Modeling and Simulation of Physical Mobility within the Nexus Framework

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Synopsis

The Nexus framework anticipates the influence of the general computational development towards the socalled state of Ubiquitous Computing, where not only most electronic devices but even those objects which today have no electric supply will be aware of their position and context and be able to communicate among each other. This will be a kind of a by-product of the ongoing miniaturization of electronic components, growing performance of processors and memory and at the same time dropping prices. By using this communication capability and the collected spatial information, the Nexus framework will provide a platform for multiple spatial aware services for mobile people and self organizing applications.

For making mobile objects spatial aware, numerous sensors are needed, which can be placed on the object itself as well as in the stationary environment. In addition, simulation of physical mobility helps to bridge the lack of spatial sensor data or in a first phase generates virtual sensor data for test purposes for Nexus components and applications.

The simulation and modeling research in our subproject focuses on two aspects of user mobility: the mobility of pedestrians and the mobility of motor-driven vehicles. As for our proposal to the SIIV Congress, we want to place emphasis on aspects of microscopic simulation of pedestrian mobility.

As it is one of the most important objectives of the Nexus project, our main concern in the field of pedestrian simulation is the scalability of our modeling methods to large areas, e.g. to large (pedestrian) networks. Therefore we mainly focus on two dimensional cellular automata. By this way, we can reduce long range effects in a complex simulation environment to a relatively small calculation rule set. We are integrating several, partly already well known approaches to pedestrian simulation based on cellular automata into a comprehensive simulation platform. This allows us to reproduce phenomena like lane formation, crossing behavior, oscillations at doors and even panic situations.

We are testing our simulation platform on a rather complex geometric environment (the campus of the University of Stuttgart), which comprises indoor, as well as outdoor situations. Because of the need of more precise geometric data, we are also developing an interface for the exchange between CAD and GIS formats and the spatial world model format of Nexus.

Up to now, there are unfortunately no satisfying ways of evaluating the quality of microscopic pedestrian simulations. The most frequently used method is a heuristic comparison between the visualization of the simulation and the reality. For that reason, a comprehensive microscopic census of pedestrian mobility within the campus area, comprising video analysis and detailed counts will be carried out in the first half of 2005. As for the SIIV Congress, we will present first results of the evaluation through real pedestrian mobility data of our microscopic pedestrian simulations based on cellular automata.

Modeling and Simulation of Physical Mobility within the Nexus Framework

The subproject "Modeling and Simulation of Mobility" is part of the Center of Excellence 627 named "Nexus" which is a research project financed by the German Research Foundation [Deutsche Forschungsgemeinschaft - DFG 2005]. Since January 2003, ten research groups at seven institutes of the University of Stuttgart are working together to build up a framework for a generic digital world model of the physical world by joining existing and future sources of sensor and spatial aware data. As the major part of the subprojects is concerned with the development of a powerful and fast-working architecture of the framework, some of them are designing exemplary applications, like "Smart Factories" [Jendoubi et al. 2004] or "Orientation Aids for the Blind" [Hub et al. 2004]. The knowledge on the dynamic positions of mobile, spatial aware objects is an important task for the overall project. Simulating the mobility of these objects can complete spatial information in case of a lack of sensor data and in addition allows short time prognosis. This paper focuses on "Modeling and Simulation of Physical Mobility" within the Nexus Framework.

THE NEXUS VISION

It is rather rare, that long term predictions become true. And if they do so, their creator had often just a lucky hand. Gordon Moore, later one of the founders of Intel was certainly one of them. In a paper printed in 1965, Moore predicted that in the field of computer technology the exponential growth would continue and that the number of transistors would double every two years [Moore 1965]. His prediction, which has come to be known as the famous Moore's Law, has held true, and Intel's current Itanium 2 processor has 410 million transistors. Proud of that fame, Intel went to spend \$10,000 to the owner of an original issue of that magazine in April 2005.

This development, which is said to be going on for at least an other ten to fifteen years, is leading to the socalled state of Ubiquitous Computing, where miniature systems are integrated in large numbers into everyday objects making these objects "smart" and able to communicate [Mattern 2005]. Thereby, everyday objects can gather information about their state and their environment. By embedding this information into a model of the real world, which nowadays can be modeled very realistically using sophisticated 3D modeling techniques, it is possible to generate powerful digital world models. Not only can existing objects of the real world and their state be mapped into these world models, but additional information can be linked to them. The result is a symbiosis of the real world and digital information spaces. Ubiquitous Computing is nowadays an important research field not only in computer science but also in engineering and especially in transportation research.

