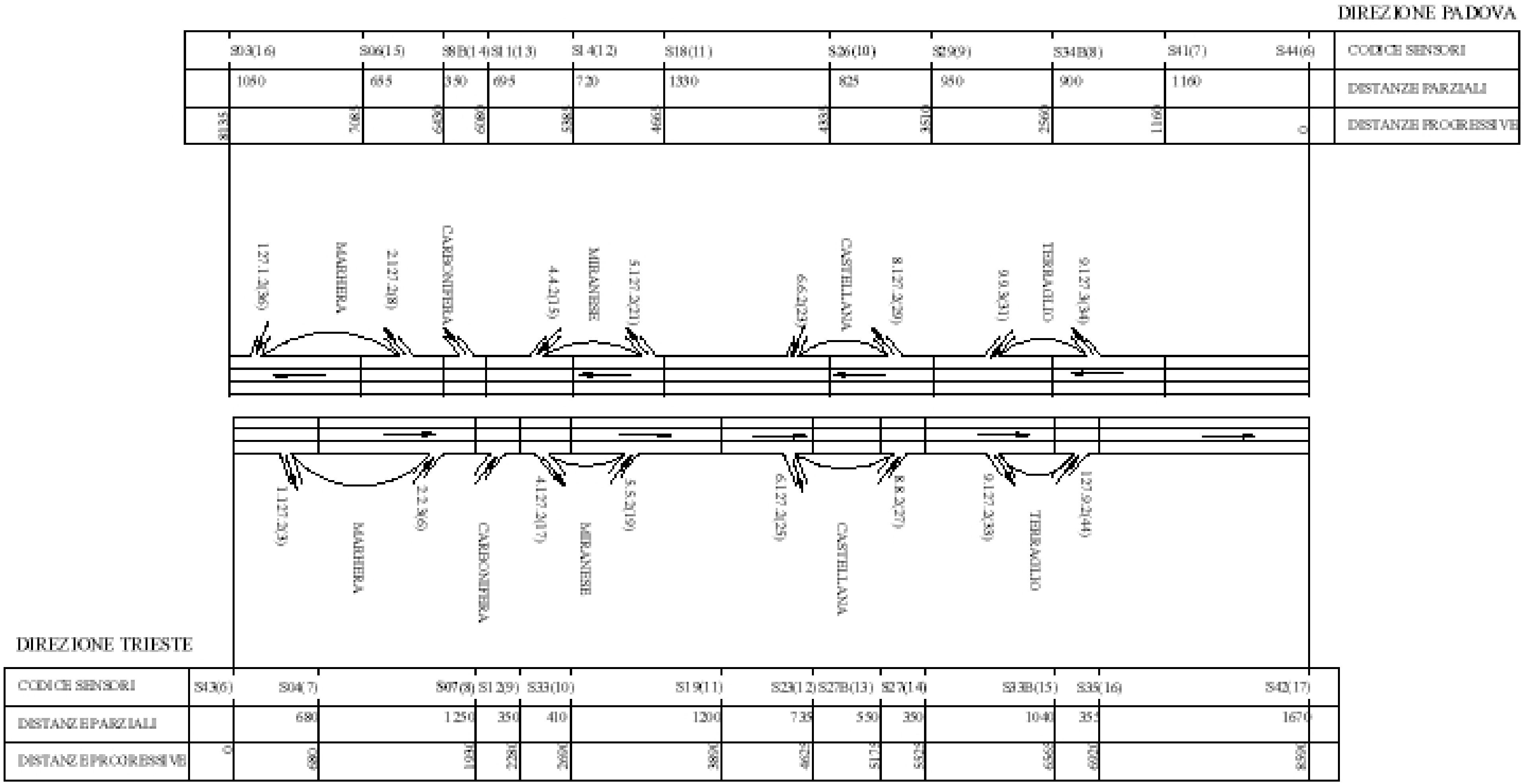
DIREZIONE PADOVA
CODICE SENSORI
DISTANZE PARZIALI
DISTANZE PROGRESSIVE
TERRAGLIO
CASTELLANA
MIRANESE
CARBONIFERA
MARHERA
DIREZIONE TRIESTE

Figure 9

$$s = -\frac{1}{\tau}\left[q - \left(\frac{1}{\rho} - \frac{1}{R}\right)^{-1} \cdot w\right]$$

Above, τ has the dimension of a time. It represents the readiness with which drivers tend to adjust their speed to the speed limit w, remark that it is *not* related to the reaction time. We set it to 0.1 sec in the simulations below.

The results of two comparisons are summarized in the figures below.

