The global reliability of paths between a pair O-D

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Synopsis

One of the possible definitions of reliability of paths between a pair O-D on a road network is given by the probability that a user could reach a destination D using his own vehicle starting from an origin O, in a given time "t_o" considering that an ensemble of events which could delay march may occur.
The following study suggests a model for the valuation of the path reliability between two nodes O-D of a road network by starting from a given network configuration, from well-known values of volumes of traffic and well-known time frequencies of the above events.

1. THE CURRENT OUTLINE OF ROAD RELIABILITY

It is known that a well-established definition of reliability is the property of any system, equipment, device, service, to not "break" during its functioning.
On an engineering point of view, reliability arises from the necessity of ensuring the continuity of functioning of several systems that both science and technology allowed to carry out in the second middle of the last century.
The limit brought out by the previous methodologies which faced up the problem in terms of factors of safety or using redundancies stimulated the development of methods of reliability.
This particularly happened in mechanical, electronic and electric engineering but soon after, the studies on reliability spread to other engineering fields, not least engineering of structures and transportation and so on.
Recently, some specific aspects of reliability have been treated particularly in the field of road engineering.
The most important are:
a) the reliability of connectivity;
b) the reliability of capacity;
c) the travel time reliability .
Generally the reliability of connectivity is used to analyse exceptional situations such as earthquakes and floods. This lets the verification of only two states for each link of network or its complete functioning or its break. Consequently this kind of analysis only let to value the reliability for each pair of nodes (O-D) of a network only in a state of full functionality or full inefficiency of links [1, 2, 3].
The reliability of capacity is expressed by the probability that the capacity of a network of infrastructures of transportation could be able to make a certain demand of travels flow between the several nodes O-D, in order to keep an acceptable level of service [5, 7]. The level of complexity of such a question is remarkably above the level of reliability of connectivity. Several scholars have studied this subject and some aspects are still object of research. Such an interest derives from the importance of such a topic in the problems of a road network management. The same matters concern other areas of engineering dealing with problems of network flow, in particular with the maximum one. Moreover, in the case of road networks the maximum flow is not only a function of the capacity of single links but of users' behaviour, too.
Finally, the travel time reliability is defined by the probability that a journey between a pair of nodes O-D occurs in prefixed time. In particular in the last ten years several authors [4, 9] have analysed some aspects of this topic.
Such index assumes a certain interest in the global evaluation of road network performances and its correlation not only with the reliability of capacity but also with other factors which generally can characterise the state of the single links of a network, is well known.

2. THE RELIABILITY INDICATOR ASSUMED FOR A GLOBAL VALUATION

Each of the above three methodologies of reliability may be used to value the reliability of path subject to a series of events which could happen.
Tab. 1 points out some examples.

Tab. n. 1. Relations among kinds of reliability and space of events

Space of events	**Kinds of reliability**		
	connectivity	**capacity**	**travel time t_o**
Structural decay and collapse	Disconnectedness in the link of network	Reduction of the road section for the sliding surface of slopes.	Increase of t_o for decay of paving.
Congestion	Interruption of flow for rigid conditioning (stop and go)	Increase of demand at a local level	Increase of t_o for increase of flow
Accident	Interruption of journey or of flow	Reduction of capacity for the closing of one lane	Increase of t_o for reduction of capacity.

The choice of the most suitable method will depend on the in-depth analysis we are going to make on the probable events on the functioning of network.
The evaluation of the probability of connection of network will be obtained using the method of the reliability of connectivity.

The evaluations concerning the possibility of transit of a certain volume of traffic in an established space of time, can be carried out using the method of reliability of capacity.
Predictions on the level of satisfaction of users who go along a path between a pair O-D in a prefixed time "t_o", can be valued through the reliability of the travel time reliability.
This study faces the problem of the reliability of the travel time reliability between a pair O-D of a network taking into account the probability that some events may obstruct the march of the vehicle till the total stop of flow and the use of alternative routes.
The events which may delay the march of vehicles are many:

- atmospheric events (rain, snow, ice, fog)
- catastrophic events (earthquake, floods. etc.)
- fire side of the road
- accidents on the roads
- cases of congestion
- execution of works (planned or 24-hour intervention)

Such events will increase the travel time in a different way.
The evaluation of the "global reliability" of travel time represents an efficient indicator to display the function of the network, its level of service and to value the benefits of users in the form of reduction of travel time. The distribution of probability of moving time for each link, path or pair O-D is described by the probability density curve *φ(t)* (fig.1).
The value t_{min} represents the time which the fastest vehicle takes in the best environmental conditions and in lack of traffic. Its superior limit is indefinite and can be very big in relation to the probability that a series of unfavourable events may happen and delay the flow of the vehicular stream until its full stop.
Given a time t_o it is possible to value the reliability "A" that such a time could not be exceeded, such as:

$$A = \int_{t_{min}}^{t_o} \varphi(t)\, dt \tag{1}$$

The function *φ(t)*, in relation to the different fields of application, to the data bank at our disposal and to their quality, is generally determined choosing a known distribution which fits on the available data in an optimal way.

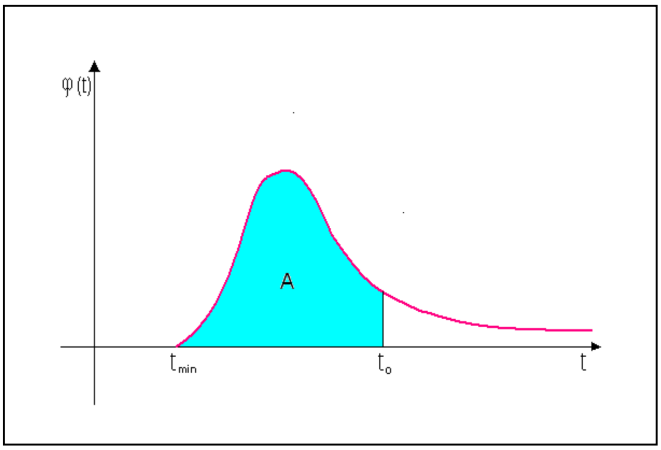

Figure 1

If data lack, it is necessary to assume a distribution beforehand, in relation to the physical nature of the possible "break" or under previous experiences. *φ(t)* is a continuous function even if more or less complex.
In the case of systems constituted by a certain number of components, the reliability of the whole strictly depends on the reliability of the single component. The analytic function of reliability either is very complex or does not exist and it can be valued either in discrete way or by determining only the extreme values of the whole field of variability.
Moreover, the problems of evaluation of the network systems reliability in relation to the number of the component links, to their interconnection and to the combinatorial nature of their treatment, may easily cause situations of intractability on the point of view of calculus.