The rapid development and proliferation of mobile communication provides a high potential for a broad spectrum of innovative applications. This potential is further enhanced by the availability of mobile, multifunctional devices, which not only integrate functionalities for communication and computation, but also different kinds of sensors, for example, for retrieving position information. The result of this technological progress is a range of new and interesting application areas, one of which is the area of "context-aware systems". These systems take parameters of their environment into account to adapt themselves to a particular situation. The most significant parameter of the environment, which plays a major role in "location-aware" systems, is the user's current position.

World Models

The interpretation of most of the context parameters requires a more or less detailed model of the environment. The combination of many such models leads to the creation of digital world models. The long term goals of the Nexus Center of Excellence are the development of concepts and techniques for the realization of comprehensive and detailed world models for mobile context-aware applications. World models comprise stationary as well as mobile objects of the real world and they can be augmented by virtual objects and services.

Modeling the physical world, which can be visualized by a three-dimensional representation using current technologies already yields a model of the environment. In addition, everyday objects can acquire information about their own state and their environment and embed this information into the world model. Finally, objects of the real world can be linked to additional information, and virtual objects can be added as well. The result is the so-called "Augmented World Model", which is an aggregated model of the real world and digital information spaces, as shown in Figure 1. The complexity of these world models ranges from simple geometric models and digital street maps to highly complex three-dimensional models of buildings, considering the objects' relations, semantic aspects etc.



As for the Nexus architecture, static information of a certain area (like streets, walking areas, land use, etc.) is represented in so called Augmented Areas (AA) which are characterized by their spatial extension, their type of information (e.g. GIS-data), their self-consistency and the fact that all objects belonging to an Augmented Area are stored at the same "Context Server", a server similar to a web server with a defined Nexus interface, which stores static objects like roads, buildings etc. Each Augmented Area has to be registered with some meta data at the Area Service Register (ASR) which is a repository of general information about the covered areas and which knows the object types and the addresses of the corresponding Context Servers (Figure 2).



Figure 2: Exemple of three different Augemented Areas

The Nexus Platform

The architecture of Nexus is subdivided in three tiers with clearly defined functions and interfaces. Context Servers as described before and all kinds of data providers are located in the service tier (Figure 3). They can access information from the platform or other Context Servers but mainly provide information. They can further access external databases which are not connected to the platform. A Context Server in Nexus is similar to a web server in the World Wide Web.

The federation tier is the innovation in the platform compared to the World Wide Web. It takes care of a consistent view on the different data sources with a direct reference to the location. It consists of a multitude of Nexus nodes. Each Nexus node includes a federation component and a set of value added services that have a special access to the federation and deliver a kind of basic functionality (e.g. navigation service).

Finally any application can join the Nexus platform via defined interfaces and can access any available information. The applications can be similar to web applications like applets in the World Wide Web. One can imagine nearby every location based application like meeting services or real-time traffic information and warning, depending on the available sensor data. For more detailed information on the Nexus architecture see [Nicklas et al. 2001].



In the modeling and simulation of mobility we now can make use of the data available in the platform and are vice versa potential data providers regarding traffic estimation, road conditions and warnings.

SIMULATION OF PHYSICAL MOBILITY WITHIN THE NEXUS FRAMEWORK

Concerning the work in the Nexus project, our aim is the integration of models and applications of physical mobility and in consequence the provision of traffic related cross-sectional data like e.g. density or average speed. Therefore one general approach is the evaluation of the suitability of traffic flow models for a platform like Nexus. For that purpose we are investigating the different microscopic traffic flow models for pedestrian and motor-driven traffic with respect to large heterogeneous networks. In the following section we are depicting some fundamental modeling approaches in the field of the simulation of motorized and non-motorized traffic. For a more comprehensive overview on microscopic simulation tools and models see [Barceló et al. 2004] and [Gartner et al. 2002].

Motorized Traffic

In the field of simulation of motorized traffic a large part of microscopic modeling approaches can be subsumed under the term of "Car Following Models". Leaving beside lane changing effect, a vehicle driver always tries to conform to the preceding vehicles' behavior. He responds to speed changes of his forerunner by accelerating or decelerating (e.g. breaking) his own vehicle. In this interplay, distance in time and space usually never stay the same.



