A bilevel programming model to optimising the modal distribution of charge in urban environments with congestion: the case of the new port of Laredo.

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Synopsis

In this paper a bilevel programming model is presented for the resolution of the modal distribution in the charge provisioning of construction works in urban environments. The model is applied to the construction of the new port of Laredo where the modal distribution is optimized between trucks and barges, for different periods.

The used methodology is based on the minimization of the total cost of the system that is composed by the cost of the operators of trucks, cost of the operators of barges and the cost of the drivers. The drivers will be increased their level of costs as consequence of the rising congestion induced by the flow of trucks circulating on the network.

Therefore, we solved a problem of bilevel optimization, in which the upper-level corresponds to the total cost of the system, conformed by the three agents: operators of trucks, operators of barges and drivers, and in the lower-level it is considered a users equilibrium model (cars) that responds to the first principle of Wardrop (transformed of Beckman). The problem consists on determining the optimized frequency of trucks and of barges, in such a way that the total costs of the system are minimized.

Additionally, environmental constrains are considered, as for the example the maximum level of flow in the links of the net, assuring that the flow of more trucks the flow of automobiles in the links is not superior to the critical value of flow, in reference to emission of pollutants. This way, 4 atmospheric pollutants and the levels of noise are analyzed.

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The growing volume of transport is contributing to a bigger pressure on the environment. The measures adopted at the present time to mitigate this tendency, in the best of the cases, only reduce lightly the acceleration of the rate of growth.

As positive aspect, to highlight that the technological progresses are allowing to reduce, in spite of the growing traffic volumes, the levels of atmospheric contamination generated by road transport.

However, it is needed more to solve the problem of the atmospheric contamination and acoustics in the urban environment, both increased by the constant growth of the congestion in the traffic of the cities. In this scenario is where the environmental negative impacts generated by the transport are concentrate mostly.

In this environment, the transport of travellers and the transport of goods in urban environments, besides the daily problems that suffer the cities, it is important to study a particular case: the problem that generates the transport of provisioning of materials to big infrastructural works that are developed inside the cities.

This event type, in spite of its temporary character, affects to the environmental of the city in two ways. The first one, contributing to the growth of the traffic congestion for the automobile users, increasing the derived problems, so much of atmospheric contamination as acoustics, besides the consequent increment of the cost for the user by a bigger consumption of time in the realization of their daily trips. The other form is due to the own effect of contamination and noise generated by the heavy vehicles of supply to the work.

This last type of environmental problems generated by the transport in the city constitutes the spine of this study. The objective will be to optimize the system of transport of provisioning of a great infrastructural work in urban environment (new port of Laredo), based on the minimization of total costs of the system, allowing to get a sustainable activity from the social, economic and half environmental perspective.

Is important to emphasize that this model is framed for the urban environment. This comes motivated because, like is known, is in the city where more and in more grade the important problems generated by the transport are presented: traffic congestion, atmospheric contamination, noise, etc.. Besides the above-mentioned is necessary not forget that is in the cities where the whole human activity is concentrate and therefore where more urgent is to act in the reduction of the previous mentioned impacts.

For big infrastructural works we understand each other those that hopelessly will cause considerable impacts on the transport and the mobility of the area where these works are located. It is therefore a relative term with two face concepts: volume of supply materials required by the work in front of transport possibilities.

In definitive, the model proposed will give answer to three basic variables to organize a system of materials provisioning to the infrastructural work that is able to minimize the total system cost.

The first considered variable is the mode or the transport operator. Will be defined which modes will intervene in the provisioning charge system to the infrastructural work. This characterization can be particularized per period, for example can contemplate the possibility that a marine transport mode (barge) alone is feasible of using in summery time.

Later will value the routes, in those modes in which their vehicles circulate on the streets of the city (terrestrial modes), the frequencies of this transport modes are the most important variables to achieve the main objective of minimization of total system costs. Also is possible to particularize this information and define routes per period, for example can happen that doesn't interest to use a route determined in summery time by circulate next to a beach and if to use in winter.

Lastly, will define the program of frequencies for each mode or operator and for modelling period considered. That is to say, the model will give as main output a complete calendar, for the total of duration of the work. In this program the optimal frequencies for the modes and routes previously preset are determined in base of minimization of total system cost.

In section 1 a brief introduction to bilevel optimization problem is presented. In section 2, the main operational characteristics of the system and the general methodology and variables used in the model are defined. In section 3 the production function is formulated. Section 4 defines the cost function. Section 5, specifies the bilevel programming model. The application and the main conclusions are presented in section 6.

INTRODUCTION TO BILEVEL PROGRAMMING

The bilevel programming constitutes one of the most important areas in the global optimization. The programs of bilevel optimization (or programming of two levels) present specific properties, some related with their high grade of no-convexity and non non differentiation. This motivates that resolution is particularly difficult and a challenged of considerable interest. They are countless problems of practical application that take advantage of their own structure hierarchical to outline and solve formulations through bilevel programming.

Is possible to define the bilevel programming like "a mathematical program that contains a problem of optimization in the restrictions". For the perfect understanding, is necessary to focus simultaneously from two points of view: on one hand, as logical extension of the mathematical programming, and for other, as generalization of a problem peculiar of the theory of games (Game of Stackelberg).

In the Stackelberg’s equilibrium a player special denominated leader, that can know the reactions from the rest of players to his strategy, exists. The rest of players are denominated *followers*. The leader can choose his strategy inside a certain group, independently of the strategies of his followers, while each follower can choose a strategy inside a group of them parametrically for the election made by the leader. The strategy of a follower depends on the leader's strategy, and his utility also depends so much of the strategies of the other followers, like of the leader.

Many problems of transport planning and urban transport networks design are formulated through a problem of Stackelberg’s equilibrium, because their hierarchical structure is adapted to reflect the process of takings of decisions. The system operators (leader) plan or design the transport system keeping in mind the behavior of the users (followers) before their decisions about administration policy or investment. In the superior level the costs (social, economic, environmental, etc.), derived of the operators policy are minimized , while in the inferior level the behavior of the users is described in the transport system intervened.