3. SUGGESTED METHODOLOGY

3.1 Synthesis of method

Hereafter we expose a short synthesis of the logic sequence which characterises the methodology suggested postponing the exposition and the operational in-depth analyses to the following paragraphs.
The principal phases are:

a) *Definitions of input:* network of structure (represented by a planar graph) characteristics of links in normal conditions of run (represented by time-flow curves), link flows, definitions of events and determination of the relative probabilities.
b) *Valuation of the consequences of events.* In this phase the state of each single link described by the time-flow curves is methodically defined for each event or set of simultaneous events.
c) *Valuation of the probability of the conditions of run of links.* Each link is characterised by a certain state for which probability and travel time are determined.
d) *Valuation of travel conditions probabilities for the principal path O-D,* understood as the route of minimal time used by drivers without the happening of the considered events. By combining the states of the single component links, we obtain the whole set of states for the principal path and we value the probabilities and the travel time related to them.
e) *Research of competitive paths:* comparison among the states of the principal path with the possible states of the alternative paths. Consequently we individualise the *competitive paths,* which will be used by drivers because they represent minimum time paths.
f) *Definition of the states of paths O-D.* By combining the states of the principal path with those of the competitive paths, the states of the pair O-D are defined and for each of them we value probability and the travel time to move from O to D.
g) Given the time t_o, the total reliability indicated by the expression (1) is valued in discrete way, in the form of summation.

3.2. The considered events and their probabilities

The considered events are principally those which have a more frequent cadence in relation to the environmental context in which the network is realised, to the geometric characteristics of roads and to the conditions of traffic.
Hereafter we have a possible list of the categories of events to be considered and a qualitative description which lets the valuation of their probability.
Atmospheric events: the probability is given by the ratio between the statistically determined time where a state of wet, snow, ice remains on the carriageway and the time of total reference.

This last one may be taken up to a whole year or to a fraction in the hypothesis of determining a specific reliability (for example wintry) of network in the period when the incidence of the examined phenomena is greater. We assume a scale of reference to value the intensity of events. Generally such scales may refer either to numerical values, for instance the height of snow, or only to situations expressed on a qualitative way.
If we consider the possible effects of snow on the increase of the travel time or on the interruption of traffic flow, it is sufficient to consider a two-value scale: *average snow* and *high snow* and to give a lower value of the path speed to the first one and the disconnectedness of links to the second one. As regards rain it is sufficient to assume two or three typical situations (wet, rain of average intensity and rain of strong intensity) associating to them the values of an average speed of path.
Fire side the road: we evaluate the effects on the practicability of road from the delay to the stop of flow. Even in this case it is sufficient to assume a scale of reference having some typical values to which the corresponding values of speed must be associated. It could be proper to define a seasonal reliability (for instance summer) making reference to the period when such events are more frequent.
Catastrophic events such as earthquakes and floods: these events have not been considered in this study because the values of their probabilities may not be compared with those of other considered events and consequently they have to be treated separately.
Accident: we use the statistics of the accidents located on the considered roads. The number of accidents is referred to the atmospheric conditions and to other conditions, too, in order to go on with the calculus of the probabilities of connected events, that is to say of subordinated probabilities.
Execution of maintenance works: for a probability evaluation it is possible to have recourse to the historical series of fulfilled interventions on the one hand, to get information on intervention programs at manager institutions on the other hand. Even for that category of events we may use the subordinated probabilities, if works are executed when weather is good.

Changes of demand. At first, we assign a volume of traffic related to a well-established period of the day to each link of the network. We can also consider particular values of flow at seasonal and weekly levels. Starting from normal conditions of the network functioning, we can consider the increases of flow related to exceptional events (sporting, artistic, religious, folk events and meetings) which determine the movement of great masses of persons by vehicle.

The events previously described are treated simultaneously by considering the possible subordinate probabilities, too.

Consequently we go on with the definition and the assignment of inputs following the order we indicate hereafter:

a) we assume the temporal space of reference;

b) we define the network (planar graph) and the characteristics in normal conditions of functioning, that is to say in the absence of particular events for each link "a" expressed by the following relation:

$$t_a = f_{ao}(q_a) \tag{2}$$

which links the time of path t_a to the capacity q_a (time-volume curves);

b) on the basis of the prefixed time, we define the link flows in normal conditions of functioning;

c) we consider a set of categories of events C_1, C_2, …C_j, … C_n which affect the conditions of the link paths;

d) in the context of each category C_j we consider the events $E_{j,1}$, $E_{j,2}$, …. $E_{j,i}$, …. $E_{j,mj}$ mutually exhaustive and exclusive. In order to be that we have to consider the "state of absence", too. For example in the *road accident* category we can consider the following events: *absence of accidents, not serious accidents, serious accidents;*

e) we determine the $p(E_{j,i})$ probability for each event $E_{j,i}$. If the events we have considered in the context of each category are mutually exhaustive and exclusive, we have:

$$\Sigma_i\, p(E_{j,i}) = 1 \qquad \text{for } i = 1, 2 \ldots.. m_j \tag{3}$$

In fig. 2 the C_j categories, the $E_{j,i}$ events linked to them and the related $p(E_{j,i})$ probabilities are pointed out.

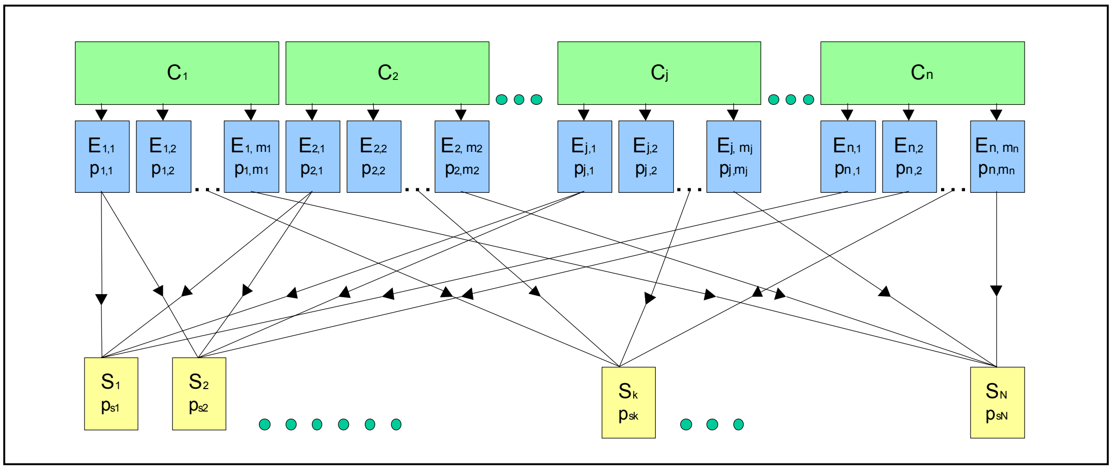

Fig. n. 2. Generation of states related to a link of the network.