Figure 4: Psycho-Physical Spacing Model [Wiedemann 1974]

The Psycho-Physical Spacing Model [Wiedemann 1974] augments classical car following models with psychological phenomena, where drivers are subjected to certain limits on stimuli to which they respond (Figure 4). The core statement of this model is first, that at large spacings the driver of a following vehicle is not influenced by the size of the difference of velocity, and second, that at small spacings there are combinations of relative speeds and distance-headways for which there is no response of the driver of the following vehicle because of the too small relative motion.

Most car Following Models can mathematically be described as closed analytic system, never the less a numerical adoption of that idea seems to be performing better, particularly concerning the scalability of a simulation system. The kind of Cellular Automata Models Steven Wolfram names as big revolution in analytical mathematics [Wolfram 2004] at least had some impact in traffic theory.



Figure 5: Traffic jam waves in a one dimensional Cellular Automata

In the Cellular Automata Model of Nagel and Schreckenberg [Nagel et al. 1992] the road is thought to be subdivided into cells, each 7.5 m long, which corresponds to the mean frontbumper-frontbumper distance between two consecutive cars captured in a jam. A cell is either empty or occupied by only one vehicle with a discrete velocity $v_i \in \{0, v_m\}$ where v_m is the maximum velocity. All speeds are measured in cells per time step. The motion of the vehicles is determined by rules for collision-free acceleration, randomization and movement. A time step usually corresponds to $\Delta t = 1$ sec. The result of a simple implementation of a Cellular Automata for a single lane road can be seen in Figure 5. In this example, six speed steps from 0 to 5 are provided, which under consideration of the cell's length implicates a maximum speed of 135 km/h. The speed of the agent for the next time step t+1 is the minimum out of v_t +1, the cell distance to the forerunner and v_{max} . In that case, every agent would tend to reach the maximum speed, no congestions would appear. That's why a stochastic "dawdle factor" is introduced, in which with a certain probability the current speed is reduced by one step. This factor allows the appearance of congestions waves, as they can be observed in Figure 5.

Pedestrian Traffic

Microscopic non-motorized traffic simulation implicates, that every pedestrian (and every biker as well) is treated as an individual. Up to now, the main part of pedestrian researches has been done on a macroscopic level. This implicates, that the interaction between pedestrians can not be considered. Furthermore macroscopic models are not suitable for prediction of pedestrian flow performance in pedestrian areas or buildings.

On the other side, microscopic simulation models are in need of a very performing software implementation (and at least a performing computer system) if they should preserve their scalability towards bigger areas. Actually in this research field two main directions are persecuted.

The first main direction of modeling methods, the so-called "Force Models", can be described as a closed system of equations. They are all based on the idea of physically described forces, which cause distracting effects among moving persons themselves on one hand and between moving persons and obstacles on the other hand. The direction of the intended destination has attracting effects. One example for this kind of model is the Social Force Model [Helbing et al. 1995], where a pedestrian is subjected to social forces that motivate him to move in a certain direction. The model is based on the assumption that every pedestrian has the intention to reach a certain destination at a certain target time. The summation of the forces acting upon a pedestrian i with mass m creates acceleration dv/dt as:

$$m\frac{dv(t)}{dt} = m\frac{v_{o}\overset{\rho}{e} - \overset{\rho}{v_{i}(t)} + \overset{\rho}{\xi_{i}(t)}}{\tau} + \sum_{j(\neq i)} f_{ij}^{\varpi}(\overset{\rho}{x_{i}(t)}, \overset{\rho}{x_{j}(t)}) + f_{b}^{\rho}(\overset{\rho}{x_{i}(t)})$$

The first term on the right hand of this equation represents the motivation to reach the goal. The direction is introduced by a unit vector e from a particular location to the destination point. The ideal speed v_0 is equal to the remaining distance per remaining time. The remaining distance is the difference between the destination point and the location at that time, while the remaining time is the difference between target time and the simulation time. The accelerating force for reaching the ideal speed is minimized, if v_0 is equaled by the individual speed v_i plus its individual fluctuation ξ_i . The second term on the right hand side of this equation is designated for interaction among pedestrians (f_{ij}) and between pedestrians and obstacles and boundaries (f_b). The summation of all three types of forces, the force of acceleration to reach the optimal speed, the forces of interaction with other pedestrians and the forces of interaction with boundaries leads to the resulting force acting currently on each pedestrian.