The mathematical formulation of Stackelberg's equilibrium games is known as mathematical programs with equilibrium restrictions (MPEC). A mathematical program with restrictions of equilibrium is a optimization model in which certain group of restrictions is defined by means of an inequality variation.

In definitive, the general structure of a MPEC in a transport planning problem is:

- Superior level. It defines the objective of the transport system planner.
- Inferior level. It represents the behaviour of the users in the transport network by means of an equilibrium assignment problem.

A problem of bilevel programming (BLP), in their more general form, present the following formulation:

$$\min_{x,y} \quad F(x,y) \tag{1}$$

$$s.a. \quad x \in X \tag{2}$$

$$g(x,y) \leq 0 \tag{3}$$

$$y \in \arg\min\{f(x,y) : y \in Y, h(x,y) \leq 0\} \tag{4}$$

Where X and Y are subset of $\Re^n$ y $\Re^m$ respectively, F y f are real functions, such that $F, f : \Re^{n+m} \to \Re$, g and h are real vectorial functions, such that $g : \Re^{n+m} \to \Re^{p1}$ and $h : \Re^{n+m} \to \Re^{p2}$ with n and $m \in \mathrm{N}$ and p_1 and $p_2 \in \mathrm{N} \cup \{0\}$.

The optimization problem for the first level of the BLP problem consists on minimizing in x and y the function $F(x, y)$ with subject restrictions (2), (3) y (4). The optimization problem for the inferior level consists on the minimization in y, parametrically by x, and it is denominated as $P(x)$ problem of second level in y:

$$\begin{array}{ll} \min_{y \in Y} & f(x, y) \\ s.a. & h(x, y) \leq 0 \end{array} \tag{5}$$

This way, x is designated by a variables vector of the first level and y by a variables vector of the second level. In a same way, $g(x, y) \leq 0$ and $h(x, y) \leq 0$ represent the restrictions of the first (upper) and the second (lower) level. The function $F(x, y)$ is denominated objective function of the first level, as for $f(x, y)$ is designated as objective function of the second level.

One class of bilevel optimization problems more frequent in the literature is conform by the convex bilevel problems optimization.

A binivel optimization problem is said convex (BLPC) when $X = \Re^n$, $Y = \Re^m$ and when they are convex in y the functions $f(x,\cdot)$ y $h(x,\cdot)$ for all x in $\Re^m$.

The main advantage of working with BLPC is that with an appropriate qualification of the restrictions, the lower level can be replaced by Karush-Kuhn-Tucker conditions (KKT), to obtain an equivalent mathematical problem of a single level. On this last aspect it is recommended to consult Bazaraa and Shetty (1979).

The more common particular cases of the BLPC problem that exists in the current literature, are:

- BLLP - lineal programs of two levels in which all the implied functions are lineal.
- BLLQP - lineal-quadratic programs of two levels in the one which the functions F, g and h are lineal and the objective function of the second level f is quadratic and strictly convex in y.
- BLQP - quadratic programs of two levels in which the objective function of the first level is also quadratic (convex or not convex)

Are several the applications described in the literature that have been modeled through the bilevel programming. Among the applications more frequently, can be studied

Application to networks design, where this type of models is characterized to use in the inferior level the traffic assignment model formulated by means of the TAP. Concerning this type of lineal bilevel programming exist applications as in Ben-Ayed et all (1992), applications of networks design keeping in mind congestion effects on the network like in Marcotte (1986), diverse algorithms and heuristic implementations as in Marcotte (1988) and Marcotte and Marquis (1992) and no-lineal binivel programming as in Suh and Kim (1992).

Another type of habitual application is the problem of estimate the demand, like in Florian and Chem (1991) where a bilevel programming is presented for estimate matrix O-D with traffic counts in some links. These models use data of traffic volumes, conforming a more economic information, in opposition of the expensive domicile survey.

The problem of space localization is another frequent application of the bilevel programming. In Miller, Friesz and Tobin (1992) heuristic algorithms are presented for localization problems.

The difficulty of these models is fundamentally its application to reality, due to big dimensions problems, of the order of several dozens of thousan. Also due to bad mathematical properties of the models, like non-differentiation and the non-convexity. This has made that the methodology developed to solve these problems is heuristic.

DEFINITION OF THE SYSTEM AND GENERAL METHODOLOGY

To solve the modal distribution in the charge provisioning to the development of the infrastructural works of the new port of Laredo we should define three variables of the system: transports modes have to use, routes in the modes that circulate on the network of the city and the calendar of frequencies of the vehicles, for the total duration of the work.

The outlined model allows determining the vector of optimal frequencies that minimize the total cost of the system, therefore, the routes of operators of transport that circulate for the network and the considered modes are previous data to the execution of the model.

Also, the demand of transport is fixed. This demand is the group of materials to transport that requires the development of the work. Habitually through the construction projects, to have the chronogram of development of the works. Therefore, it is possible to define an initial calendar of frequencies of vehicles based on the information of the project. Given the flexibility of the model is possible to determine the frequencies as the work is executed and to make modifications on the initial calendar.

We have considered so many modelling periods like periods of different demand level (certain with the chronogram of works) in combination with the different defined periods of traffic. In the case of Laredo, 3 periods of traffic have been determined: a period of winter, a period of summer and peak hour and a period of summer in off peak hour.

The considered modes of transport, to find a solution to the problem of the optimal modal distribution, have been: trucks and barges. In this case, the trucks contribute flow to the network and therefore we should define the routes for where they will circulate. In the case of Laredo, and after an analysis of different possible routes, there are two possible routes for the trucks.

Given this scenario, the outlined model is based on the minimization of the total cost of the system, made up of the cost of truck operators, barge operators and drivers, whose level of costs will increase as a result of higher congestion produced by truck traffic.

In order to determine the optimal level of charge distribution between trucks and barges, the cost structure of both operators and the one of the users of the system are considered simultaneously. The optimal distribution will be that which minimizes the total cost of the system defined by the three agents described (trucks, barges and drivers).