3.3. Evaluation of the probabilities of the travel time of links.

Evaluation of the consequences of events. First of all we systematically define the set of the states of each single link "a" $\mathbf{S_a}$, in relation to all the combinations of events $E_{1,i1}$, $E_{2,i2}$, … $E_{j,ij}$, … $E_{n,in}$, we have obtained associating an event of the first category to the second one and so on in order to use up all the categories.

$$\mathbf{S_a} = \mathbf{S_a}\{ E_{1,i1}, E_{2,i2}, \ldots E_{j,ij}, \ldots E_{n,in} \} \qquad \text{with } i_j = 1, 2, \ldots m_j \tag{4}$$

The N number of such combinations is given by (fig. 2):

$$N = \Pi_j\, m_j \qquad \text{per } j = 1,2,\ldots.n \tag{5}$$

The state of the link S_{ak} will be defined by considering the influences of all the events which happen simultaneously. It is characterised by the relative time-flow curve that is to say the relation which links the time of path *t* to the volume of traffic *q*:

$$t_a = f_{ak}(q_a) \tag{6}$$

the probability of events which has determined the S_{ak} state represents the probability that the relation (6) exists. It describes the travel conditions of the link "a" in the state S_{ak} and we obtain it by starting from the time-flow relation (2) related to the normal conditions of functioning.
Generally, the considered events modify the time-volume curve of the link by moving the course of the generic function $t_a = f_a(q_a)$ up or diminishing the capacity or producing both the effects. Finally, other events determine the interruption of link.
Fig. 3 shows an example of the rise of the time-flow curve as an effect of the decrease of speed and an example of how the same changes because of a decrease of capacity. As the two events occur simultaneously, the effects merge and give rise to an ulterior time-flow curve.

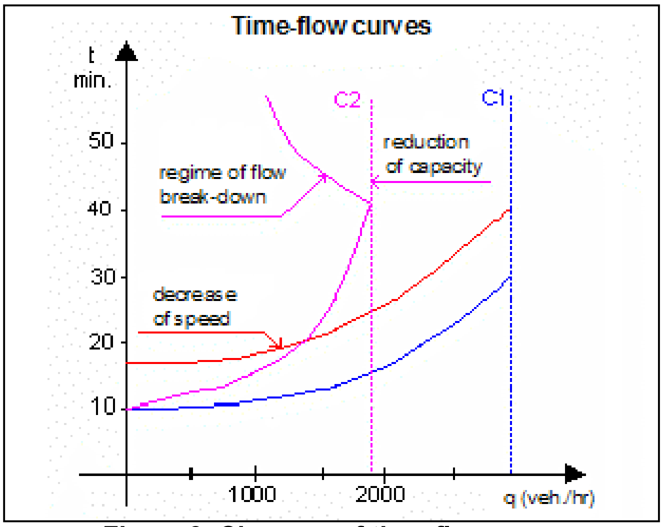

Fig. n. 3. Changes of time-flow curves

In conclusion, on the base of the events related to the S_{ak} state, we obtain a new time-flow curve, where the travel time related to that state is determined, according to the volume of traffic.
In the case of capacity reductions, the flow of traffic which runs the road segment may result greater than that new limit. In that case user will travel as flow break-down regime (*stop and go*).
If one of the events related to the S_{ak} state consists of an increase of flow, the travel time will be determined on the time-flow curve made according to the other concomitant events, in correspondence with the new flow.
We estimate the *probability* that the state S_{ak} of the link could happen. As we know, if events are *independent,* the $p(S_{ak})$ probability of the state of the link is given by:

$$p(S_{ak}) = p(E_{1,i1})\, p(E_{2,i2}) \; \ldots\; p(E_{j,ij}) \;\ldots\; p(E_{n,in}) \tag{7}$$

where $p(E_{j,i})$ represents the probability of the event $E_{j,i}$.
If all events are mutually correlated, the probability of the state is given by:

$$p(S_{ak}) = p(E_{1,i1}/\, E_{2,i2},..E_{j,ij},\ldots E_{n,in})\; p(E_{2,i2}\, /\, E_{3,i3},\ldots E_{j,ij},\ldots E_{n,in}) \;\ldots p(E_{j,ij}\, /E_{j+1,ij+1},\ldots E_{n,in}) \;\ldots\; p(E_{n,in}) \tag{8}$$

where $p(E_{j,ij}\, /E_{j+1,ij+1}\ldots E_{n,in})$ is the probability of the $E_{j,ij}$ event subordinated to the happening of the events $E_{j+1,ij+1},\ldots\, E_{n,in}$.
If, in general, some events are independent, let's suppose the first *"g"* events, and the other "*n-g"* mutually correlated, the probability of the state is given by:

$$p(S_{ak}) = p(E_{1,i1})\, p(E_{2,i2}) \;\ldots p(E_{g,ig})\; p(E_{g+1,ig+1}\, /E_{g+2,ig+2},\ldots E_{n,in}) \;\ldots p(E_{n,in}) \tag{9}$$

As we have mutually exhaustive and exclusive events, we have for each state of the link:

$$\Sigma_k\, p(S_{ak}) = 1 \qquad \text{for } k = 1, 2 \,\ldots\, N \tag{10}$$

3.4 Evaluation of the probabilities of the travel times of paths

There are cases in which the probabilities of single events on a link are strictly referred to the probabilities of events on another link.

In the evaluation of the *compound probabilities,* we consider the *subordinated probabilities* of the events related to the specific links because some types of events such as, for example the atmospheric ones, are not independent from those of the other links.

For example, if we define the *low snow* probability for a single *x* link (or an homogeneous set *X* of links), it is necessary to value the probability of events of the same category subordinated to the fact that the event on the *x* link (or on the set *X* of links) could come true, for all the other links of the considered network.

In conclusion, probabilities of events considering the effective state of stochastic dependence must be assigned to the single link.

We go on with the calculus of the probabilities of state for the *principal path* and for the probable alternative competitive ones.

We also define the *states* for the paths, which we obtain by combining those of the single component links. The set of the states of the path *"p",* $\mathbf{S_p}$, which derives from all the combinations of the states of the component links S_{a1,k_1}, S_{a2,k_2},… S_{al,k_l},… S_{an,k_n}, is obtained by associating to a state of the first link *"a_1"* one state of the second *"a_2"* and so on till the last one link *"a_n"*.

$$\mathbf{S_p} = \mathbf{S_p}\,\{ S_{a1,k_1}, S_{a2,k_2},\dots S_{al,k_l},\dots S_{an,k_n}\} \qquad (11)$$

If m_l shows the number of states (exhaustive and mutually exclusive) of the l-esim link, the number *M* of those combinations will be given by (fig. 2):

$$M = \Pi_l\, m_l \qquad \text{for } l = 1,2, \dots n \qquad (12)$$

We associate the travel time of path obtained by adding the travel times of the component links in link states $S_{al,k}$, to the state S_{pi}:

$$t_{pi} = \Sigma_l\, t_{al} = \Sigma_l\, f_{al}\,(q_{al}) \qquad \text{for } l = 1, 2, \dots\dots n \qquad (13)$$

where t_{al} represents the time of the link *"a_l"* in the state S_{al,k_l}, $f_{al}\,(.)$ the relation time-flow of the link *"a_l"* in the state S_{al,k_l} and q_{al} the flow.