While in the field of "Force Models", at least space, often time and speed are continuous and thus physically exact, in the other main research field of pedestrian modeling methods space and in consequence time and speed are cut in discrete steps. Following the modeling of motorized traffic, pedestrians (and bikers) are treated as moving "Agents" in a Cellular Automata, in this case not with one but two dimensions. By considering the example for motorized traffic in Figure 5 this means, that a cell transition is not only possible to one but potentially to eight neighboring cells, assuming quadratic cells with each the size to house one pedestrian and the allowance of vertical, horizontal and diagonal movements. Each cell has a certain transition probability, assigned by the surroundings and by the motivation to reach the goal, transition is done by a probabilistic decision. As modeling pedestrian behavior by Cellular Automata does not implicit an analytical solution, simulation is mandatory. Concerning pedestrian movement, this research field is quite new but developing fast, as quality and efficiency of Cellular Automata can be enhanced step by step.

One of the latest refinements of Cellular Automaton models for pedestrian dynamics is introducing the socalled "Floor Fields" [Schadschneider 2003]. Inspired by the principles of chemotaxis used e.g. by ants to guide other members of their population to food places, the dynamic floor field modifies the transition rate to neighboring cells. Its strength is influenced of time and space effects like diffusion and decay, which leads first to a dilution and finally to the vanishing of the trace. The dynamic floor field overlays the static floor field, which represents the constant properties of the surroundings. This step allows the implementation of very well performing simulation tools, as it is translating the long ranged spatial interaction between pedestrians into a local interaction with a certain memory.

FOOT TRAFFIC SIMULATION FRAMEWORK

Our main task within the Nexus project is the provision of (faster than) real-time microscopic data of physical mobility. As what is concerning pedestrian mobility, we are building up a comprehensive simulation framework which comprises the three modules Geometry, Simulation an Evaluation (Figure 6).



Figure 6: Schema of the implementation of pedestrian simulation for the Nexus project

Unfortunately up to now, geometric data of pedestrian footways often does not exist in a sufficient quality. For instance city maps usually focus on road and building alignments and not on the precise reproduction of footway geometries. Therefore we are in need of a certain editing of the input data before it can be used in our simulation module. Relating to our results, we will be able to deliver them in the World Modeling Language format, which is a Nexus adopted XML-Format.

Input Formats

The simulation module needs detailed information about the footway geometry. For instance it is necessary to distinguish if a line is an obstacle which cannot be crossed or if it represents a curb which separates a footway from a carriage way. In addition varying walking resistances are related to differnt walkable surfaces. The preparation effort is depending on the data source. Actually, for two reasons we are focusing on CAD exchange formats (e.g. dxf). On one hand we can access construction plans of our test bed, the campus of the University of Stuttgart, on the other hand most GIS applications support the dxf-format for data export. Finally Autodesk AutoCAD[®] provides a programming interface which eases the external access of geometric data.

Data Processing

To learn more about simulation quality and performance of different pedestrian modeling approaches, we want to modularly integrate different simulation models. Beginning with our test bed, we will calculate on a single computer, later by simulating bigger areas, we will scale-out on our institute owned simulation laboratory, which will allow us parallel computing.

Output Formats

As for the simulation results, there are several analysis potentialities. During the simulation, the developing traces of each agent will be saved. They contain the agent's ID, time stamp and the position coordinates. Based on this information, we can generate mobility characteristics like source-destination matrices with medium walking times or local pedestrian densities and capacities of pathways, exits and so on. While we provide trace information in a simple ASCII format, we can deliver e.g. tracking results of the agent's projected position in the World Modeling Language format

VALIDATION ON THE REAL WORLD

Regardless of which simulation model is applied, validation of model assumptions, calibration of the model parameters and finally endorsement of the simulation results are the key challenges in successfully modeling and simulating pedestrian movements. Calibration of model parameters can be done for instance by extracting them from the literature [Alrutz et al. 1999], but one common obstruction for many research projects in microscopic simulations. Only few research groups can rely on such a database. Thereby real microscopic empirical data comprises often only small-scaled areas like one floor of a building or even just a single room.

In 2003, Klüpfel compared evacuation exercises in a primary school with simulation results [Klüpfel et al. 2003]. But as for the data ascertainment, no real microscopic data has been collected. The initial distribution for the simulation of the pupils in the classrooms has been taken from the statistical records of the headmaster; the egress time for leaving each class room has been measured by the class teacher. This leads to the dissatisfying situation, that due to the collection of macroscopic data, only aggregated simulation results can be compared with the reality. It is not possible to derive any conclusions about the grade of realism of the microscopic behavior of the simulated agents. Up to now, there is no automatic detection method available for collecting microscopic empirical data and coupling them to microscopic simulation of pedestrian motion. Together with our partners in the Nexus research group and the DLR - Institute of Transport Research, Berlin, we are to find new ways for microscopic data enquiries.