The methodological approach deals with solving a bi-level mathematical programming problem. At the upper level, a structure of total costs of the system, made up of the three relevant agents (truck operators, barge operators and drivers) is defined. Drivers are relevant here because their costs would increase as a result of the congestion produced by trucks. In the event of there being no congestion, the analysis would be carried out straight from the costs of both operators (trucks and barges), as drivers would not be affected. Moreover, a series of constrains are taken into account at the upper level, preventing the combined volume of cars and trucks not exceeding the limits of pollution emissions (PM_{10}, CO, NOx and SOx) and noise pollution.

On the other hand, an optimization problem is considered at the lower level, providing as a result a car assignment according to Wardrop’s first principle. However, truck traffic will affect the Wardrop’s equilibrium of the drivers implied in the congestion.

In order to define the upper level’s objective function, it is necessary to determine analytic expressions for the costs of the three agents involved: truck operators, barge operators and drivers.

As mentioned above, the total cost of the system is defined as the total of the operation cost of trucks, barges and drivers

$$C_{truck} = \sum_{t} c_c \cdot \left(T_c^{\,t} \cdot f_c^{\,t}\right) \cdot F_t \qquad (6)$$

$$C_{barges} = \sum_t c_b \cdot \left(T_b^{\,t} \cdot f_b^{\,t}\right) \cdot F_t \tag{7}$$

$$C_{drivers} = \sum_t \sum_a \left(c_a\left(f_{a,v}^{\,t}, f_{a,c}^{\,t}\right) \cdot f_{a,v}^{\,t}\right) \cdot F_t \tag{8}$$

where:

c_c : average unit cost of operating a truck (€/hour).

$f_c^{\,t}$: frequency of trucks during the modelling period *t* (trucks/hour).

T_c^t : travel time of the truck, including return, plus times for loading and unloading (hours).

c_b : average unit cost of operating a cargo barge (€/hour).

f_b^t : frequency of barges during the modelling period *t* (cargo carriers/hour).

T_b^t : travel time of the barge, including return, plus times for loading and unloading (hours).

F_t : weighting factor representing the duration of each modelling period in the total time of the work completion.

$c_a\left(f_{a,v}^{\,t}, f_{a,c}^{\,t}\right)$: total cost of operation in the link *a* during the modelling period *t* (€/hour).

$f_{a,v}^{\,t}$: flow of vehicles in the link during the modelling period *t* (vehicles/hour).

$f_{a,c}^{\,t}$: flow of trucks in the link during the modelling period *t* (lorries/hour).

Note that $f_{a,c}^{\,t}$ equals $f_c^{\,t}$.

PRODUCTION FUNCTION

In order to transport a total charge of Q during a specific period, it is feasible to transport Q^t in a sub-period *t*, so that:

$$Q = \sum_t Q^t \tag{9}$$

Charge Q^t can be transported by truck or barge, and therefore:

$$Q^t = q_c^t + q_b^t \tag{10}$$

In order to transport q_c^t units during a *t* period with trucks with an average capacity of K_c , we will need the following frequency of trucks $f_c^{\,t}$:

$$q_c^t = f_c^{\,t} \cdot K_c \tag{11}$$

Analogously, in order to transport q_b^t units during the *t* period with barges with an average capacity of K_b , we will need the following frequency f_b^t :

$$q_b^t = f_b^{\,t} \cdot K_b \tag{12}$$

Then, by substituting, from (11) and (12), in (10), we obtain:

$$Q^t = f_c^{\,t} \cdot K_c + f_b^{\,t} \cdot K_b \tag{13}$$

Thus, we clearly observe the perfect substitution condition between trucks and barges. And, depending on the costs of each mode of transport and the impact that would be generated on drivers, how much to transport for each option should be determined.

In (13), the demand of every period Q^t is constant. On the other hand, we suppose that the trucks and barges will go loaded until their capacity (K_c, K_b). Therefore the variables are the frequencies, of truck and barge.

COST FUNCTION

For this case practical and once analyzed application the possible access routes by truck, two routes have been determined (see Figure 1). A first route by the more direct access road to the port but with high level of congestion. This has motivated define a second route, a little longer in distance and arrive to the port for the nearest streets to the beach.

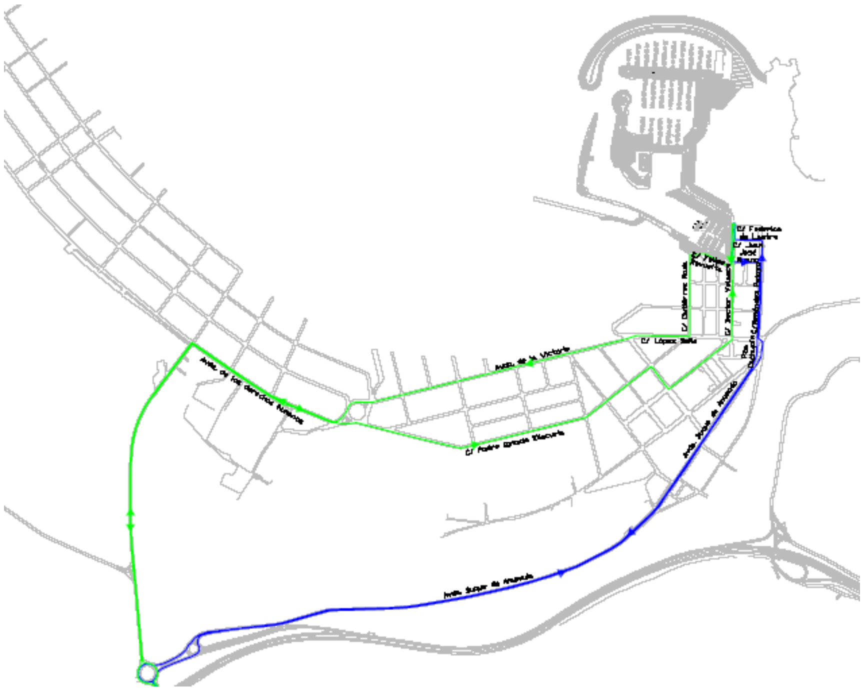

Figure 1. Routes for trucks.