If, in general, some states are independent, let's suppose the first "*g*" states and the other *"n-g"* mutually correlated, the probability of the state of path will be given by:

$$p(S_{pi}) = p(S_{a1,k_1})\, p(S_{a2,k_2}) \dots p(S_{ag,k_g})\, p(S_{ag+1,k_{g+1}} / S_{ag+2,k_{g+2}},\dots S_{an,k_n}) \dots p(S_{an,k_n}) \qquad (14)$$

Even in this case, as we have exhaustive and mutually exclusive events for each state of the path, we have:

$$\Sigma_i\, p(S_{pi}) = 1 \qquad \text{per } i = 1, 2,\dots\dots M \qquad (15)$$

3.5 Evaluation of the states of path in presence of alternative routes.

It is known that the events on a path may change the state (understood in terms of travel-time) of other itineraries. If at an increase of demand of traffic or at the decrease of the capacity of a link, the travel-time exceeds that of an alternative one, part of the traffic of the first one moves to the second one. As effect of the deviation of traffic from one path to another, the travel-time begins to diminish on the first path and to increase on the second one till a new situation of equilibrium has been established so that the travel time becomes equal for both of them.

Let's suppose we have individualised a competitive path for a part of the principal path. We combine the *n* states of the first path (part of the principal path) with the *n* states of the second (competitive) one generating a number of *n x m* states of *"composite paths",* of which we calculate the probability of state. For a generic state S_p of the first path and a generic state S_c of the competitive path, the resulting probability of the state of the composite path S_w is given by:

$$p(S_w) = p(S_p)\, p(S_c) \qquad (16)$$

Fig. 4 represents a road network. The principal path between the origin O=1 and the destination D=7 is represented in bold. Beside the links we have the travelling-times expressed in minutes. For some links we have the relative two-state times of the principal path. The first state corresponds to the absence of events and as a consequence the times related to it are those in normal conditions. In the second state the events have caused an increase of times. The states of the links for which only one value is indicated do not change in the passage from the first to the second state of the principal path.

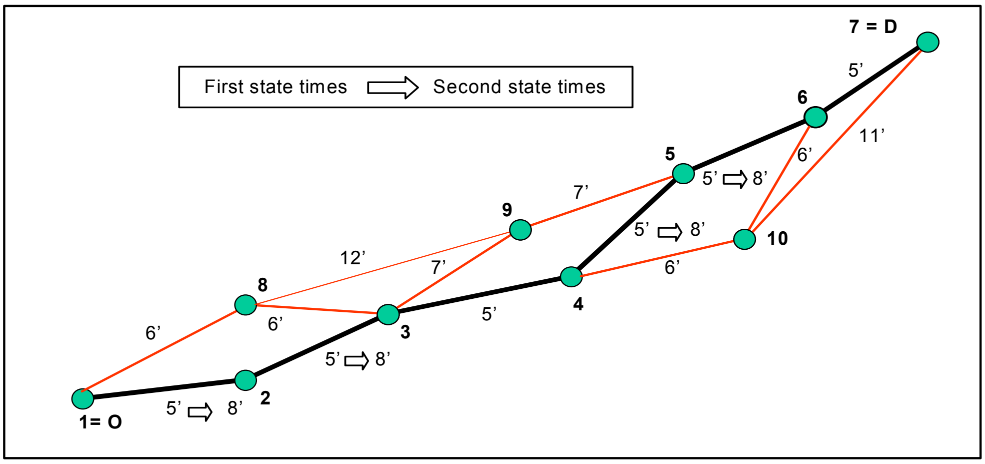

Fig. 4. Change of the principal path state

In the new state we individualise two alternative competitive paths: 1-8-3 and 4-10-6 respectively competitive to the paths 1-2-3 e 4-5-6.

In fig. 5 the two pairs of routes are pointed out in red.

The value of the time at equilibrium is determined in an iterative way after having individualised a competitive path as follows:

a) we decrease the flows on the links of the interested part of the principal path of a same value Δq small enough to not go beyond the conditions of equilibrium;
b) according to the new values of flow we determine the travel-times on the links of the principal path through the time-volume curves;
c) we add those times determining a value of the travelling-time of the principal path t_p;
d) we increase the flow of the links of the competitive path of the same value Δq, subtracted from the principal path;
e) according to the new values of flow we determine the travelling-times on the links of the competitive path through the time-value curves;
f) we add those times determining a value of the travel-time of the competitive path t_c;
g) if we have $/ t_p - t_c / > \varepsilon$ prefixed, we come back to the point *a*. Otherwise we value the time at the equilibrium t_e as arithmetic mean of the values t_p and t_c.

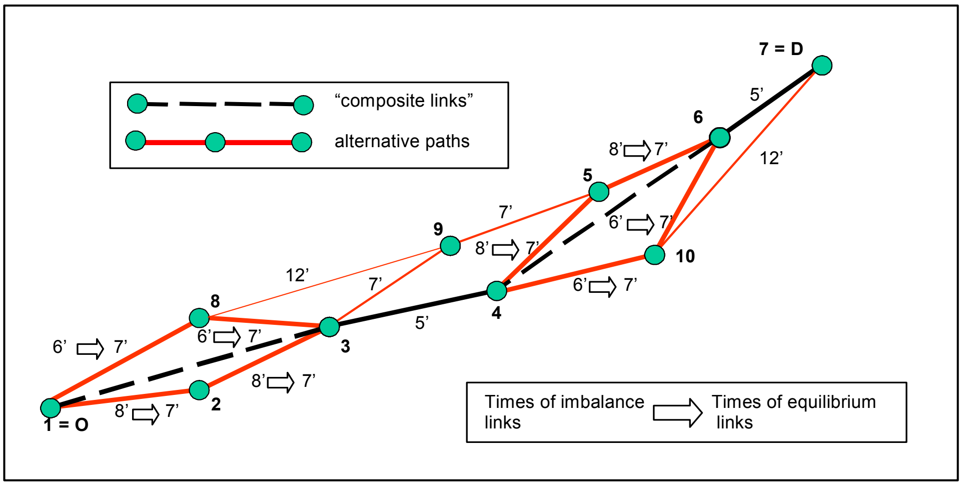

Fig. 5. Evaluation of the equilibrium state

We may never reach the $| t_p - t_c | < \varepsilon$ condition. This corresponds to the circumstance that all the traffic O-D finds advantage to run the competitive path. In that case the time t_e at equilibrium is the one determined on that path.
Each pair (generally each set) of alternative paths may be substituted by a *"composite link"* to which we assign the t_e value as travelling-time. In fig.5 the composite links are represented by broken lines.
The *research of the competitive itineraries* of the principal path or of its segments is made according to their travel time in the S_{c1}, S_{c2},… S_{cj},…. S_{cm} states.
The *definition of the states of the O-D paths* is made in the following way:

a) Each state of the principal path, associated to each state of the competitive paths individualised generates the composite paths. If m composite links have been individualised with $n_1, n_2, ….. n_m$ states, the generated states are $n_1 \times n_2 \times …..n_m$.
b) Let be S_{wq} the state of a composite path obtained by substituting to the partial path made by the sequence of links { $a_f, a_{f+1}, … a_g$ } of the principal path in the S_{pi} state, the composite link corresponding to the state S_{cq} of a competitive path. We obtain the probability $p(S_{wq})$ by multiplying the $p(S_{pi})$ probability calculated by (14) by the $p(S_{cq})$ probability:

$$p(S_{wq}) = p(S_{pi})\, p(S_{cq}) \qquad (17)$$

As the states S_{c1}, S_{c2},…S_{cq},….S_{cm} of the competitive path are mutually exclusive and exhaustive, we have:

$$\Sigma_q\, p(S_{cq}) = 1 \qquad \text{by } q = 1, 2 ….. m \qquad (18)$$

This implies that:

$$\Sigma_q\, p(S_{wq}) = p(S_{pi}) \qquad \text{by } q = 1, 2 ….. m \qquad (19)$$

c) The travelling-time of the composite path in the S_{wq} state is calculated by substituting the terms of the summation in (13) which refers to the links { $a_f, a_{f+1}, … a_g$ } the travel-time t_{eq} related to the composite link corresponding to the state S_{cq} and determined through the iterative way we have already seen.

$$t_{wq} = t_{a1,k1} + t_{a2,k2} + …….+ t_{af-1,kf-1} + t_{eq} + t_{ag+1,kg+1} + …..+ t_{an} \qquad (20)$$

At the end of the operations we have:

a) A set of the states of the principal path, for which alternative competitive routes have not been individualised, each having its own probability and its own travel-time. They can be considered *states of the pair O-D,* because the traffic O-D will run the principal path when those state come true.
b) A set of states of composite paths obtained by the states of the principal path when we individualised routes which are competitive to it. The probabilities and the travel-time are determined for those states, too. They can be considered *states of the pair O-D* because the traffic O-D will be distributed on those paths.

3.6 Evaluation of the reliability related to the pair O-D

If starting from the set of the states of the principal path, the states for which competitive paths have been individualised are substituted by the states of the composite paths made in the described way, the sum of the probabilities of the new set of states will be equal to the unity. In fact let us have *M'* states S'_{pi} of the principal path not substituted and *M"* states $S"_{pi}$ each substituted by m_i states of composite paths S_{wqi} according to (15) and (19) we have:

$$\Sigma_{(i=1,…M')}\, p(S'_{pi}) + \Sigma_{(i=M'+1,…M)}\, \Sigma_{(q=1,…mi)}\, p(S_{wqi}) =$$
$$= \Sigma_{(i=1,…M')}\, p(S'_{pi}) + \Sigma_{(i=M'+1,…M)}\, p(S"_{pi}) = \Sigma_{(i=1,…M)}\, p(S_{pi}) = 1 \qquad (21)$$

It follows that the states of the obtained set are not only mutually exclusive but also exhaustive of all the field of events. They unequivocally represent all the states of the paths which have the probability to be run by the traffic moving from *O* in order to reach *D.*
Moreover, we have seen that each state of path (principal or composite) S_w of the obtained set corresponds to a travel-time t_w. The probability of the state of path is also the probability that the time taken could be t_w.
Once fixed a t_0, time, we can calculate the total reliability that that time could not be exceeded by simply adding the probabilities of those states for which $t \leq t_0$:

$$A_{to} = \sum_{(t \leq t_o)} p(S_w) \qquad (22)$$

4. CASE STUDY

An application of the method to the directrix which from the central area of Bari leads to the centre of Monopoli has been made. The *principal path* is made by the SS 16 (main road) called “Adriatic” for all the whole development and from the radial lines of penetration into the two urban centres which imply the shortest route (fig. 6).

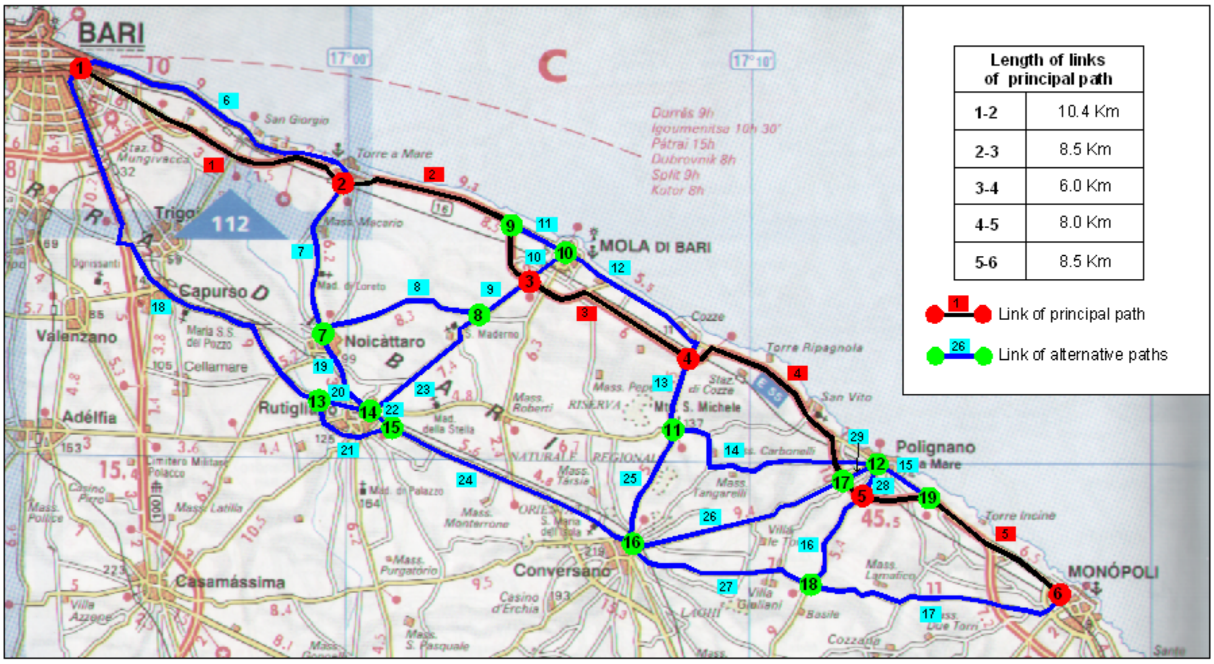

Fig 6. The considered road network

The SS (main road) 16 presents for all the considered development a conformation as two separate carriageways, each having two lanes which can be classified in the category B according to the Rules of the National Research Council.