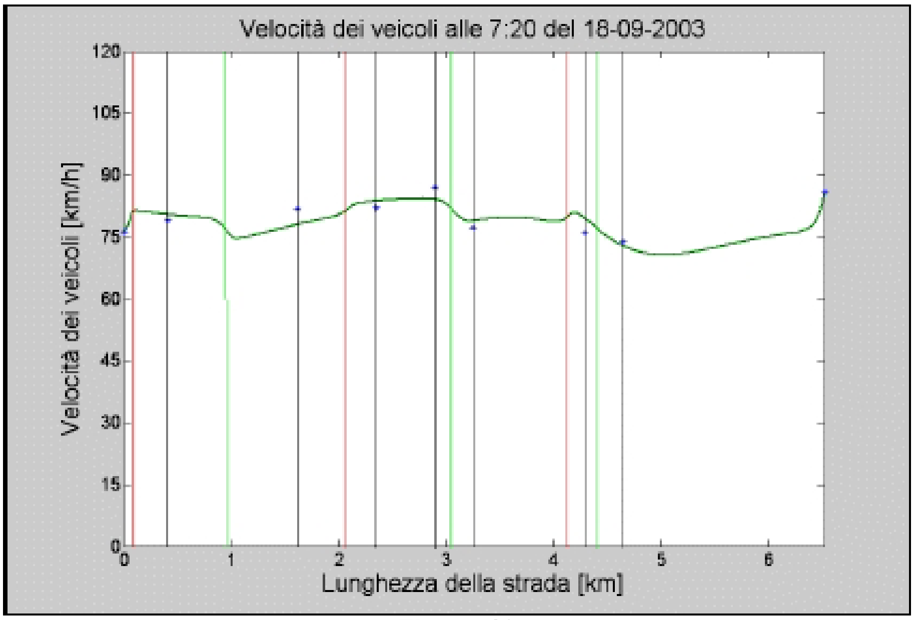

Figure 10

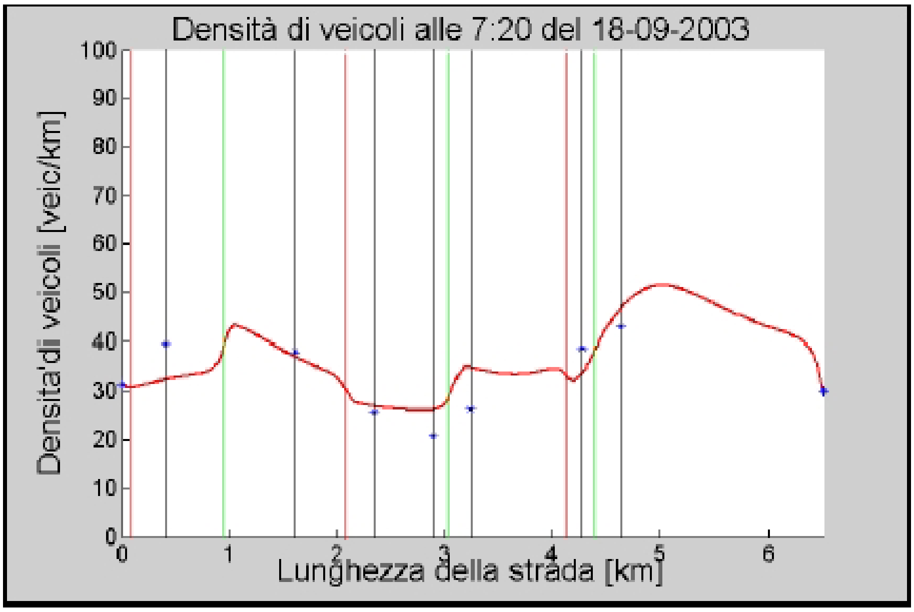

Figure 11

In Figures 10 and 11 we consider the flow measured on September 18, 2003 between 7:15 and 7:20. We have the space variable on the horizontal axis and vehicles' density (Figure 10) or speed (Figure 11) on the vertical one. The vertical straight lines represent entries (when green), exits (red) and the location of loop detector on the freeway (black). On these latter lines, the stars denote the measured data. The continuous graph is what the model forecasts. The numerical integration catches the behaviour of the experimental data.

A further experiment is shown in Figures 12 and 13. Here, a longer time is chosen, namely from 7:15 to 7:45. The notation is the same of the previous situation. Note that here measures speed at 7:45 varies more than in the previous case, but the model keeps catching the behaviour of the measured data.