Since two possible circulation routes are valued for the trucks, the definitive function of costs of the system can be defined as follows:

$$C_{Total} = \sum_t c_c \cdot \left(T_{c1}{}^t \cdot f_{c1}^t + T_{c2}{}^t \cdot f_{c2}^t\right) \cdot F_t + \sum_t c_b \cdot \left(T_b{}^t \cdot f_b^t\right) \cdot F_t + \sum_t \sum_a \left(c_a^t \left(f_{a,v}^t, f_{a,c1}^t, f_{a,c2}^t\right) \cdot f_{a,v}^t\right) \cdot F_t \tag{14}$$

In the previous function of costs it is possible to substitute the production function, this way we have:

$$C_{Total} = \sum_t c_c \cdot \left(T_{c1}{}^t \cdot f_{c1}^t + T_{c2}{}^t \cdot f_{c2}^t\right) \cdot F_t + \sum_t c_b \cdot \left(T_b{}^t \cdot \left(\frac{Q^t - K_c \cdot (f_{c1}^t + f_{c2}^t)}{K_b}\right)\right) \cdot F_t + \sum_t \sum_a \left(c_a^t \left(f_{a,v}^t, f_{a,c1}^t, f_{a,c2}^t\right) \cdot f_{a,v}^t\right) \cdot F_t \tag{15}$$

where:

c_c : average unit cost of operating a truck (€/hour).

f_{c1}^{t} : frequency of trucks, circulating in route 1, during the modelling period *t* (trucks/hour).

f_{c2}^{t} : frequency of trucks, circulating in route 2, during the modelling period *t* (trucks/hour).

T_{c1}^{t} : travel time of the truck, circulating in route 1, including return, plus times for loading and unloading (hours).

T_{c2}^{t} : travel time of the truck, circulating in route 2, including return, plus times for loading and unloading (hours).

c_b : average unit cost of operating a cargo barge (€/hour).

T_b^t : travel time of the barge, including return, plus times for loading and unloading (hours).

F_t : weighting factor representing the duration of each modelling period in the total time of the work completion.

$c_a\left(f_{a,v}^{t}, f_{a,c1}^{t}, f_{a,c2}^{t}\right)$: total cost of operation in the link *a* during the modelling period *t* (€/hour).

$f_{a,v}^{t}$: flow of vehicles in the link during the modelling period *t* (vehicles/hour).

$f_{a,c1}^{t}$: flow of trucks, circulating in route 1, in the link during the modelling period *t* (lorries/hour).

$f_{a,c2}^{t}$: flow of trucks, circulating in route 2, in the link during the modelling period *t* (lorries/hour).

Then, we must find a distribution which minimizes expression (15). Therefore, as truck transport increases, the cost of this mode increases and the cost of barges decreases (and vice versa). Moreover, if there is congestion, the costs for users will increase as the truck charge increases.

To notice that given the perfectly substitute character of the production function, the only variables to determine are the frequencies of trucks, obtaining for complementarily the barge frequencies.

In definitive, the number of variables to determine (frequencies) it will be the product of the number of routes considered for the trucks multiplied by the number of modelling periods considered.

BI-LEVEL OPTIMIZATION PROBLEM

Given the scenario outlined above, in order to determine the optimal distribution of charge between trucks and barges, the following bi-level optimization problem must be solved:

Upper level:

$$\min_{\{f_c^t\}} Ctot = \sum_t c_c \cdot \left(T_1^t \cdot f_{c1}^t + T_2^t \cdot f_{c2}^t\right) \cdot F_t + \sum_t c_b \cdot \left(\frac{Q^t - k_c \cdot \left(f_{c1}^t + f_{c2}^t\right)}{k_b}\right) \cdot T_b^t \cdot F_t + \sum_t \sum_a \left(c_a\left(f_a^t, f_{c1}^t, f_{c2}^t\right) \cdot f_a^t\right) \cdot F_t \quad (16)$$

subject to

$$\left\{\begin{array}{l}
Emission(f_{c1}^t + f_{c2}^t + f_a^t) \leq Emission_{\lim ite}^{SOx} \\
Emission(f_{c1}^t + f_{c2}^t + f_a^t) \leq Emission_{\lim ite}^{CO} \\
Emission(f_{c1}^t + f_{c2}^t + f_a^t) \leq Emission_{\lim ite}^{NOx} \\
Emission(f_{c1}^t + f_{c2}^t + f_a^t) \leq Emission_{\lim ite}^{PM10} \\
Emission(f_{c1}^t + f_{c2}^t + f_a^t) \leq Emission_{\lim ite}^{Noise}
\end{array}\right. \quad \textit{for all links of the network} \quad (17)$$

Lower level:

$$Z = \sum_{a \in A} \int_0^{f_a} c_a(x)dx$$

s. a

$$\begin{cases} \sum_{p \in P_w} h_p = T_w \,, & \forall w \in W \\ f_a = \sum_{p \in P} \delta_{ap} h_p & \forall a \in A \\ h_p \geq 0 \,, & \forall p \in P \,,\ \forall w \in W \end{cases}$$

Wardrop's equilibrium (18)

The optimization problem considered at the lower level, providing as a result a car assignment according to Wardrop's first principle. However, truck traffic will affect the Wardrop's equilibrium of the drivers implied in the congestion (18). There are so many problems of optimization, at the lower level, as number of modelling periods considered.

Additionally, the optimal charge distribution between the different studied alternative modes of transport (truck and barge), will keep in mind the environmental impact that will produce to add the heavy vehicles to the network.

For each type of pollution emitted by the traffic of vehicles and trucks in the network, there is a constraint at the upper level of the optimization level (17). This constrains are taken into account at the upper level, preventing the combined volume of cars and trucks not exceeding the limits of pollution emissions: PM10 (suspended particles), CO (monoxide of carbon), NOx (derived of the nitrogen), SOx (derived of the sulfur) and acoustic pollution. There are as many environmental restrictions as the product of number of links of the network and number of studied pollutants.