Cordon Census

As mentioned before, the test bed of our pedestrian simulation framework is the Campus of the University of Stuttgart as shown in Figure 7. A first approach towards the collection of empirical microscopic data of pedestrian dynamics is the so-called Cordon Census. In the research field of motorized traffic, this kind of counting is already well-known since several decades. In that case, the registration plates of motor vehicles on all radial highways towards the study area are listed during a defined period, usually a whole day. Later, by matching the listed registration plates, an analysis of source, destination and through traffic can be done for the whole study area as well as for each counted radial highway.

For analysis of (closed) public transport systems, the Cordon Census is also an often used instrument. Here, when entering a bus, tramway or subway system at a certain station, a passenger gets a counting badge of for example a certain color and is asked for giving it back when he is leaving the transport system at a second station. By this way, a source-destination matrix can be drawn out of the counting results, indeed a "classical" macroscopic result.

For our project, we want to augment the Cordon Census towards a more microscopic character by not only evaluating the charge on different connections but by evaluating the exact walking times. We are doing this by giving a unique ID to each pedestrian when he is leaving a building respectively the rapid-transit railway (called "S-Bahn" and hence labeled with a black "S" in the centre of Figure 7) and registering it a second time when entering a building or the "S-Bahn" station. At each registration, an exact time stamp is assigned to the

ID. As during the morning peak hour between 8 and 9 pm, more than 1500 passengers are leaving the train station, this aim implicates a big technical effort. We have already determined the advantages and disadvantages of several census technologies, from handing out little ID-tickets to RFID tags with unique IDs saved on it. Actually, we are persecuting two methods: Barcode scanning of barcode IDs and detection of active Bluetooth devices. While the first method is self explaining, the latter one needs some more explanation.



Figure 7: Campus Vaihingen of the University of Stuttgart with Counting Cordon

Most of today's mobile devices dispose of a Bluetooth device, a wireless network standard, which allows easy data exchange among Bluetooth devices. While possessing a high security standard for communication clearance, activated but passive Bluetooth devices are sending their ID every 1.28 seconds. Haase [Haase et al. 2005] provides a tool, which easily allows the detection of the unique 48 bit MAC address of active Bluetooth devices. Yet we are expecting, that about 5 to 10% of the pedestrians on campus own a mobile device with an activated Bluetooth interface. Supposing equal distribution and by counting the total number of pedestrians in the same time, this will allow us to reason a complete source-destination matrix with real walking times and day-time distributions.

Orthogonal Video Analysis

Beside the Cordon Census, we are prosecuting a second way for obtaining completely microscopic data of pedestrian mobility. This is in particular important for the validation of effects of lane formation or group and evasion behavior. Actually, there are existing several technologies for detecting pedestrian movement like microwave detectors and infrared or ultrasonic sensors. Unfortunately, these detectors perform the best on a distance up to ten meters respectively relatively small sections and hence are adopted the best inside of buildings.

Another possible way for gaining these data is to make photos or single-step videos. Unfortunately up to now, there have been no satisfying algorithms for a computer based extraction of pedestrian movement and a manually extraction of pedestrian traces especially in rather big areas is not worth to be done. The Institute of Transportation Research of the German Aerospace Center (DLR) has yet first positive experiences with an automatic video detection of road traffic movements [Dalaff et al. 2003]. We have now projected to make an orthogonal video record of the main part of the campus using a camera system placed under a carrier balloon. The algorithms for automatic motorized mobility detection will be refined in a way that they are also able to detect pedestrian mobility. This will result in an important database of empirical microscopic pedestrian mobility data, which will allow numerous validations of our simulation framework in future.

CONCLUSION

The main task of mobility simulation within the Nexus Framework is the provision of actual and predicted positions of mobile objects and persons as well as the generation of (virtual) sensor data. Concerning pedestrian mobility for that purpose, we are building up a simulation framework which allows us to test several modeling methods on pathway geometries gained from many different sources. We are providing position information by exporting it to the World Modeling Language format after e.g. a query of a Nexus Service. Due to our efforts in the field of collecting microscopic empirical data, we will dispose of important validation possibilities for our simulation framework. First results of simulations and its validation will be presented on the SIIV Congress.

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