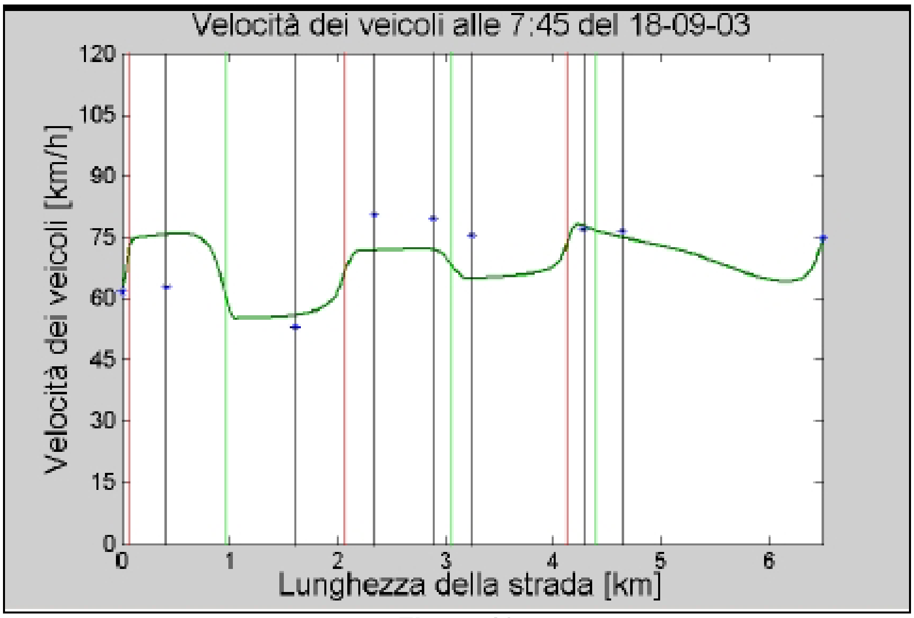

Figure 12

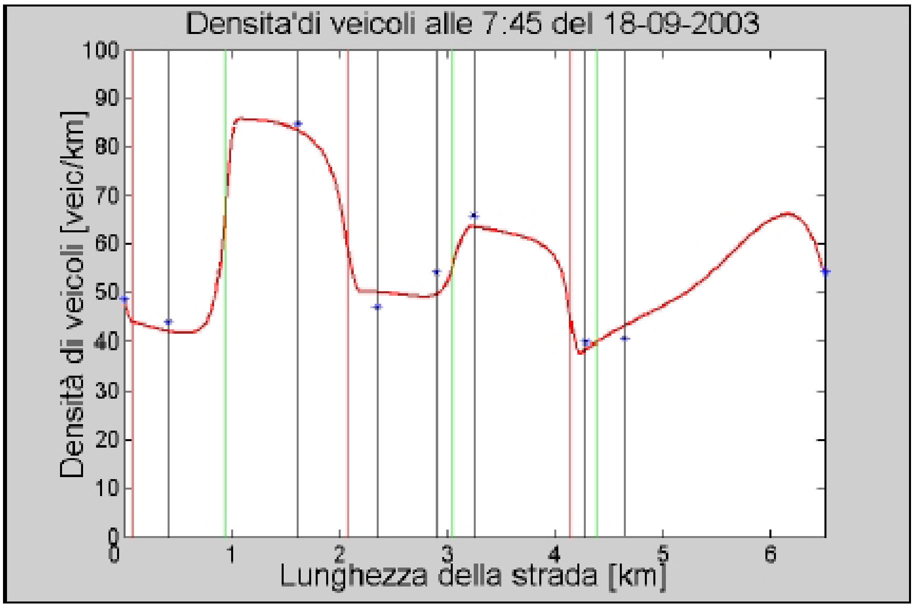

Figure 13

All the details of the various constants and parameters are in [P]. Here we only add that these integrations were effected without the usual optimizations that, at different levels, may dramatically reduce the time of computing. In spite of this, the full integration needed less than an hour on a single 2GHz Pentium 4. The use of a better integration method, the optimization of the code, the use of a compiled language and a more powerful computer may well change the order of magnitude of the computation time.

IMPLEMENTATION OF THE HYPERBOLIC MODEL FOR SAFETY PURPOSES

In a suburban environment

When applied to a motorway-type dual carriageway infrastructure, the hyperbolic model described in the previous paragraphs makes it possible to simulate traffic volumes during unstable periods, before congestion occurs. It is well known that unstable traffic flows generate a considerably higher risk of road traffic accidents. We therefore need to be able to simulate this phase correctly, so as to be able to prevent any negative consequences.

The experiment described in the previous paragraph highlights how the critical nature of traffic flows on the Venice freeway is a serious problem, causing a high number of accidents (one of the highest percentages on the entire Italian motorway network). In order to prevent motorway congestion, the resulting interruption of the traffic flow and the highly dangerous “stop and go” phenomenon, it could be possible to reduce the number of vehicles accessing the motorway by using the barriers at the motorway tollgates. After receiving real-time information on the number of vehicles in circulation from the traffic monitoring devices in service on that section of motorway, the hyperbolic model is able to calculate the maximum flow that could enter the motorway without giving rise to congestion. The flow passing through the tollgates could be regulated by working on the barrier opening times.