APPLICATION AND CONCLUSIONS

Finally, and such and like was shown previously, in reference to the material type to transport, the main group of provisioning material (see Figure 2 and Table 1): material of everything-one, selected stone and granulate filler , all considered important in the design of the modal distribution in the charge provisioning between truck and barge with destination the works of the port of Laredo.

Figure 2. Port of Laredo and future sport-fishing port.

That is to say, a total of 1.150.000 m^3 to what it is necessary to add the necessary materials for the construction of the concrete blocks (almost 170.000 m^3).

Unit of Work	Material	Quantity
Dike breakwater	*m^3 material everything-one*	*510.000 m^3*
	m^3 selected stone	*103.000 m^3*
	m^3 granulate filler material	*58.000 m^3*
Dikes and jetties of the fishing port	*m^3 selected stone*	*94.000 m^3*
	m^3 selected stone	*25.000 m^3*
	m^3 granulate filler material	*190.000 m^3*
Dikes and jetties of the sport port	*m^3 selected stone*	*57.000 m^3*
	m^3 selected stone	*13.000 m^3*
	m^3 granulate filler material	*100.000 m^3*
TOTAL		***1.150.000 m^3***

Table 1. Quantity and material type for work unit.

As for the traffic and based on the taken data and their later analysis, the existence of one pick hour and one valley hour is determined in the period of summer, while in winter exists a medium hour when not having a clear distinction pick-valley (see Figure 3).

Periodo	Meses	Horas
Summer – pick hour	*1 Jul y - 30 September*	*9:00 at 11:00, 14:00 at 16:00 and 19:00 at 21:00*
Summer – valley hour	*1 July - 30 September*	*00:00 at 9:00, 11:00 at 14:00, 16:00 a 19:00 and 21:00 at 24:00*
Winter	*1 October - 30 June*	*00:00 at 24:00*

Figure 3. Periods of traffic considered for the practical application in Laredo.

Arrived to this point, it is feasible to combine the requirements of materials periods of supply to the work with the different traffic periods considered. This combination will determine those periods of modelling that should be considered in the model, periods with different quantities of material or different conditions of the traffic in the city.

This way, the modelling periods are shown in the Table 2 and the Figure 4.

Periodos	Duración
Periodo 1	*Del 1 de Febrero de 2006 a 1 de Marzo de 2.006.*
Periodo 2	*Del 1 de Marzo de 2006 a 1 de Julio de 2.006.*
Periodo 3	*Del 1 de Julio de 2006 a 1 de Octubre de 2.006. Horas punta.*
Periodo 4	*Del 1 de Julio de 2006 a 1 de Octubre de 2.006. Horas valle.*
Periodo 5	*Del 1 de Octubre de 2006 a 1 de Marzo de 2.007.*

Table 2. Modelling periods for application.

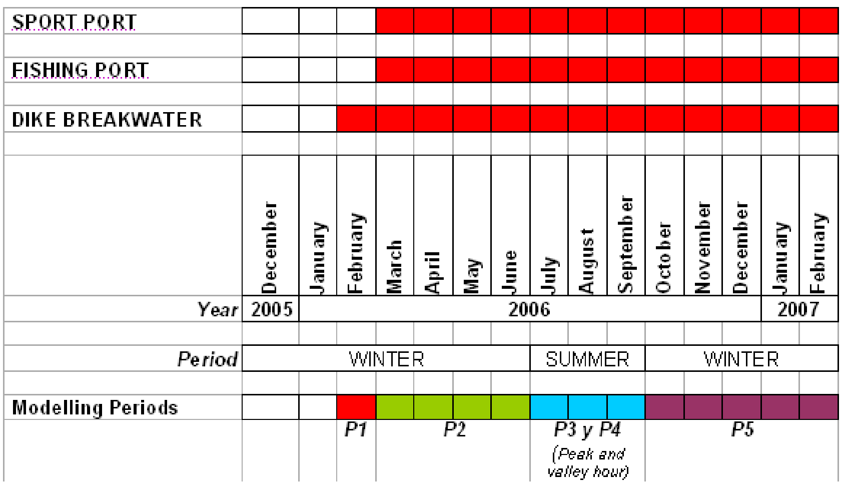

Figure 4. Modelling periods for application..

On the other hand, we have decided to take as means truck for the transport of materials, the truck of 20m^3 375 CV and 25 tons.

Since the material to transport have similar density, with a simple calculate is determined that the real capacity of the trucks is 13,25 m^3.

As for operation cost trucks, and being based on prices of principles of the year 2.005 for Cantabria, this is 45 €/hora.

The barge type to consider in this analysis will be a barge of 200 m^3 of capacity (appropriate size to work in the environment of the work) with a operation cost of 220 €/hora.

Regarding the final times of cycle they are:

	Route 1 for trucks	Route 2 for trucks
Time from point of supply to entrance in Laredo	*75 min.*	*75 min.*
Time of interior route in Laredo	*15 min.*	*16,8 min.*
Times of charge and discharges	*22,8 min.*	*22,8 min.*
CYCLE TOTAL	*3,13 hours*	*3,16 hours*

	Route for barges
Time from point of supply to shipment port	*17,47 min.*
Time of shipment port to Laredo	*225 min.*
Times of charge and discharges	*103 min.*
CYCLE TOTAL	*9,8 hours*

Table 3. Times of cycle for two alternatives of transport.

As for the resolution algorithm and for the special characteristics of the problem, the solution methods used should not require differential calculations of the function objective. This situation forces to use solution methods that don't require of an analytic expression of the gradient of this function and therefore it is appealed to methods and algorithms of heuristic type.

Inside these methods, we can find the algorithm of Hooke-Jeeves among whose advantages highlight that it doesn't demand any special attribute of the function objective. This algorithm has been used for the resolution of the problem of optimization.

This algorithm was programmed *ad hoc* in Borland-Delphi (see Figure 5) and in combination with software of analysis of transport systems (used to solve the assignment of vehicles to the network) a solution was obtained that minimized the costs.