The study aims to define the *global reliability* of the pair O-D Bari-Monopoli relative to the period autumn-winter and to the space 7:00 a.m – 10:00 p.m of a working day.

4.1 Evaluation of the probability of the states of link

Tab. 2 represents the length of the atmospheric states and of the reductions of the carriageway for roadworks which is expressed in days and detected for the period autumn-winter (1st October- 31st March) in the space of six years. We have obtained the frequencies of the states by dividing those values by 182 that is the number of the days of the period we have considered. We have regarded frequencies as estimation of the probabilities of the states.

Tab. 2. Length of states

	T: Average length of the states in the period autumn-winter (expressed in days)				
LINK	Light rain	Heavy rain	Low snow	Not intense fog	Roadworks
1-2	28,31	12,13	0,15	0,21	7,08
2-3	28,31	12,13	0,15	0,21	7,08
3-4	28,31	12,13	0,15	0,21	5,06
4-5	30,33	13,14	0,24	0,27	4,04
5-6	30,33	13,14	0,24	0,27	3,03

Tab. 3 displays the average number of the accidents happened during the period autumn-winter in the working days (holidays and days before holidays not included), from 7:00 a.m. till 10:00 p.m. separated according to seriousness of accidents and to the atmospheric conditions.

Tab. 3. Number of accidents

	N: Average number of accidents, period autumn-winter, working days, from 7 a.m. till 10 p.m.							
LINK	Clear sky Not serious	Clear sky Serious	Rain Not serious	Rain Serious	Fog	Intense fog	Low-snow	High snow
1-2	10,40	0,36	4,00	0,00	0,00	0,00	0,00	0,00
2-3	3,71	0,00	1,21	0,35	0,00	0,00	0,00	0,00
3-4	1,60	0,00	1,04	0,17	0,00	0,00	0,00	0,00
4-5	1,21	0,00	0,18	0,00	0,00	0,00	0,00	0,00
5-6	2,06	0,00	1,47	0,00	0,00	0,00	0,00	0,00

We suppose that each not serious accident determines a state of reduction of capacity which is consequent to the closing of a lane to traffic and we suppose that that state persists for two hours. Consequently the *p* probability of the state of link due to the accident is valued as follows:

$$p = 2\,N_l / [T \times 15 \times (125/182)]$$

where N_l is the average number of not serious accidents detected in the period autumn-winter from 7:00 a.m. till 10:00 p.m. and displayed in tab. 3, T is the average length of the atmospheric state in the period and displayed in tab. 2, 125 are the days of that period, holidays and days before holidays not included, 15 are the hours of the daily space we have considered.
Similarly we have calculated the probability of the state of the link related to a serious accident.
We have supposed that each serious accident causes the stop of the flow for two hours and determines a reduction of capacity for the following two hours.
We have considered the atmospheric events as mutually exclusive. The event "roadwoks" has been supposed as independent of the atmospheric events and of the events of "accident". These last ones separated as "serious" and "not serious" have been supposed as correlated to the atmospheric conditions and therefore we have valued and used the probabilities subordinated to the fact that these could occur.
Fig. 7 displays the diagram of flow of information related to a link of the principal path. Here we have the effects caused by the considered events.
These ones have contributed to the development of five states of link which are described by the relative time-volume curves.

- The first one is related to the absence of events and to the normal conditions of travel.
- The second one is determined by light rain.
- The third one is determined by heavy rain, snow and fog.
- The fourth one is related to the reduction of capacity as a consequence of a lane closed to traffic and it is determined by the happening of not serious accidents and by the state of roadworks. The greatest possible speed allowed at such conditions is 50 km/h, as limit imposed.
- For the above suppositions, that state persists for two hours as a consequence of the not serious accident and for the two hours which follow the state of interruption of the flow as a consequence of serious accidents.
- The fifth one is related to the interruption of flow. It is determined by serious accidents and persists for two hours after those events. Moreover it is determined by the simultaneous happening of the two "light accident " and "roadworks" events.

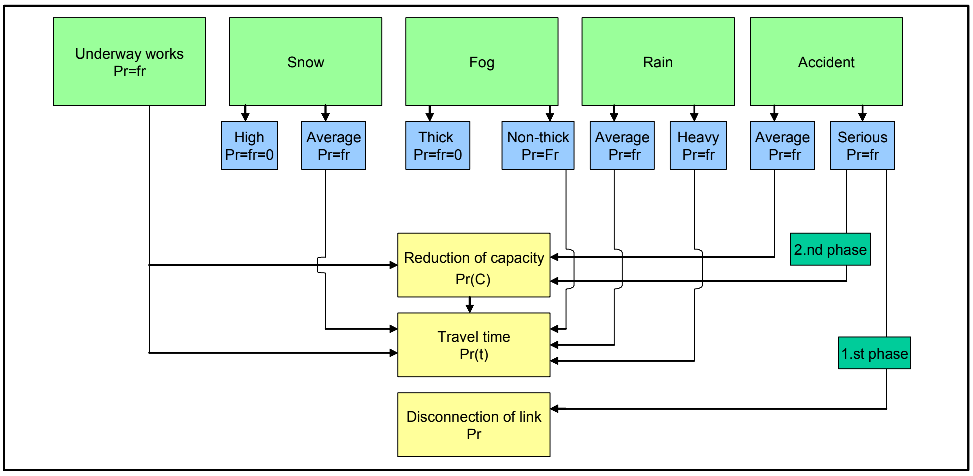

Fig. 7. Flow of information related to a network link

Fig. 8 displays the time-volume curves related to the considered states, for the first of the five links where the principal path has been divided. We have determined similar curves for the others.

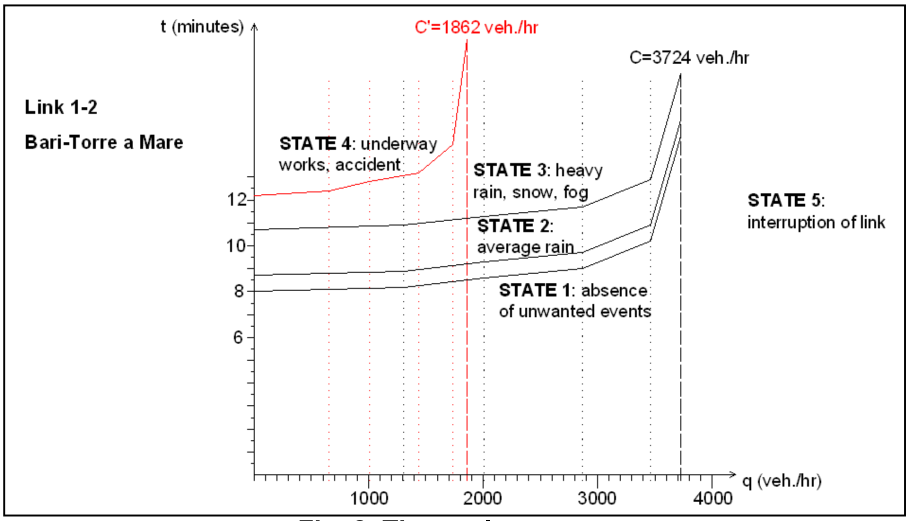

Fig. 8. Time-volume curves

Venn diagram in fig. 9 points out the way in which events contribute to the making of those states.