Even in the case of an accident, the detection of the disturbance to the vehicle flow would enable the proposed model to simulate the critical nature of the traffic or congestion in real-time. It could provide drivers with this information through variable message signs, indicating alternative routes in serious cases or, in less serious cases, the optimal speed that drivers should adopt so as not to increase vehicular flow problems.

During application of this hyperbolic model to the Venice freeway, it was found that this model is able to simulate the period of unstable traffic correctly. Given the consistently high volumes of traffic and congestion throughout several hours of the day, it allows the hard shoulder to be used as another carriageway on the section of road in question during the most critical points of the day. The proposed model makes it possible to predict when and for how long it is necessary to use the hard shoulder as an extra carriageway.

The same considerations we have applied to motorways are also valid for the main suburban dual carriageways, although it is impossible to regulate vehicular entry as there are no barriers. The optimal driving speed information given to drivers on the variable message signs, which should be located near the main entry points, remains the same.

On secondary suburban roads, it is important that the speed adopted by drivers is suitable for the volume of traffic flow for safety purposes. When there are very few vehicles in circulation, the vehicle speed is generally adequate, unless there are structural or infrastructural deficiencies on the road. However, in the case of intense traffic, drivers adopt speeds that are not suitable for the critical circulation situation. In these cases it is particularly important to inform vehicle drivers of the most suitable driving speed for the current road conditions.

The speed limit, which is lower than the maximum limit applied to the road and imposed by permanent signs, derives from the geometry and situation of the traffic during rush hours. This generally means that drivers, during quieter periods, do not observe the set speed limit because they do not understand it and therefore they adopt a speed on the basis of their experience and their judgement of the traffic situation, as was highlighted in the previous paragraph on the application of the model on the Venice freeway. Sometimes temporary situations (such as road works) require slower speeds to be adopted over fairly long stretches or at intersections, but even after the situation that has led to this reduction is no longer valid, the same speed limits remain in place (either due to negligence or because the organization responsible for the road believes that it is better for them), with the result that drivers are encouraged not to observe the signs. It would be best for the speed limits to vary on the basis of the actual road traffic situation, so that drivers are encouraged to observe them. Variable message signs, therefore, are very useful because they indicate the ideal driving speed, as calculated in real-time by the proposed model. Drivers would notice that the set speed limit is dictated by the actual situation on the section of road that they are driving along. Making vehicles adopt the optimal driving speed depending on the traffic situation is extremely effective for safety purposes.

In an urban environment

Routes into and through urban areas are often subject to unstable, congested traffic flows for many hours of the day. Application of the hyperbolic model makes it possible to manage these routes as far as possible, correctly simulating the traffic flows during the most critical periods. A variable message sign could be placed at the start of each main road, indicating the optimal driving speed to be adopted by drivers, as calculated by the model, or alternative routes in order to avoid congestion.

It is definitely better for vehicles to drive at the same speed and not create a “stop and go” situation”, since it reduces exhaust fumes emissions and, consequently, atmospheric pollution.

Another important factor regards the placement of indicators that monitor the accident risk on a given stretch of urban road, in the presence of unstable traffic and before the accidents take place. This type of indicator is a preventive safety measure. It is well known how risk indicators generally report the average accident risk for the average traffic flow on that road. This can be useful if the road is used by a limited number of vehicles, while it fails to provide reliable information if there is a considerable amount of traffic and an unstable traffic flow. Because of this, it could be possible to link the volume of traffic during the unstable phase, as calculated by the hyperbolic model, to the road’s main geometric parameters, thereby providing a risk indicator *a priori*, without having to know the number of accidents that have taken place. In order to achieve this, a possible approach could involve calculating how the proposed indicator varies with the variation in traffic, on a stretch of road for which the traffic data, the number of accidents and their locations is known. Once the scale of values indicating the degree of risk has been obtained through this experiment, one could, for comparison, extend it to roads for which there is no accident data. This would lead to the creation of a preventive safety indicator suitable for detecting the risk whenever the traffic is unstable, thereby varying throughout the day and making it more meaningful. The simulated information would make it possible to intervene preventively on an administrative level, with more suitable speed limits for the actual road situation and suitable infrastructure operations.

In an urban environment, the hyperbolic model could be used to correctly simulate concentrated traffic flows

generated by poles of attraction, at certain times (for example, the flow of vehicles from the stadium after a football match). These large flows of vehicles generate periods of unstable traffic or congestion on the roads around the magnetic pole. Being able to simulate these situations correctly would make it possible to manage the road routes in question, providing the drivers with adequate information through variable message signs.
Therefore, the application of the hyperbolic model in an urban environment would make it possible to simulate periods of traffic instability in advance and inform the drivers in real-time through the use of variable message signs. Encouraging vehicles to drive at a suitable speed for the actual road traffic situation reduces accident levels and leads to lower exhaust fumes emissions.

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