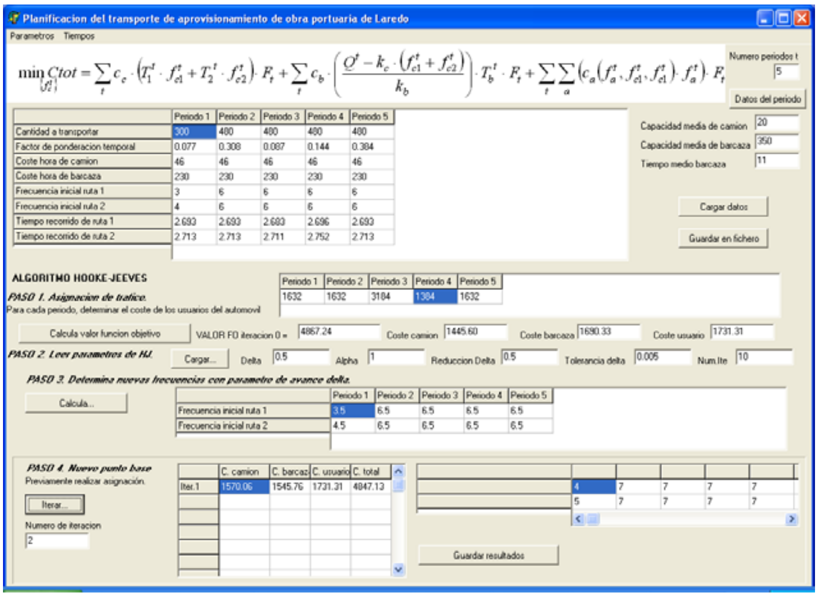

Figure 5. Software created *ad hoc* for control of the algorithm of Hooke-Jeeves.

To solve this problem of traffic assignment we use a potent software of transport systems modeling, ESTRAUS. Inside this wide computer package the module GENRED has been used for creation of network and the module ASIGNA to assign traffic private to the network.

For the confirmation of the environmental restrictions a *macro* was programmed in Excel that calculated for all links of the network the concentration of pollutants and of noise.

This evaluation is for the limit of concentration maximum per hour and yearly. In the Figure 6 one can see the methodology followed with the three participant software.

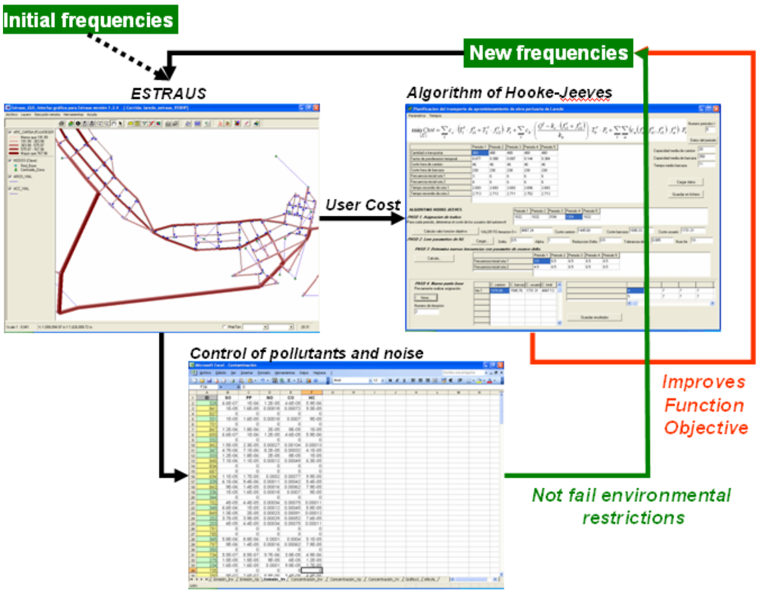

Figure 5. Methodology followed with the three participant software.

In the Table 4 we are shown the vector of frequencies, for the five periods and two routes of trucks, along the successive iterations, understanding for iteration completes every time that is assigned to the network the matrix of trips.

Iteration		**Period 1**	**Period 2**	**Period 3**	**Period 4**	**Period 5**
1	Route 1	*3 trucks/hr*	*6 truck/hr*	*12 trucks/hr*	*12 trucks/hr*	*6 trucks/hr*
	Route 2	*4 trucks/hr*	*6 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*6 trucks/hr*
2	Route 1	*4 trucks/hr*	*7 trucks/hr*	*13 trucks/hr*	*13 trucks/hr*	*7 trucks/hr*
	Route 2	*5 trucks/hr*	*7 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*7 trucks/hr*
3	Route 1	*5 trucks/hr*	*8 trucks/hr*	*14 trucks/hr*	*14 trucks/hr*	*8 trucks/hr*
	Route 2	*6 trucks/hr*	*8 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*8 trucks/hr*
4	Route 1	*6 trucks/hr*	*9 trucks/hr*	*15 trucks/hr*	*15 trucks/hr*	*9 trucks/hr*
	Route 2	*7 trucks/hr*	*8 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*8 trucks/hr*
5	Route 1	*7 trucks/hr*	*10 trucks/hr*	*16 trucks/hr*	*16 trucks/hr*	*10 trucks/hr*
	Route 2	*7 trucks/hr*	*8 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*8 trucks/hr*
				Fails environmentally		
6	Route 1	*7 trucks/hr*	*9 truck./hr*	*15 trucks/hr*	*17 trucks/hr*	*9 trucks/hr*
	Route 2	*7 trucks/hr*	*8 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*8 trucks/hr*
7	Route 1	*7 trucks/hr*	*9 trucks/hr*	*15 trucks/hr*	*18 trucks/hr*	*9 trucks/hr*
	Route 2	*7 trucks/hr*	*8 trucks/hr*	*0 trucks/hr*	*0 trucks/hr*	*8 trucks/hr*

Table 4. Vectors of frequencies along the iterations until optimal solution.

It is observed as leaving of a situation that represents 50% of transport in truck and 50% in barge, as change the values of the vector of frequencies these spread to grow, that is to say, increases the percentage of total transport in truck.