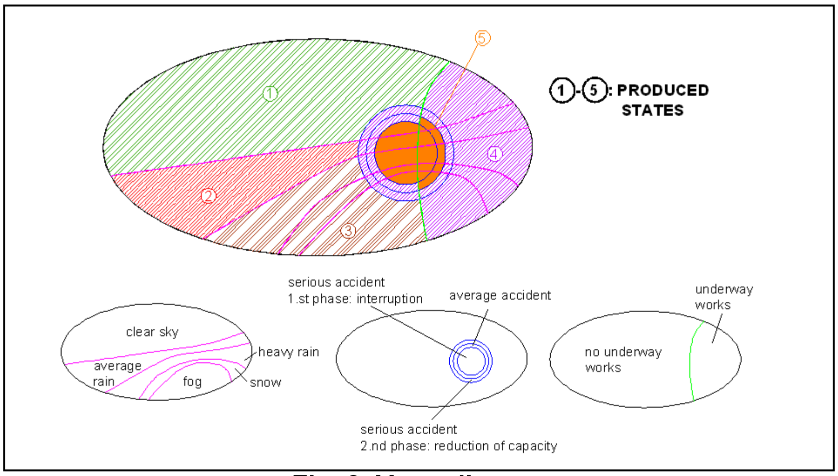

Fig. 9. Venn diagram

Through the formulas (9) we have calculated the probabilities for each state of the links of the principal path (tab. n. 4)

Tab. 4. Probabilities of the states of the links of the principal path

	State of link					
Link	1	2	3	4	5	Total
1-2	0,7269	0,1452	0,0738	0,0532	0,0010	1,000
2-3	0,7344	0,1467	0,0744	0,0440	0,0006	1,000
3-4	0,7450	0,1487	0,0754	0,0306	0,0003	1,000
4-5	0,7253	0,1641	0,0870	0,0236	0,0000	1,000
5-6	0,7285	0,1641	0,0871	0,0203	0,0001	1,000

The links which constitute the alternative paths, except a short segment of 2 km of the SS (main road) 100 included in the link (1-13), are composed by one-lane secondary roads for each direction which sometimes cross town centres. We have regarded the efficiency of these roads as independent of the atmospheric conditions and we have supposed that low speeds could not be reduced because of however light atmospheric phenomena.

Therefore we have considered only the effects of the roadworks and of the accidents occurred on the links of the alternative routes. This led to define three states for the links of the alternative paths:

- the first one is related to the conditions of normal travel.
- the second one is related to the reduction of capacity for not serious accidents and roadworks.
- the third one is related to the interruption for serious accidents.

We have determined the probabilities of the states of the alternative paths (tab. 5) using a procedure which is similar to the one we have adopted for the links of the principal paths:

Tab. 5. Probability of the states of links of alternative itineraries

	State of link			
Link	1	2	3	Total
6	0,98247	0,01727	0,00027	1,000
7	0,98767	0,01231	0,00001	1,000
8	0,98826	0,01173	0,00001	1,000
9	0,98808	0,01191	0,00001	1,000
10	0,98806	0,01193	0,00001	1,000
11	0,98767	0,01231	0,00001	1,000
12	0,98787	0,01212	0,00001	1,000
13	0,98825	0,01174	0,00001	1,000
14	0,98827	0,01172	0,00001	1,000
15	0,98766	0,01233	0,00001	1,000
16	0,98786	0,01213	0,00001	1,000
17	0,98783	0,01215	0,00002	1,000
18	0,98663	0,01314	0,00023	1,000
19	0,98766	0,01233	0,00001	1,000
20	0,98787	0,01212	0,00001	1,000
21	0,98806	0,01193	0,00001	1,000
22	0,98825	0,01174	0,00001	1,000
23	0,98805	0,01194	0,00001	1,000
24	0,98846	0,01153	0,00000	1,000
25	0,98763	0,01235	0,00002	1,000
26	0,98783	0,01215	0,00002	1,000
27	0,98804	0,01195	0,00002	1,000
28	0,98785	0,01214	0,00001	1,000
29	0,98766	0,01233	0,00001	1,000

4.2 Evaluation of the reliability between the centres O-D Bari-Monopoli

The average time of travel of the principal path in absence of events has been calculated in 32 minutes. As regards the calculus of reliability, we have fixed five values for the limit time t_o to not be exceeded: 40, 45, 50, 55 and 60 minutes.

Combining the five states of the first link among them and those of the second and of the following links till the fifth one, we obtain $5^5 = 3125$ states of the principal path. The elaboration has been executed through a program which generates combinations.

Using the same program we have calculated the probabilities of all the states *"i"* of the principal path *P* by the expression (14):

$$p(S_{pi}) = \Pi_{(l=1,..5)\ (k=1,..5)}\ p(S_{al,k}) \qquad (14')$$

where $k = 1,2,\ldots\ldots 5$, $l = 1,2,\ldots 5$ e $i = 1,2,\ldots\ldots 3125$.

The travel times of the states have been determined by the (13).

Tab. 6 represents only the first lines of the table which displays the values of probability of the 3125 states of the principal path.