This growth reflects perfectly as the barge cost diminishes (every time is transported less in barge) increasing the cost of transport in truck and the cost on the user.

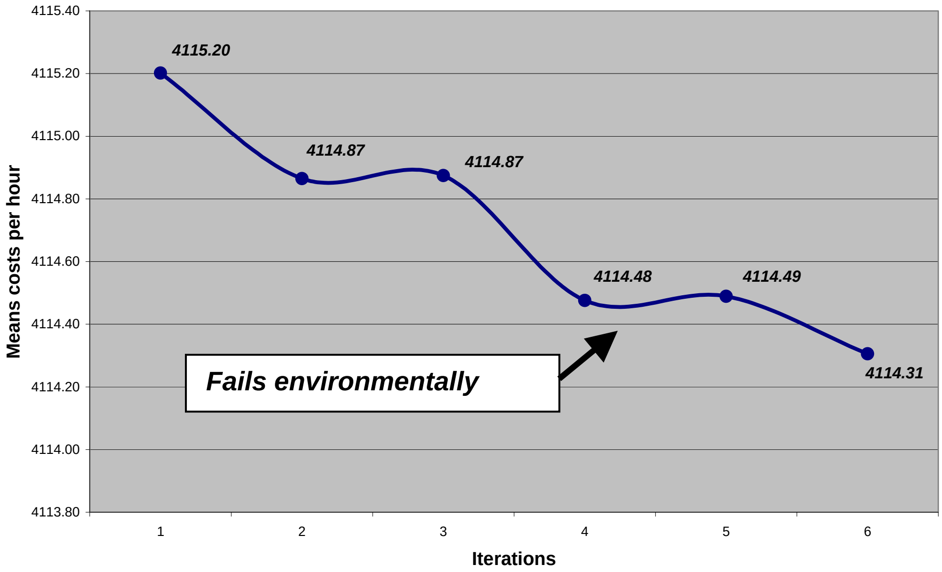

Figure 6. Evolution of means costs per hour until optimal solution.

In the previous Figure 6 can see as the algorithm in their evolution goes improving iteration to iteration the means costs per hour, until the iteration 5 where improves the cost but fail in the environmental restrictions.

If we analyze the cause of fail environmentally in the iteration number 5, we observe that is in the period of winter where the problem is presented: in the route 1 when increasing from 9 trucks per hour to 10 truck per hour. This period in spite of having less traffic intensity is bigger duration and therefore that of more weight in the cost function to optimize.

This is confirmed when observing that the route 1 has a value of frequency superior of 10 trucks per hour in other periods, even of high level of traffic, for example in the periods 3 and 4 with 15 trucks per hour (summer period).

The optimal solution represents that 76% of total of material should be transported in truck and the rest by barge.

In the Figure 7 the means costs per hour is represented (optimized value of the function objective) for all the possible charge combinations, from 0% until 100 % charge in truck.

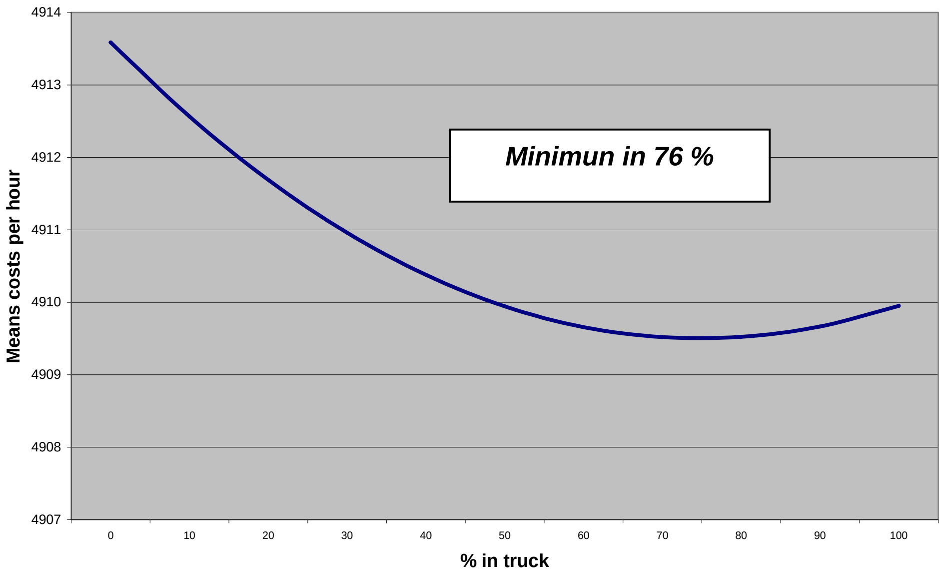

Figure 7. Evolution of the means costs per hour from 0% to 100 % charge in truck.

It is interesting to study some of the variations of the total cost attentively around the optimal solution, that is to say, carry out the analysis of sensibility.

The Analysis of Sensibility (or Post-optimalidad) study how would affect to the optimal solution obtained and the function objective the change (inside a predetermined range) of one of the parameters, maintaining fixed the remaining ones.

In the first place we analyzed the variation of the total cost in function of the variation of each one of the variables (frequencies) considering fixed the other ones.

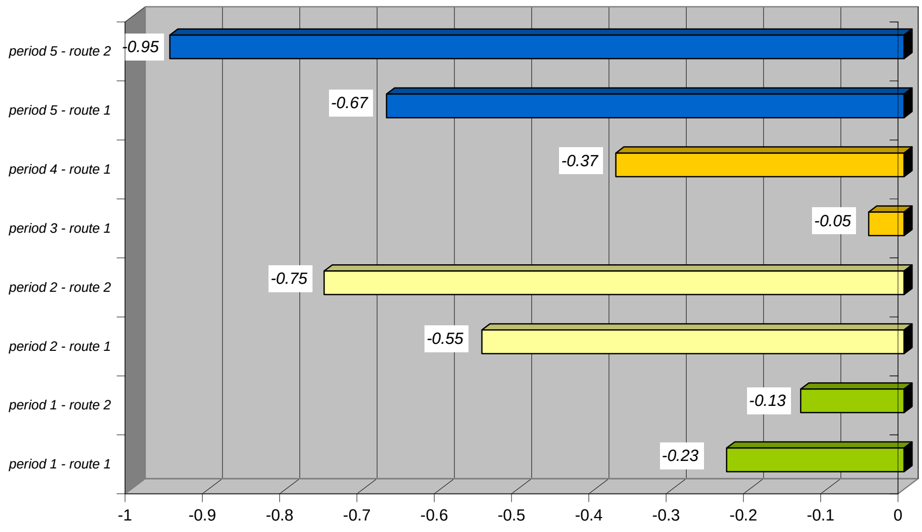

Figure 8. Change in function objective to diminish an unit the frequency.

In the Figure 8 we graph the change taken place in the function objective as consequence of diminishing in an unit the frequency of certain period and certain route, maintaining the invariable rest of frequencies.

If the previous analysis is carried out increasing an unit the frequency of the period and route in analysis, the results are those shown in the Figure 9

In every period and for all the routes, diminishing or increasing a frequency, maintaining the invariable rest, is observed that the effect on the total cost is negative, that is to say, doesn't improve the function objective, doesn't decrease the cost. This confirms that the solution is optimal for the study problem.

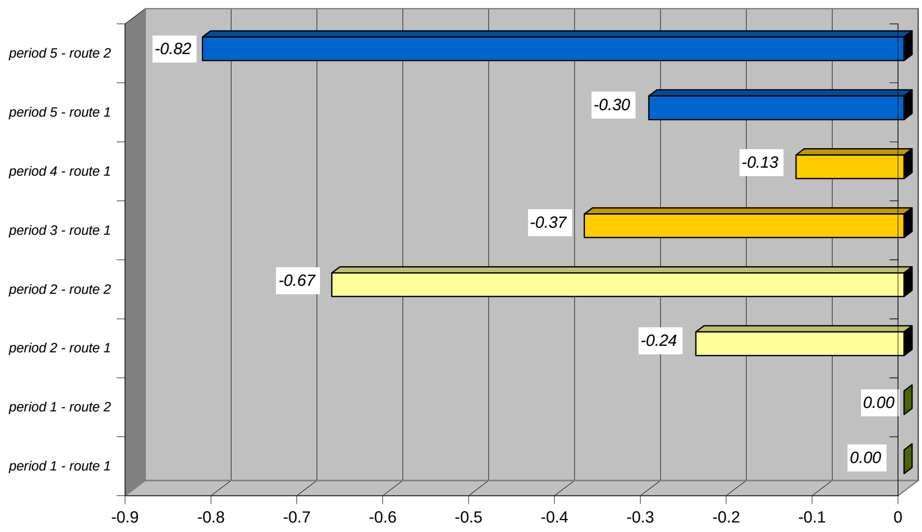

Figure 9. Change in function objective to increase an unit the frequency.

Consequence of this analysis is feasible to carry out some interesting comment in reference to the previous figures:

- The periods and routes that take null value of change in the function objective correspond with periods in those which, for the optimal solution, the total transport is carried out in truck and therefore doesn't make sense value a positive increment of this frequencies.
- In summery periods, in the pick traffic period (period 3) is more harmful to introduce a truck more than to diminish, while in the valley traffic period is worse to diminish an unit of frequency that increase it.
- In reference to the previous point, notice that any modification is harmful, but we can affirm that if we needs to introduce a truck in summer (for example motivated by an previous delays accumulation) should be in the hours valley.
- If the previous analysis is centered in the periods of winter (periods 2 and 5), in all the cases, is most interesting introduce more number of trucks (if it was necessary) in the route 1 that in the route 2.

Applying the analysis of sensibility to the operation unitary costs of trucks and barges, the results of the Figure 10 are obtained.

We can observe as the effect of reducing in 0,1 euro the unitary cost of operation of the trucks is equivalent to a reduction of 0,75 euros in the barges.

The points of court of the curve of "reduction costs in 0,1 euro in the operation of trucks" with the curves of the diverse reductions of cost of operation of barges, determine the charge percentage in truck starting from which is not interesting the transport barges and if in trucks.

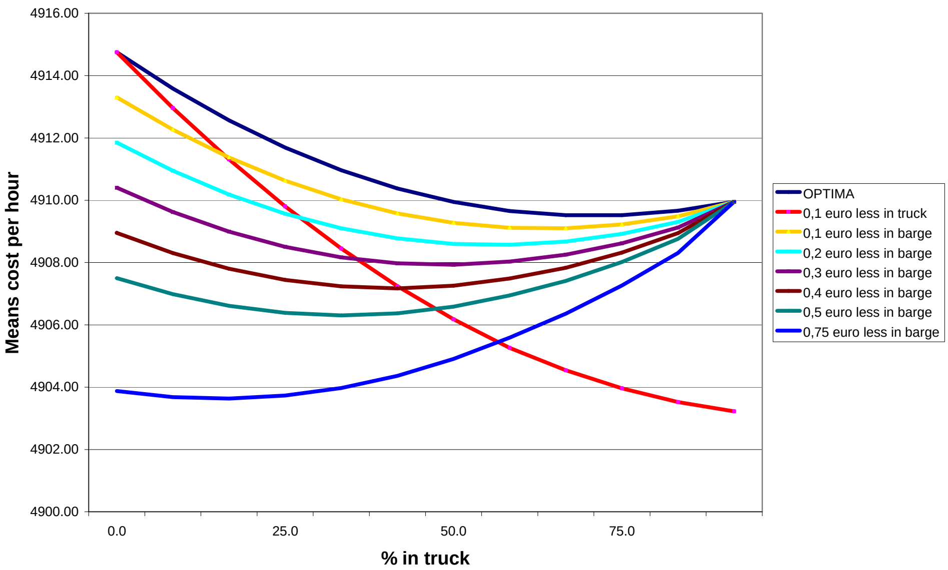

Figure 10. Curves of means costs per hour in function of the% to transport in truck for diverse scenarios of operation unitary costs of trucks and barges.

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