Tab. 6. Probability of the states of the principal path (extract)

Probability of states on single links					State probability
1	2	3	4	5	
0,7269	0,7344	0,7450	0,7253	0,7285	0,2101
0,7269	0,7344	0,7450	0,7253	0,1641	0,0473
0,7269	0,7344	0,7450	0,7253	0,0871	0,0251
0,7269	0,7344	0,7450	0,7253	0,0203	0,0058
0,7269	0,7344	0,7450	0,7253	0,0001	0,0000
0,7269	0,7344	0,7450	0,1641	0,7285	0,0476
0,7269	0,7344	0,7450	0,1641	0,1641	0,0107
0,7269	0,7344	0,7450	0,1641	0,0871	0,0057
0,7269	0,7344	0,7450	0,1641	0,0203	0,0013
0,7269	0,7344	0,7450	0,1641	0,0001	0,0000
0,7269	0,7344	0,7450	0,0870	0,7285	0,0252
0,7269	0,7344	0,7450	0,0870	0,1641	0,0057
0,7269	0,7344	0,7450	0,0870	0,0871	0,0030
0,7269	0,7344	0,7450	0,0870	0,0203	0,0007
0,7269	0,7344	0,7450	0,0870	0,0001	0,0000
0,7269	0,7344	0,7450	0,0236	0,7285	0,0068
0,7269	0,7344	0,7450	0,0236	0,1641	0,0015
0,7269	0,7344	0,7450	0,0236	0,0871	0,0008
0,7269	0,7344	0,7450	0,0236	0,0203	0,0002
0,7269	0,7344	0,7450	0,0236	0,0001	0,0000
0,7269 ……….	0,7344 ……….	0,7450 ……….	0,0000 ……….	0,7285 ……….	0,0000 ………. Total probability = 1

The state of the alternative paths has been determined on the basis of the following assumptions:

- we have the first one (conditions of normal travelling) when in all the links of the path there are not events which could lead to reductions of capacity or interruptions of flow.
- we have the second one when in at least one of the component links we have a reduction of capacity. The new capacity of the path corresponds to the minimum value of the capacity of the component links. On the basis of such value we determine the new time-volume curve.
- we have the third one when in at least one of the component links there is an interruption of flow. In this case we look for a possible third path which the users choose on the basis of the perfect knowledge of the state of all the network links.

As regards the examined network we have considered the competitive alternative paths for all the possible combinations of events which generate states of the 4 and 5 type of the links of the principal path (reduction of capacity and interruption). The states 2 and 3 do not lead to change path.
Tab. 7 displays the competitive paths near all the combinations of delays generated by the states 4 and 5 of the links of the principal path.

Tab. 7. Competitive alternative paths

Scenes of the principal path					*Alternative paths*	
Link 1	***Link 2***	***Link 3***	***Link 4***	***Link 5***	***Sequence of links***	***Sequence of nodes***
0	0	0	0	0	-	-
R	0	0	0	0	6	1-2
0	R	0	0	0	7-8-9	2-7-8-3
0	0	R	0	0	10-12	3-10-4
0	0	0	R	0	13-14-15	4-11-12-19
0	0	0	0	R	16-17	5-18-6
R	R	0	0	0	18-20-23-9	1-13-14-8-3
0	R	R	0	0	7-8-9-10-12	2-7-8-3-10-4
0	0	R	R	0	10-12-13-14-15	3-10-4-11-12-19
0	0	0	R	R	13-25-27-17	4-11-16-18-6
R	0	R	0	0	6-2-10-12	1-2-3-10-4
R	0	0	R	0	(18-21-24-27-17) (6-2-3-13-14-15)	(1-13-15-16-18-6) (1-2-3-4-11-12-19)
R	0	0	0	R	(6-2-3-4-16-17) (6-2-3-13-25-27-17)	(1-2-3-4-5-18-6) (1-2-3-4-11-16-18-6)
0	R	0	R	0	(7-8-9-3-13-14-15) (18-21-24-27-17)	(2-7-8-3-4-11-12-19) (1-13-15-16-18-6)
0	R	0	0	R	(7-19-22-24-27-17) (7-8-9-3-4-16-17)	(2-7-14-15-16-18-6) (2-7-8-3-4-5-18-6)
0	0	R	0	R	10-12-4-16-17	3-10-4-5-18-6
R	R	R	0	0	18-21-24-27-17	1-13-15-16-18-6
0	R	R	R	0	7-19-22-24-27-17	2-7-14-15-16-18-6
0	0	R	R	R	10-12-13-25-27-17	3-10-4-11-16-18-6
R	R	0	R	0	18-21-24-27-17	1-13-15-16-18-6
R	R	0	0	R	18-21-24-27-17	1-13-15-16-18-6
R	0	R	R	0	18-21-24-27-17	1-13-15-16-18-6
R	0	R	0	R	18-21-24-27-17	1-13-15-16-18-6
R	0	0	R	R	(6-2-3-13-25-27-17) (18-21-24-27-17)	(1-2-3-4-11-16-18-6) (1-13-15-16-18-6)
0	R	0	R	R	(7-8-9-3-13-25-27-17) (18-21-24-27-17)	(2-7-8-3-4-11-16-18-6) (1-13-15-16-18-6)
0	R	R	0	R	7-19-22-24-27-17	2-7-14-15-16-18-6
0	R	R	R	R	7-19-22-24-27-17	2-7-14-15-16-18-6
R	0	R	R	R	18-21-24-27-17	1-13-15-16-18-6
R	R	0	R	R	18-21-24-27-17	1-13-15-16-18-6
R	R	R	0	R	18-21-24-27-17	1-13-15-16-18-6
R	R	R	R	0	18-21-24-27-17	1-13-15-16-18-6
R	R	R	R	R	18-21-24-27-17	1-13-15-16-18-6

0 = state of the link of the type 1,2 or 3
R = state of the link of the type 4 or 5

Composing the *n* states of parts of the principal path which present states of the type 4 and/or 5 with the *m* states of the alternative paths, we have individualised *n x m* states. We have calculated the probabilities and determined the equilibrium time. For each set of alternative paths we have individualised the "composite link" (see fig. 5) to which we have assigned the probabilities of $p(S_{al,k})$ state and its travel time t_{al} which has the same value as the equilibrium time t_{el}.
We have applied the formulas (13), (14) to the "composite paths" obtaining thus the travel time and the probability of the states. By applying the formula (22) to the set formed by the not substituted states of the principal path and by the states of the composite paths we have valued the reliability of the pair O-D for several hypothesis of the limit time t_o. The results are in tab. 8

Tab. 8. Final results

Reliability of the pair O-D Bari-Monopoli				
to=40'	to=45'	to=50'	to=55'	to=60'
0,86538	0,92090	0,98532	0,99634	0,99928

5. CONCLUSIONS

After having valued the reliability of each link of the network in relation with the considered events, the methodology we used by having recourse to well known concepts of the theory of the probability, first values the reliability of the principal path as previously defined, and of the competitive alternative paths and finally the reliability between a pair of nodes O-D.

The methodology has been applied to a case study of network between the centres of Bari and Monopoli of Apulia.

The results pointed out remarkable changes of the reliability levels of the travel time which the considered usual and not exceptional events have produced.

If we consider that a little increase of the travel time generally implies high social costs, it would be appropriate to increase and deepen this kind of evaluations, both for carrying out an efficient program of maintenance interventions and for arranging all the suitable interventions which could better the conditions of safety and of travelling of a path.

The limit of the methodology we have applied, as we said before, lies in the remarkable amount of necessary elaborations which, in the case of more complex networks, can rapidly lead to the intractability of the case.

However it is also true that with the technological progress of calculus this limit progressively moves towards higher levels whose values cannot be foreseen.

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